Preface

Two decades ago the authors of this book undertook the study of the errors one makes when numerically approximating the solutions of stochastic differential equations driven by Lévy processes. In particular we were interested in the normalized asymptotic errors of approximations via an Euler scheme, and it turned out we needed sophisticated laws of large numbers and central limit theorems that did not yet exist. While developing such tools, it became apparent that they would be useful in a wide range of applications.

One usually explains the difference between probability and statistics as being that probability theory lays the basis for a family of models, and statistics uses data to infer which member or members of that family best fit the data. Often this reduces to parameter estimation, and estimators are shown to be consistent via a Law of Large Numbers (LLN), and the accuracy of an estimator is determined using a Central Limit Theorem (CLT), when possible. The case of stochastic processes, and even stochastic dynamical systems, is of course more difficult, since often one is no longer estimating just a parameter, but rather one is estimating a stochastic process, or—worse—trying to tell which family of models actually does fit the data. Examples include using data to determine whether or not a model governing a dynamical system has continuous paths or has jumps, or trying to determine the dimension of the driving Brownian forces in a system of stochastic differential equations. This subject, broadly speaking, is a very old subject, especially as concerns asymptotic studies when the time parameter tends to infinity. The novelty presented here in this book is a systematic study of the case where the time interval is fixed and compact (also known as the *finite horizon* case). Even in the finite horizon case however, efforts predate the authors' study of numerical methods for stochastic differential equations, and go back 5 years earlier to attempts to find the volatility coefficient of an Itô process, via a fine analysis of its quadratic variation, by the first author joint with Valentine Genon-Catalot. This in turn builds on the earlier work of G. Dohnal, which itself builds on earlier work; it is indeed an old yet still interesting subject.

There are different variations of LLNs and CLTs one might use to study such questions, and over the last two decades substantial progress has been made in finding such results, and also in applying them via data to delve further into the unknown, and to reveal structures governing complicated stochastic systems. The most common examples used in recent times are those of financial models, but these ideas can be used in models of biological, chemical, and electrical applications as well. In this book we establish, in a systematic way, many of the recent results. The ensuing theorems are often complicated both to state, and especially to prove, and the technical level of the book is (inevitably, it seems) quite demanding. This is a theory book, and we do not treat applications, although we do reference papers that use these kinds of results for applications, and we do indicate at the end of most chapters how this theory can be used for applications.

An introduction explaining our approach, and an outline of how we have organized the book, can be found in the Introductory Chapter 1. In addition, in Chap. 1 we present several sketches of frameworks for potential applications of our theory, and indeed, these frameworks have inspired much of the development of the theory we present in this book.

If one were to trace back how we came to be interested in this theory, the history would have to center on the work and personality of Denis Talay and his "équipe" at INRIA in Sophia-Antipolis, as well as that of Jean Mémin at the University of Rennes. Both of these researchers influenced our taste in problems in enduring ways. We would also like to thank our many collaborators in this area over the years, with a special mention to Tom Kurtz, whose work with the second author started this whole enterprise in earnest, and also to Yacine Aït-Sahalia, who has provided a wealth of motivations through applications to economics. We also wish to thank O.E. Barndorff-Nielsen, S. Delattre, J. Douglas, Jr., V. Genon-Catalot, S.E. Graversen, T. Hayashi, Yingying Li, Jin Ma, S. Méléard, P. Mykland, M. Podolskij, J. San Martin, N. Shephard, V. Todorov, S. Torres, M. Vetter, and N. Yoshida, as well as A. Diop, for his careful reading of an earlier version of the manuscript.

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