Chapter 2
The Evolution of the Universe

The expansion of the Universe and the Big Bang are shown to be derivable from equations based on the general theory of relativity. The history of the Universe is discussed, as are various observations confirming the picture of the Big Bang, such as the abundance of light elements and the cosmological microwave background radiation. Dark matter and dark energy are introduced, and their properties and the arguments for their presence are explained. Reasons for the assumption of a so-called inflationary phase shortly before the Big Bang are given. The chapter concludes with several open questions that are the subject of today's research in cosmology.

2.1 The Expansion of the Universe in General Relativity

We have seen in the introduction that galaxies are moving away from each other, as sketched in Fig. 1.1, with a velocity v that increases in proportion to their distance d:

$$v \simeq H_0\, d, \tag{2.1}$$

where H_0 is the Hubble constant. Looking back in time, this implies that all matter was compressed $\sim 10^{10}$ years ago.

This phenomenon is easier to understand if we imagine a two-dimensional world instead of our three-dimensional one. A two-dimensional world corresponds to a surface, and all physical objects (and creatures) in this surface possess a width and a length, but no height. Creatures in this surface can move only inside the surface, they measure distances inside the surface and cannot even imagine a third dimension. (The mathematicians of this two-dimensional world can of course perform calculations in three-dimensional spaces; however, they have difficulties explaining to their cohabitants what this is supposed to mean.)

Let us now imagine a surface in the form of a sphere, which represents the Universe of the two-dimensional creatures. For us, this conception poses no difficulties at all;

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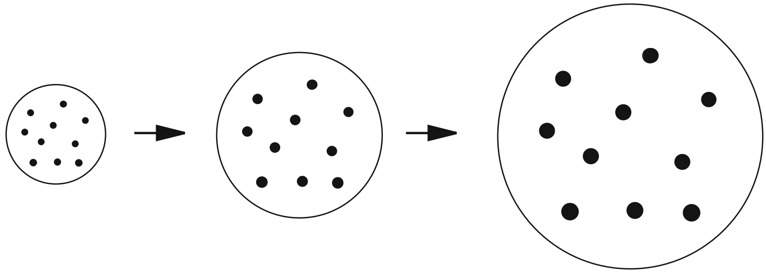

Fig. 2.1 Expanding surface of a sphere, which corresponds to a two-dimensional expanding Universe

it is, however, not conceivable for the two-dimensional creatures! In addition, we imagine that this surface is expanding, as in Fig. 2.1.

This behavior corresponds to that of our three-dimensional Universe: now, all distances between points (or galaxies) on this surface of the sphere are increasing, and the relative velocity between two points is proportional to their distance. This can be verified by a short calculation.

We introduce a dimensionless quantity $a(t)$, which is proportional to the diameter of the sphere under consideration and which increases in the course of time. $a(t)$ plays the role of a *scale factor*, i.e., all scales or lengths in the surface of the sphere are proportional to $a(t)$. We choose the convention that $a(t_0) = 1$ at the time $t = t_0$. The distance between two points measured at $t = t_0$ is denoted as Δ_0. At a later time $t > t_0$, this distance is given by

$$\Delta(t) = a(t)\Delta_0. \tag{2.2}$$

The velocity with which two points move apart from each other can be computed as follows (where $\dot{a} = da/dt$):

$$v(t) = \frac{d}{dt}\Delta(t) = \dot{a}(t)\Delta_0 = \frac{\dot{a}(t)}{a(t)}a(t)\Delta_0 = \frac{\dot{a}(t)}{a(t)}\Delta(t) = H(t)\Delta(t), \tag{2.3}$$

where

$$H(t) = \frac{\dot{a}(t)}{a(t)}. \tag{2.4}$$

Thus, $v(t)$ is indeed proportional to the distance $\Delta(t)$, but the coefficient $H(t)$ depends in general on the time t.

Here we have considered an expanding two-dimensional surface whose curvature is the same everywhere. There exist more two-dimensional surfaces with this property, which is denoted as "homogeneity": the flat plane and a surface in the form of a saddle. Equations 2.3 and 2.4 are valid in all these cases, as well as for our three-dimensional Universe: a warped three-dimensional space (or an expanding three-dimensional space) is just as unthinkable for us as a two-dimensional space with these properties for the two-dimensional creatures. Nevertheless, the calculation (2.3) above still implies a relation of the form (2.1); it suffices to replace t by $t = t_{\text{today}}$ everywhere in (2.3).

Today's experiments allow us even to measure the time dependence of $H(t)$: for very distant supernovae, the ratio of their velocity to their distance is not exactly constant, since their light was emitted very long ago and the value of $H(t)$ at that time was not exactly the same as today. Later we will discuss implications of these measurements.

We should note that an increasing scale factor $a(t)$ does *not* imply that objects in the Universe (such as stars and galaxies) are expanding: the diameter of such objects is determined by the compensation of the forces that act on their constituents (e.g. the gravitational and centrifugal forces acting on stars in galaxies). As long as these forces remain the same, the diameters of objects remain unaffected by the expansion of the Universe.

The time dependence of $a(t)$—and accordingly that of $H(t)$—can be computed in the framework of general relativity. In general relativity, space (and even space-time, see Chap. 3) is generally considered as warped (or curved). The detailed form of a warped space is determined by the distances between points everywhere in the space. The mathematical quantity describing these distances is denoted as the *metric*, which we will discuss in more detail in Chap. 3. For homogeneous spaces, the metric does not depend on the position and is completely determined by the scale factor $a(t)$ introduced above.

Einstein used the metric for the description of warped spaces in the theory of general relativity, and proposed equations that determine the metric in terms of matter (and energy) distributed in space [1].

If a homogeneous Universe is assumed, all kinds of matter (galaxies, stars, dust, atoms, elementary particles) can be considered as a homogeneous gas. In general, this gas consists of several components, but it is completely specified by its *matter density* ϱ (measured in kg/m^3) and its pressure p. For a homogeneous gas, these quantities do not depend on the position, but solely on the time t.

In general, one has to distinguish the following kinds of matter and energy:

(a) Bodies moving slowly compared to the speed of light, such as galaxies, stars, dust, and (massive and not too energetic) elementary particles. The contribution of these bodies to the density ϱ is denoted as ϱ_{nr} (where "nr" indicates non-relativistic objects with velocities $v \ll c$). The contribution of these objects to the "pressure of the Universe" is negligibly small.

(b) Massless (or light and energetic) particles that move at (or near) the speed of light provide a contribution ϱ_{r} to the density as well as a contribution p to the pressure, where p and ϱ_{r} are related by $p \sim \frac{1}{3}\varrho_{\text{r}}c^2$.

(c) Constant fields (see Chap. 4, "Field Theory", and Chap. 7, "The Weak Interaction") can generate a potential energy (density), which is called the *dark energy* or the *cosmological constant* Λ and measured in units of $(\text{kg}\,\text{m}^2/\text{s}^2)/\text{m}^3 = \text{kg}/(\text{m}\,\text{s}^2)$.

The Einstein equations lead to two equations for the time derivatives of $a(t)$, depending on $\varrho = \varrho_{\rm nr} + \varrho_{\rm r}$, p, and Λ. It is convenient to define a gravitational constant κ related to Newton's constant G:

$$\kappa = \frac{8\pi G}{c^2} \simeq 1.866 \times 10^{-26}\,\mathrm{m\,kg^{-1}}. \tag{2.5}$$

Using the standard definitions $\dot{a} = \mathrm{d}a/\mathrm{d}t$ and $\ddot{a} = \mathrm{d}^2a/\mathrm{d}t^2$, these equations are of the following form (under the assumption that the homogeneous Universe is *not* warped, which agrees best with the observations):

$$3\frac{\dot{a}^2}{a^2} = \kappa\left(\Lambda + \varrho(t)c^2\right), \tag{2.6}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = \kappa\left(\Lambda - p(t)\right). \tag{2.7}$$

These equations are also denoted as the Friedmann–Robertson–Walker equations (see, e.g., A. Friedmann in [2]).

In the present Universe the contribution of the pressure $p(t)$ to (2.7) is negligible. If one neglects Λ as well, the right-hand side of (2.7) vanishes. The left-hand side can be written in terms of the function $H(t)$ defined in (2.4), and we obtain

$$2\dot{H}(t) + 3H^2(t) = 0. \tag{2.8}$$

The general solution of this equation is given by $H(t) = 2/\big(3(t - \bar{t})\big)$, $\bar{t}$ arbitrary, and it is convenient to choose $\bar{t} = 0$ for the "origin of time". Then we get

$$H(t) = \frac{2}{3t}. \tag{2.9}$$

Now, $a(t)$ can be determined from (2.4):

$$a(t) = a_0 t^{\frac{2}{3}}, \tag{2.10}$$

where a_0 is an arbitrary constant. Consequently $a(t)$ increases with t, corresponding to an expanding Universe.

$\varrho(t)$ and $p(t)$ always satisfy a relation that follows from the conservation of energy (or from a combination of (2.6) and (2.7)):

$$\dot{\varrho}(t) = -3\frac{\dot{a}}{a}\left(\varrho(t) + p(t)/c^2\right). \tag{2.11}$$

In the case $p(t)=0$, it follows that

$$\varrho(t) = \frac{\varrho_0}{a^3}, \tag{2.12}$$

where ϱ_0 is a free constant.

Assuming $\Lambda = 0$, (2.6) and (2.10) or (2.12) now allow the (complete) matter density $\varrho(t)$ to be determined:

$$\varrho(t) = \frac{4}{3\kappa c^2 t^2} = \frac{\varrho_0}{a_0^3 t^2}. \tag{2.13}$$

Accordingly the matter density decreases, which is understandable as the volume of the Universe increases as a^3. (Eq. (2.13) can be considered as an equation for a_0 for a given constant ϱ_0: $a_0^3 = \frac{3}{4}\kappa\varrho_0 c^2$.)

2.2 The History of the Universe

As an apparent consequence of (2.13), the matter density $\varrho(t)$ was very high in the early Universe (for small t). According to the laws of thermodynamics, the temperature increases in a compressed gas. Thus, the temperature in the early Universe was very high. A high temperature of a gas implies high average velocities of its components. Collisions between these components can break them up into their subcomponents: with increasing temperature and density first molecules into atoms, then atoms into electrons and nuclei, then nuclei into baryons (protons and neutrons), and finally even baryons into quarks.

If the evolution of the Universe is described by (2.6) and (2.7), all this happened in reverse order: at the beginning, the Universe was extremely dense and hot, filled with elementary particles such as quarks and electrons. (As long as the average velocities of these particles are close to the speed of light, they contribute to the pressure $p(t) \sim \frac{1}{3}\varrho_r c^2$. Then, (2.6) and (2.7) imply—under the assumption $\Lambda \sim 0$—that $a(t) \sim a_0\sqrt{t}$ instead of (2.10) during this early stage.) This Universe sort of exploded: it expanded very rapidly, whereupon its temperature and density decreased. This process is known as the "Big Bang". In the course of time, the baryons, nuclei, atoms, molecules, and ultimately the stars and galaxies formed.

Using (2.6) and (2.7), the laws of thermodynamics (which allow the temperature to be determined as a function of the density and the pressure), and the known interactions between quarks, baryons, nuclei, and electrons, the history of the Universe can be reconstructed quite precisely, and observable consequences of this scenario can be predicted.

During the first 10^{-12} seconds the temperature was so high (above 10^{15}°C) that the processes that occurred depended on the properties of very massive—still unknown—elementary particles. (Very massive elementary particles can not yet have been produced at present accelerators, see Chap. 8.) This period is the subject of ongoing research in particle physics and cosmology. One topic of particular interest is the origin of the disequilibrium between matter and antimatter (the present Universe contains practically no antimatter); for the generation of this disequilibrium, processes that occur at such temperatures can play an important role.

After about 10^{-6} seconds (at a temperature of about 10^{12}°C), the quarks formed protons and neutrons.

After about 10 seconds (at a temperature of 10^9–10^{10}°C), the protons and neutrons formed the nuclei of light elements such as deuterium, helium, the isotope helium-3, and lithium. (Hydrogen, whose nucleus consists of just one proton, remained the most frequent element after this stage.)

After about 4×10^5 years (at a temperature of about 3000°C) the atoms were built out of nuclei and electrons.

After about 10^8 years (at a temperature of about 30 kelvin (30 K)) the stars and galaxies formed. In the interior of these stars, and during the first explosions of supernovae, nuclei of heavy elements such as iron and uranium were generated.

After about 10^{10} years (at a temperature of about 6 K) the solar system formed. It contains heavy elements, mainly in the planets, which were produced during the previous period.

Today the Universe has an age of about 1.4×10^{10} years, and has cooled to a temperature of 2.73 K.

Are there implications of this history of the Universe that are observable today?

The first of the processes described above leading to a verifiable prediction is the formation of the light elements. The relative abundance of protons to neutrons at that time (about 7:1) is calculable, and allows the determination of the relative abundance of the light elements hydrogen, helium, lithium, and their isotopes. The results of these calculations agree well with the measurements of the relative contributions of these elements ($\sim$75% hydrogen, $\sim$24% helium, see Exercise 2.2) to the density of gaseous clouds that originated during the primitive Universe.

Until the formation of atoms by nuclei and electrons, the constituents of the gas filling the Universe carried electric charges; subsequently the electric charges of the nuclei and electrons became neutralized in the atoms. The high temperature of the gas corresponds to chaotic motions at high velocities, and high accelerations generated by collisions. Under such circumstances, charged particles at temperatures above about 1000°C emit electromagnetic radiation corresponding to visible light. (A flame is a gas at a temperature high enough that electrons are ripped off atoms as a result of violent collisions. This gas contains ionized atoms and free electrons; it is called a *plasma*. When ionized atoms capture electrons, light is emitted.)

Hence, the Universe was full of electromagnetic radiation interacting with charged particles (i.e., being emitted, absorbed, or scattered), until the electrons and nuclei combined to neutral atoms. After the formation of (neutral) atoms the production of electromagnetic radiation ceased.

What became of the light originating from this period? A large fraction has not been absorbed up to now, and is still present in today's Universe. However, between the moment this light was produced and now, the Universe has expanded by about a factor of 1000. Simultaneously, the wavelength of the radiation in the Universe has been stretched by the same amount. Originally, this wavelength corresponded to $\lambda_{light} \sim 7 \times 10^{-7}$ m; accordingly it corresponds to microwave radiation today. It is also known as cosmic background radiation, and glares uniformly from all directions in the sky. The dependence of the intensity of the radiation on wavelength agrees with the calculations to a relative precision of 10^{-5}, and corresponds to the electromagnetic radiation of a body of a temperature of 2.73 K. For this reason we can consider this temperature as the temperature of the Universe: every object in empty space (sufficiently far from radiating stars and galaxies) will cool down to this temperature.

The cosmic background radiation following from the theory of the Big Bang was predicted, amongst others, by R. Dicke and G. Gamow, and detected by A.A. Penzias and R.W. Wilson in 1964–1965, for which they were awarded the Nobel prize in 1978.

The genesis of stars and galaxies after about 10^8 years took place under the action of gravity, which could play a role only after the motions generated by temperature had sufficiently died away. The formation of lumps of matter under the influence of gravity required, however, small density fluctuations in the gas at that time. We can deduce, from the order of magnitude of the density variations at that time, the density variations during the much earlier epoch when the atoms formed. These density variations of electrons and nuclei manifest themselves, in turn, in the form of inhomogeneities of the radiation (the light) at that time, which leads to inhomogeneities of the presently observed cosmic background radiation. This implies that the intensity of the cosmic background radiation observed today should depend weakly on the direction in the sky; the predicted relative intensity variations $\Delta I/I$ of the order of 10^{-5} were first detected in 1992 by instruments placed on the COBE (Cosmic Background Explorer) satellite, for which the Nobel prize was awarded to J.C. Mather and G.F. Smoot in 2006.

Hence, the theory of the Big Bang—at least as of 10^{-6} seconds after the origin of the Universe—has been confirmed by several observations and measurements that are based on very different physical phenomena.

2.3 Dark Matter and Dark Energy

Let us return to the Friedmann–Robertson–Walker equations (2.6) and (2.7), from which we will draw additional conclusions. The solutions (2.9) for $H(t)$, (2.10) for $a(t)$, and (2.13) for $\varrho(t)$ have been derived under the assumption that the contributions from the pressure $p(t)$ and the cosmological constant Λ can be neglected in (2.6) and (2.7). Even though the pressure played a role in the early Universe, the solution (2.9) allows quite a precise estimate of the age of today's Universe.

The age of the Universe t_{today} can be determined from the present value $H_0 \simeq 70\,\text{km/s} \times 1/\text{Mpc}$ of the Hubble constant. After converting megaparsecs into kilometers we obtain from (2.9)

$$t_{\text{today}} \equiv t_0 \sim 1.4 \times 10^{10}\ \text{years}, \tag{2.14}$$

which also corresponds approximately to the age of the oldest stars and galaxies.

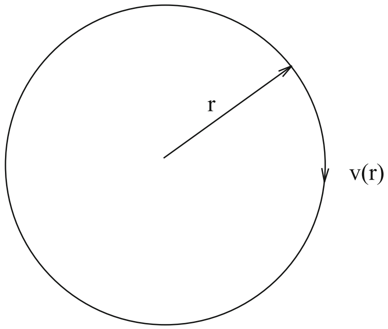

Fig. 2.2 Radius r and rotational velocity $v(r)$ of a star rotating around the center of a galaxy

Then we obtain for the matter density $\varrho(t_0)$ from (2.13)

$$\varrho(t_0) \sim 2 \times 10^{-27}\,\mathrm{kg\,m^{-3}}. \tag{2.15}$$

This value can be compared to the density of galaxies and intergalactic dust. The corresponding density of known matter ϱ_{known} is smaller than the value (2.15):

$$\varrho_{\text{known}} \sim \frac{\varrho(t_0)}{6}. \tag{2.16}$$

This means that, besides the known matter, there should exist an unknown form of "dark matter" ("dark" since, evidently, it does not emit light). The contribution of dark matter to the total matter density seems to be about five times the contribution of known matter.

At this point we should discuss a phenomenon related to the dynamics of stars inside galaxies: the nearly circular motion of stars around the center of galaxies is caused by the gravitational attraction between the stars. From the known form of the gravitational force we can compute the rotational velocity $v(r)$ of a star (sketched in Fig. 2.2), which depends on its distance r to the center of the galaxy and the mass $M(r)$ inside a fictitious sphere with radius r (G is Newton's gravitational constant):

$$v^2(r) = \frac{GM(r)}{r}. \tag{2.17}$$

In practice we can measure the rotational velocities $v(r)$ of stars at different distances r to the galactic center for a large number of galaxies and estimate $M(r)$. Surprisingly, the observations do not agree with (2.17): either the measured values of $v(r)$ are systematically too large, or the estimates of $M(r)$ are systematically too small! (Notably for large r, where the density of stars decreases and where $M(r)$ should hardly increase with r, $v(r)$ does *not* decrease as $1/\sqrt{r}$, but remains approximately constant.) This discrepancy suggested, already before cosmology, the existence of additional dark (invisible) matter, which contributes to $M(r)$ and thus to the attractive gravitational force of galaxies. Hence, there exist two independent reasons for the hypothesis of dark matter.

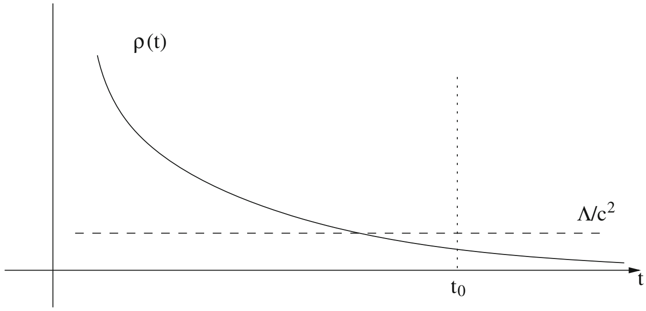

Fig. 2.3 Schematic time dependences of $\varrho(t)$ ($\sim 1/t^2$) and Λ (constant)

In recent years, very distant supernova explosions whose light was emitted a very long time ago have been successfully observed [5–8]. Through measurements of their radial velocities, with the help of the Doppler effect, and their distances, with the help of the known luminosity of such supernova explosions, the time dependence of $H(t)$ was determined for the first time, allowing a comparison to solutions of the Friedmann–Robertson–Walker equations. It appeared that $\dot{H}(t)$ is somewhat larger than expected from the above solution, which was derived under the assumption $\Lambda = 0$. The measured value of $\dot{H}(t)$ is compatible only with a positive value of Λ (a "dark energy") in (2.6) and (2.7):

$$\Lambda \sim 4 \times 10^{-10}\,\mathrm{kg\,s^{-2}\,m^{-1}}. \tag{2.18}$$

The Nobel prize 2011 was awarded to Saul Perlmutter, Adam Riess and Brian Schmidt for this observation.

First of all we have to verify whether this value invalidates the results obtained above under the assumption $\Lambda = 0$. Fortunately this is not the case. Comparing the two terms on the right-hand side of (2.6) one finds that, on the one hand, both are of the same order today:

$$\Lambda \sim 2 \times \varrho(t_0)c^2. \tag{2.19}$$

However, the time dependences of the two terms are very different: $\varrho(t)$ behaves as $1/t^2$, but Λ can be considered as constant (see Fig. 2.3).

Correspondingly, at earlier times, i.e., for $t \ll t_0$, $\varrho(t)$ was much larger than Λ/c^2, and Λ was numerically negligible. (In fact, this is also true for (2.7), as we can verify explicitly for $p(t) = 0$ and inserting the above solution for $a(t)$.) For this reason, Λ has had an impact on the evolution of the Universe only in recent times; corresponding small corrections have already been taken into account in the value (2.14) for the age of the Universe.

Because of the different time dependences of $\varrho(t)$ and Λ, it appears a remarkable coincidence that—as noted in (2.19)—$\varrho(t_0)c^2$ and Λ are of the same order today. Accordingly, we live in a kind of transition period: in the (still very far) future the evolution of the Universe will be determined nearly exclusively by the Λ terms in (2.6) and (2.7), whereupon $a(t)$ will increase exponentially with t, in contrast to (2.10) (see the next section). Then, the Universe becomes infinitely large, empty and cold. However, before that (in about five billion years) our Sun will balloon to a red giant star.

2.4 Inflation

In fact, the practically homogeneous distribution of galaxies and the cosmic background radiation within the presently observable part of the Universe poses a puzzle. The homogeneous distribution of galaxies and the cosmic background radiation implies that the hot and compressed gas of elementary particles that the Universe consisted of at its beginning was also distributed very uniformly.

However, a gas can spread uniformly only if its constituents can flow back and forth. Independently of their precise nature, the flow velocity of these constituents is always limited by the speed of light. Therefore these constituents can move at most a distance $\Delta d = c\Delta t$ within a given time interval Δt.

At the beginning of the Universe, during the period of the Big Bang, this distance was not large enough in order to encompass all of the Universe observable today. (Even light needs billions of years to cross the present Universe.) Since the gas at that time could not spread uniformly within the presently observable region of the Universe, the presently almost homogeneous distribution of galaxies and the cosmic background radiation is a paradox at first sight.

In order to resolve this paradox, so-called *inflation* was invented [9, 10], which corresponds to the following behavior of the early Universe. At first we content ourselves with the fact that the original gas can spread uniformly, within the time interval Δt, only within distances $\Delta d = c\Delta t$, where Δd is very much smaller than the present Universe. Subsequently we can make use of the behavior of the solutions of the Friedmann–Robertson–Walker equations (2.6) and (2.7) in the case where the parameter Λ is much larger than $\varrho(t)$ and $p(t)$. Then, the time dependence of the scale factor $a(t)$ is no longer given by (2.10) but—as can be easily checked—by

$$a(t) = a_0 \mathrm{e}^{\sqrt{\kappa\Lambda/3}t}. \tag{2.20}$$

This implies an extremely fast—exponentially growing—expansion of the Universe, much faster than described by (2.10) before. (Such a Universe is known as a *de Sitter Universe*.)

Thereby, the distance Δd inflates to $\mathrm{e}^{\sqrt{\kappa\Lambda/3}t}$ times its original value as well! This process is called *inflation*. If the period of inflation continued for a time interval Δt with $\sqrt{\kappa\Lambda/3}\Delta t \gtrsim 60$, the initial distance Δd within which the original gas was uniformly distributed expanded sufficiently in order to encompass the presently visible part of the Universe.

On the one hand, this process would explain the present homogeneous distributions of galaxies and the cosmic background radiation. However, we know that the Universe has no longer been expanding exponentially for about 1.4×10^{10} years, otherwise all previous results would no longer be valid. Hence we have to assume that the inflationary phase ceased after Δd had sufficiently inflated. This implies that the parameter Λ had to shrink from a relatively large value to the relatively small value given by (2.18).

Thus we have to understand how the parameter Λ can change with time. This is comprehensible in the context of field theory as used in particle physics: in field theory one obtains contributions to the potential energy that, in turn, depend on the presence of a constant field. The minimization of such a potential energy as a function of the so-called Higgs field will play an important role in Sect. 7.3 on the weak interaction. In cosmology, this potential energy acts precisely as a parameter Λ in the Friedmann–Robertson–Walker equations (2.6) and (2.7). Once a field varies in time (since it always tries to minimize its potential energy), the potential energy can decrease from a large value to a small value. (We will come back to this behavior at the end of Sect. 7.3.) Such a mechanism explains the end of an inflationary period, and now all results obtained previously are to be interpreted in the era "after the end of inflation".

Actually, such an end of an inflationary epoch through the variation of a field has additional consequences: before a field settles down to a new value (which minimizes the potential energy), it wiggles a little bit and radiates energy in the form of particles, which generates—albeit tiny—density fluctuations. This fits well with the considerations at the end of Sect. 2.2, in which small density fluctuations of the original matter are a necessary condition for the possibility that matter lumps together to form stars and galaxies under the action of gravity.

This also leads to relative intensity variations $\Delta I/I$—depending on the direction in the sky—of the cosmic background radiation observed today: if we measure the intensities of the cosmic background radiation in different directions in the sky, separated by an angle θ, they differ by about 0.001%. In addition we can now compute how this difference depends, on average, on the angle θ. This θ dependence of the intensity variations has been measured by instruments on the WMAP satellite (see the internet address given in the appendix), and it agrees well with the inflationary model.

2.5 Summary and Open Questions

The standard model of cosmology including the Big Bang has led to various predictions that agree very well with measured observables: the temperature and the (tiny, but measurable) variations of the cosmic background radiation as well as the relative abundance of light elements; the most relevant observation is, of course, that the radial velocity of galaxies increases with their distance. However, several questions still remain open.

(a) What does dark matter consist of? Practically all forms of known matter (e.g., cold, invisible stars, dust, or gas) are excluded, since they would absorb too much light if their abundance or density should explain all of dark matter. One possibility would be a new species of elementary particles (so-called *WIMPs*, weakly interacting massive particles), which should be: (1) neutral, in order not to absorb too much light; (2) stable, in order not to have decayed yet; (3) relatively heavy such that their average velocity is much smaller than the speed of light—otherwise they would contribute to the pressure term $p(t)$ in (2.7) (which is not observed), and $M(r)$ in (2.17) could not depend on r in the observed way. None of the known elementary particles satisfies all these conditions! We believe for this reason, amongst others, that there exist new elementary particles still to discover, which are the constituents of dark matter (see also Sect. 12.2 on supersymmetry).

(b) What is the origin of the dark energy (or the cosmological constant)? As we already mentioned above, its present numerical value—of the same order as the matter density $\varrho(t_0)$—is a coincidence that is difficult to explain. A real problem appears in the context of field theory mentioned above: in this theory we obtain contributions to the potential energy (or "vacuum energy") that correspond to the cosmological constant but which exceed its value given in (2.18) by many orders of magnitude (by a factor 10^{54} in the framework of weak interaction; see the end of Sect. 7.3). The fact that a large value of Λ was actually desirable during an inflationary epoch does not facilitate an explanation of its relatively small value today. Either we have not yet understood an essential aspect of the relevant theory, or there are many different contributions to Λ that cancel nearly exactly after the end of the inflationary epoch. However, at present nobody is aware of a mechanism that would lead to such a compensation of different contributions; this problem is called the "problem of the cosmological constant".

(c) Normally we should assume that, after the Big Bang, the Universe contains as many particles as antiparticles. However, the observable part of the Universe contains practically no antimatter, just "ordinary" matter. That is, evidently processes occurred that break the matter–antimatter symmetry. Indeed, we have already observed a violation of this symmetry in decays of certain particles (see the so-called CP violation in Sect. 7.4). However, at present it is not clear whether this symmetry violation suffices to explain the present disequilibrium between matter and antimatter; to this end we need a better understanding of processes that took place at a time before 10^{-12} s (at a temperature above 10^{15}°C).

(d) Did the Universe really undergo an inflationary epoch? If yes, what precisely did it look like? (See also the end of Sect. 7.3.) Which field, or which potential energy, was responsible for it? Is it really true that an oscillating field at the end of an inflationary epoch is responsible for the density fluctuations at the origin of the formation of stars and galaxies? In order to learn more about this inflationary epoch, a better knowledge of the angular dependence of the intensity variations of the cosmic background radiation would be very helpful. We hope to gain such information with the help of instruments placed on the Planck satellite, which was launched in 2009.

(e) What was the origin of the Big Bang? What happened at times before 10^{-12} s, or even before $t = 0$? Equations (2.6) and (2.7) can no longer be valid in the limit $t \to 0$, and the answers to these questions depend on how these equations are modified. Different theories beyond the Einstein equations lead to different modifications, but nobody knows at present whether—or which of—such theories (amongst others theories in which space-time is higher dimensional, see Sect. 12.3) are realistic.

Exercises

2.1. Solve both Friedmann–Robertson–Walker equations (2.6) and (2.7) for $\Lambda = 0$, $p(t) = w\varrho(t)c^2$ for arbitrary constants w. ($w = 0$ corresponds to a Universe dominated by massive particles and $w = 1/3$ to a Universe dominated by massless particles. Show that $w = -1$ is equivalent to $p(t) = \varrho(t) = 0,\ \Lambda \neq 0$.)

2.2. Assume that, before the formation of light nuclei, the Universe contains free protons and neutrons with a ratio 7:1. Assume, in addition, that only the particularly stable helium nuclei ${}^4_2$He form, but free protons ${}^1_1$H (hydrogen nuclei) remain as well. Derive the ratio of densities $\varrho_{\mathrm{H}} : \varrho_{\mathrm{He}}$ after the formation of light nuclei.

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