Chapter 8
Damage of Solids Glued on One Another: Coupling of Volume and Surface Damages

Let us consider two pieces of concrete glued on one another. Both the concrete and the glue responsible for the adhesion, can damage due to external actions. The volume and surface damages are coupled. The interesting part of the predictive theory is the set of equations on the contact surface, [99, 100]. Mathematical results are reported in [43].

8.1 State Quantities and Quantities Describing the Evolution

We neglect the thermal effects and do not take the temperature into account. The state quantities and quantities describing the evolution are:

in Ω_1 and in Ω_2

$$E_1 = \{\varepsilon_1, \beta_1, \mathrm{grad}\,\beta_1\}, \delta E_1 = \left\{\frac{\partial \varepsilon_1}{\partial t}, \frac{\partial \beta_1}{\partial t}, \mathrm{grad}\,\frac{\partial \beta_1}{\partial t}\right\},$$

$$E_2 = \{\varepsilon_2, \beta_2, \mathrm{grad}\,\beta_2\},\ \delta E_2 = \left\{\frac{\partial \varepsilon_2}{\partial t}, \frac{\partial \beta_2}{\partial t}, \mathrm{grad}\,\frac{\partial \beta_2}{\partial t}\right\},$$

where the ε's are the small deformations and the β's the volume damages;

in $\partial\Omega_1 \cap \partial\Omega_2$

$$E_s = \{\mathbf{u}_2 - \mathbf{u}_1, \beta_s, \mathrm{grad}_s\,\beta_s, \beta_1, \beta_2\},$$

$$\delta E_s = \left\{\mathbf{U}_2 - \mathbf{U}_1, \frac{\partial \beta_s}{\partial t}, \mathrm{grad}_s\,\frac{\partial \beta_s}{\partial t}, \frac{\partial \beta_1}{\partial t}, \frac{\partial \beta_2}{\partial t}\right\},$$

with

$$\mathbf{U}_1 = \frac{\partial \mathbf{u}_1}{\partial t},\ \mathbf{U}_2 = \frac{\partial \mathbf{u}_2}{\partial t},$$

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where the $\mathbf{u}$'s are the small displacements with velocities $\mathbf{U} = \partial \mathbf{u}/dt$, β_s is the surface or glue damage. On the contact surface, we use the gap $\mathbf{u}_2 - \mathbf{u}_1$ and its velocity as it is usual in contact mechanics. But we will have to deal with damage resulting from the elongation or the stretching of the contact surface. In an elongation where

$$\forall \mathbf{x},\ \mathbf{u}_2(\mathbf{x}) - \mathbf{u}_1(\mathbf{x}) = 0,$$

the gap is 0 because the two displacements are equal. Thus we choose to introduce a non local deformation quantity

$$g(\mathbf{y},\mathbf{x}) = 2(\mathbf{y}-\mathbf{x})\cdot(\mathbf{u}_2(\mathbf{y}) - \mathbf{u}_1(\mathbf{x})),$$

which does not vanish in an elongation.

$$E_{s,1,2} = \{g(\mathbf{y},\mathbf{x}) = 2(\mathbf{y}-\mathbf{x})\cdot(\mathbf{u}_2(\mathbf{y}) - \mathbf{u}_1(\mathbf{x})), \beta_s(\mathbf{x}), \beta_s(\mathbf{y})\},$$
$$\delta E_{s,1,2} = \left\{D_{1,2}(\mathbf{U}_1,\mathbf{U}_2), \frac{\partial \beta_s}{\partial t}(\mathbf{x}), \frac{\partial \beta_s}{\partial t}(\mathbf{y})\right\},$$

with

$$D_{1,2}(\mathbf{U}_1,\mathbf{U}_2)(\mathbf{x},\mathbf{y}) = 2(\mathbf{y}-\mathbf{x})\cdot(\mathbf{U}_2(\mathbf{y}) - \mathbf{U}_1(\mathbf{x})).$$

The last quantity is the velocity of the non local deformation $g(\mathbf{y},\mathbf{x})$. Let us note that this velocity is 0 in any rigid body velocity, $\mathbf{V} + \varpi \times \mathbf{x}$. This property is obvious because $D_{1,2}$ is the time derivative of the square of the distance of two material points

$$D_{1,2}(\mathbf{U}_1,\mathbf{U}_2)(\mathbf{x},\mathbf{y}) = \frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{y}(t) - \mathbf{x}(t))^2.$$

8.2 Equations of Motion

They result from the principle of virtual power which involves the powers of the interior forces, of the exterior forces and of the acceleration forces, [112].

8.2.1 Virtual Power of the Interior Forces

Both volume damage and surface damage result from microscopic motions whose power is taken into account in the power of the interior forces. We choose the velocities $\partial\beta/dt$ to account for the microscopic velocities at the macroscopic level. In order to take into account local interactions, we introduce the gradients $\mathrm{grad}(\partial\beta/dt)$. Assuming $\mathbf{V} = (\mathbf{V}_1, \mathbf{V}_2)$ and $\gamma = (\gamma_1, \gamma_2, \gamma_s)$ to be macroscopic and microscopic virtual velocities, the virtual power of the interior forces, which is linear function of the virtual velocities, is chosen as

$$
\begin{aligned}
\mathscr{P}_{int}(\mathbf{V},\gamma) = &-\int_{\Omega_1} \sigma_1 : \mathsf{D}(\mathbf{V}_1) d\Omega - \int_{\Omega_1} \{B_1\gamma_1 + \mathbf{H}_1 \cdot \operatorname{grad}\gamma_1\} d\Omega \\
&-\int_{\Omega_2} \sigma_2 : \mathsf{D}(\mathbf{V}_2) d\Omega - \int_{\Omega_2} \{B_2\gamma_2 + \mathbf{H}_2 \cdot \operatorname{grad}\gamma_2\} d\Omega \\
&-\int_{\partial\Omega_1\cap\partial\Omega_2} \mathbf{R}\cdot(\mathbf{V}_2 - \mathbf{V}_1) d\Gamma \\
&-\int_{\partial\Omega_1\cap\partial\Omega_2} \{B_s\gamma_s + \mathbf{H}_s \cdot \operatorname{grad}_s\gamma_s + B_{1,s}(\gamma_1-\gamma_s) + B_{2,s}(\gamma_2-\gamma_s)\} d\Gamma \\
&+\int_{\partial\Omega_1\cap\partial\Omega_2}\int_{\partial\Omega_1\cap\partial\Omega_2} M(\mathbf{x},\mathbf{y}) D_{1,2}(\mathbf{V}_1,\mathbf{V}_2)(\mathbf{x},\mathbf{y}) d\mathbf{x}d\mathbf{y} \\
&+\int_{\partial\Omega_1\cap\partial\Omega_2}\int_{\partial\Omega_1\cap\partial\Omega_2} \{B_{s,1}(\mathbf{x},\mathbf{y})\gamma_s(\mathbf{x}) + B_{s,2}(\mathbf{x},\mathbf{y})\gamma_s(\mathbf{y})\} d\mathbf{x}d\mathbf{y}.
\end{aligned}
$$

The different quantities which contribute to the power of the interior forces are products of kinematic quantities by interior forces. The kinematic quantities are the quantities that intervene in the motion we intend to describe. Their choice is of paramount importance since they determine the whole predictive theories. They are chosen by experimenting phenomena which are volume and surface deformations together with volume and surface damage, i.e., micro-voiding and micro-cracking. Thus there are quantities with surface and volume densities depending on the quantities we have chosen to describe the evolutions or the deformations of the system. Some of them are classical others are new. Most of them are local but a few are non local because there is a kinematic quantity which is non local. Let us comment on the different power densities:

- The usual strain rate D introduces the stress σ.
- The damage velocity, $\partial\beta/dt$ is a scalar, thus the associated interior force is also a scalar, B. It is a work, the internal damage work which is responsible for the evolution of the damage in the volume and in the surface.
- The gradient of the damage velocity, $\operatorname{grad}(\partial\beta/dt)$ is a vector, thus the interior force is a vector, $\mathbf{H}$. It is a work flux vector which is responsible for the interaction of the damage at a point on the damage of its neighbourhood. Its physical meaning is to be given by the boundary condition of the equation of motion as the physical meaning of the stress is given by the boundary condition of the equation of motion.
- The gap velocity $\mathbf{U}_2 - \mathbf{U}_1$ on the contact surface introduces the classical macroscopic interaction force $\mathbf{R}$.
- In the same way the difference between the damage velocities $\partial\beta_i/dt - \partial\beta_s/dt$ introduces a damage work flux on the surface, $B_{i,s}$ which describes the interaction of volume damage and surface damage.
- The elongation velocity, $D_{1,2}(\mathbf{U}_1,\mathbf{U}_2)(\mathbf{x},\mathbf{y})$ which is non local, introduces a non local scalar $M(\mathbf{x},\mathbf{y})$ interior force. It describes the effects of the elongation. It results in the equations of motion as a classical force. The interaction

macroscopic mechanical force has a non local part and a classical local part, the force $\mathbf{R}$ (see formula (8.2) below). Since we are going to assume the interior force $M(\mathbf{x},\mathbf{y})$ depending on the surface damage β_s, it is wise to add an extra non local power depending on damage velocity $\partial\beta_s/dt$. It describes the effect of damage at point $\mathbf{x}$ on damage at point $\mathbf{y}$. The interior forces $B_{s,i}(\mathbf{x},\mathbf{y})$ have the same effect than M: they introduce a non local internal source of damage work. The microscopic mechanical force has a non local part and three local parts, B_s due to the glue and the two $B_{i,s}$ due to the interactions with the volumes (see formula (8.1) below).

Let us note that even if the interior forces are numerous and some of them are unusual, all of them are simple and precisely committed to take into account a particular aspect of the coupling of volume and surface, and of microscopic and macroscopic evolution of the system.

8.2.2 Virtual Power of the Exterior Forces

We assume no exterior microscopic, either surface or volume, source of damage such as radiative, electrical or chemical damaging actions. Thus we have

$$\begin{aligned}\mathscr{P}_{ext}(\mathbf{V},\gamma) = &\int_{\Omega_1} \mathbf{f}_1 \cdot \mathbf{V}_1 d\Omega + \int_{\partial\Omega_1 \setminus (\partial\Omega_1 \cap \partial\Omega_2)} \mathbf{g}_1 \cdot \mathbf{V}_1 d\Gamma \\ &+ \int_{\Omega_2} \mathbf{f}_2 \cdot \mathbf{V}_2 d\Omega + \int_{\partial\Omega_2 \setminus (\partial\Omega_1 \cap \partial\Omega_2)} \mathbf{g}_2 \cdot \mathbf{V}_2 d\Gamma,\end{aligned}$$

where the $\mathbf{f}$ and $\mathbf{g}$ are the body and surface exterior forces.

8.2.3 Virtual Power of the Acceleration Forces

For the sake of simplicity, we assume a quasi-static problem. Thus

$$\mathscr{P}_{acc}(\mathbf{V},\gamma) = 0.$$

8.2.4 The Principle of Virtual Power

It is

$$\begin{aligned}\forall \mathbf{V} &= (\mathbf{V}_1, \mathbf{V}_2),\ \gamma = (\gamma_1, \gamma_2, \gamma_s), \\ \mathscr{P}_{acc}(\mathbf{V},\gamma) &= \mathscr{P}_{int}(\mathbf{V},\gamma) + \mathscr{P}_{ext}(\mathbf{V},\gamma).\end{aligned}$$

8.2.5 The Equations of Motion

They result from the principle. By choosing convenient virtual velocities, we get easily for domain Ω_1

$$\operatorname{div}\sigma_1 + \mathbf{f}_1 = 0,\ -B_1 + \operatorname{div}\mathbf{H}_1 = 0,\ in\ \Omega_1,$$
$$\sigma_1\mathbf{N}_1 = \mathbf{g}_1,\ \mathbf{H}_1\cdot\mathbf{N}_1 = 0,\ in\ \partial\Omega_1\backslash(\partial\Omega_1\cap\partial\Omega_2).$$

They are the volume equations of motion accounting for macroscopic and microscopic effects. The equations of motion for domain Ω_2 are the same. For the sake of simplicity, we consider the domains Ω_1 and Ω_2 have the same mechanical properties and give only the mechanical relationships for domain Ω_1. The related boundary conditions and the equation of motion on $\partial\Omega_1\cap\partial\Omega_2$ involve non local forces.

$$\begin{aligned}
\sigma_1\mathbf{N}_1(\mathbf{x}) &= \mathbf{R}(\mathbf{x}) + \int_{\partial\Omega_1\cap\partial\Omega_2} 2(\mathbf{x}-\mathbf{y})M(\mathbf{x},\mathbf{y})d\mathbf{y},\ \mathbf{x}\in\partial\Omega_1\cap\partial\Omega_2,\\
\sigma_2\mathbf{N}_2(\mathbf{y}) &= -\mathbf{R}(\mathbf{y}) + \int_{\partial\Omega_1\cap\partial\Omega_2} 2(\mathbf{y}-\mathbf{x})M(\mathbf{x},\mathbf{y})d\mathbf{x},\ \mathbf{y}\in\partial\Omega_1\cap\partial\Omega_2,\\
\mathbf{H}_1\cdot\mathbf{N}_1 &= -B_{1,s},\ \mathbf{H}_2\cdot\mathbf{N}_2 = -B_{2,s},\\
&\quad -B_s(\mathbf{x}) + \operatorname{div}_s\mathbf{H}_s(\mathbf{x}) + B_{1,s}(\mathbf{x}) + B_{2,s}(\mathbf{x})\\
&\quad -\int_{\partial\Omega_1\cap\partial\Omega_2} B_{s,1}(\mathbf{x},\mathbf{y}) + B_{s,2}(\mathbf{y},\mathbf{x})d\mathbf{y} = 0,\ \mathbf{x}\in\partial\Omega_1\cap\partial\Omega_2, \qquad (8.1)\\
\mathbf{H}_s\cdot\mathbf{n}_s &= 0,\ on\ \partial(\partial\Omega_1\cap\partial\Omega_2),
\end{aligned}$$

where $\mathbf{n}_s$ is the normal vector to the boundary $\partial(\partial\Omega_1\cap\partial\Omega_2)$ of $\partial\Omega_1\cap\partial\Omega_2$. Constitutive laws are needed for the numerous interior forces. As usual, we choose to define them with free energies depending on the state quantities E and pseudopotential of dissipation depending on the velocities δE.

Remark 8.1. Function $M(\mathbf{x},\mathbf{y})$ is not symmetric. Because there are at a distance interactions, there is no clear difference between the contact surface considered as a structure and contact surface considered as a material. In case a subdomain of $\partial\Omega_1\cap\partial\Omega_2$ is considered, the non local actions result in interior non local actions and exterior non local actions. This point of view is developed in [112].

8.3 The Constitutive Laws

Because the thermal phenomenon are not taken into account, the second law of thermodynamics is

$$\frac{d\Psi_1}{\partial t}(\varepsilon_1, \beta_1, \operatorname{grad}\beta_1) \le \sigma_1 : \mathsf{D}(\mathbf{U}_1) + B_1 \frac{\partial \beta_1}{\partial t} + \mathbf{H}_1 \cdot \operatorname{grad}\frac{\partial \beta_1}{\partial t},\ in\ \Omega_1,$$

$$\frac{d\Psi_s}{\partial t}(\mathbf{u}_2 - \mathbf{u}_1, \beta_s, \operatorname{grad}_s \beta_s, \beta_1 - \beta_s, \beta_2 - \beta_s) \le \mathbf{R}\cdot(\mathbf{U}_2 - \mathbf{U}_1) + B_s \frac{\partial \beta_s}{\partial t}$$

$$+\mathbf{H}_s \cdot \operatorname{grad}_s \frac{\partial \beta_s}{\partial t} + B_{1,s}(\frac{\partial \beta_1}{\partial t} - \frac{\partial \beta_s}{\partial t}) + B_{2,s}(\frac{\partial \beta_2}{\partial t} - \frac{\partial \beta_s}{\partial t}),\ on\ \partial\Omega_1 \cap \partial\Omega_2,$$

$$\frac{d\Psi_{s,1,2}}{\partial t}(g(\mathbf{y},\mathbf{x}), \beta_s(\mathbf{x}), \beta_s(\mathbf{y})) \le -M(\mathbf{x},\mathbf{y}) D_{1,2}(\mathbf{U}_1, \mathbf{U}_2)(\mathbf{x},\mathbf{y})$$

$$-B_{s,1}(\mathbf{x},\mathbf{y})\frac{\partial \beta_s}{\partial t}(\mathbf{x}) - B_{s,2}(\mathbf{x},\mathbf{y})\frac{\partial \beta_s}{\partial t}(\mathbf{y}),\ in\ (\partial\Omega_1 \cap \partial\Omega_2) \times (\partial\Omega_1 \cap \partial\Omega_2).$$

These relationships are used to define the constitutive laws with pseudopotential of dissipation. The $+$ or $-$ signs appearing in the constitutive laws result from the $+$ or $-$ signs which are in the right hand sides of the inequalities. The right hand sides are the opposite of the densities of the actual powers of the interior forces.

The free energy and pseudopotential of dissipation are the same for each domain. For domain Ω_1, they are

$$\Psi_1(E_1) = \Psi_1(\varepsilon_1, \beta_1, \operatorname{grad}\beta_1) = w_1(1-\beta_1) + \frac{k_1}{2}(\operatorname{grad}\beta_1)^2 + I(\beta_1)$$

$$+\frac{\beta_1}{2}\left\{\lambda_1 (tr\varepsilon_1)^2 + 2\mu_1 \varepsilon_1 : \varepsilon_1\right\},$$

$$\Phi_1(\delta E_1) = \Phi_1\left(\frac{\partial \beta_1}{\partial t}\right) = \frac{c_1}{2}\left(\frac{\partial \beta_1}{\partial t}\right)^2 + I_-\left(\frac{\partial \beta_1}{\partial t}\right),$$

where w_1 is the damage threshold, k_1 the damage interaction parameter which quantifies the influence of volume damage on its neighbourhood, λ_1 and μ_1 the Lamé parameters. They are the more simple energies coupling elasticity and volume damage. They give the constitutive laws

$$\sigma_1 = \frac{\partial \Psi_1}{\partial \varepsilon_1} = \beta_1\{\lambda_1 tr\varepsilon_1 1 + 2\mu_1 \varepsilon_1\},$$

$$B_1 = \frac{\partial \Psi_1}{\partial \beta_1} + \frac{\partial \Phi_1}{\partial(\partial \beta_1/dt)}$$

$$= -w_1 + \frac{1}{2}\left\{\lambda_1 (tr\varepsilon_1)^2 + 2\mu_1 \varepsilon_1 : \varepsilon_1\right\} + \partial I(\beta_1) + c\left(\frac{\partial \beta_1}{\partial t}\right) + \partial I_-\left(\frac{\partial \beta_1}{\partial t}\right),$$

$$\mathbf{H}_1 = \frac{\partial \Psi_1}{\partial(\operatorname{grad}\beta_1)} = k_1 \operatorname{grad}\beta_1,$$

where 1 is the identity matrix.

The free energy and pseudopotential of the glued contact surface are

$$\begin{aligned}\Psi_s(E_s) &= \Psi_s(\mathbf{u}_2-\mathbf{u}_1,\beta_s,\mathrm{grad}_s\,\beta_s,\beta_1-\beta_s,\beta_2-\beta_s)\\ &= w_s(1-\beta_s)+\frac{k_s}{2}(\mathrm{grad}_s\,\beta_s)^2+I(\beta_s)+I_-((\mathbf{u}_2-\mathbf{u}_1)\cdot\mathbf{N}_2)\\ &\quad+\frac{\beta_s\hat{k}_s}{2}(\mathbf{u}_2-\mathbf{u}_1)^2+\frac{k_{s,1}}{2}(\beta_1-\beta_s)^2+\frac{k_{s,2}}{2}(\beta_2-\beta_s)^2,\\ \Phi_s(\delta E_s) &= \Phi_s(\frac{\partial\beta_s}{\partial t})=\frac{c_s}{2}\left(\frac{\partial\beta_s}{\partial t}\right)^2+I_-\left(\frac{\partial\beta_s}{\partial t}\right),\end{aligned}$$

where w_s is the surface damage threshold, k_s the surface damage interaction parameter, $\hat{k}_s$ is the surface rigidity, $k_{s,1}$ and $k_{s,2}$ are the damage surface-volume interaction parameter, c_s is the damage viscosity. The function $I_-((\mathbf{u}_2-\mathbf{u}_1)\cdot\mathbf{N}_2)$ takes into account the impenetrability of the two pieces of concrete on their contact surface. The surface free energies and pseudopotential of dissipation are also the more simple we may choose. They account for elastic, viscous and damage properties. They give the constitutive laws

$$\mathbf{R} = \frac{\partial\Psi_s}{\partial(\mathbf{u}_2-\mathbf{u}_1)}\in\beta_s\hat{k}_s(\mathbf{u}_2-\mathbf{u}_1)+\partial I_-((\mathbf{u}_2-\mathbf{u}_1)\cdot\mathbf{N}_2)\mathbf{N}_2, \tag{8.2}$$

$$B_s = \frac{\partial\Psi_s}{\partial\beta_s}\in -w_s+\frac{\hat{k}_s}{2}(\mathbf{u}_2-\mathbf{u}_1)^2+\partial I(\beta_s)+c_s\frac{\partial\beta_s}{\partial t}+\partial I_-\left(\frac{\partial\beta_s}{\partial t}\right),$$

$$\mathbf{H}_s = \frac{\partial\Psi_s}{\partial(\mathrm{grad}_s\,\beta_s)}=k_s\,\mathrm{grad}_s\,\beta_s,$$

$$B_{1,s} = \frac{\partial\Psi_s}{\partial(\beta_1-\beta_s)}=k_{s,1}(\beta_1-\beta_s),$$

$$B_{2,s} = \frac{\partial\Psi_s}{\partial(\beta_2-\beta_s)}=k_{s,2}(\beta_2-\beta_s).$$

The force in $\partial I_-((\mathbf{u}_2-\mathbf{u}_1)\cdot\mathbf{N}_2)\mathbf{N}_2$ is the impenetrability reaction. The non-local free energy on the glued contact surface is

$$\Psi_{s,1,2}(E_{s,1,2}(\mathbf{x},\mathbf{y}))=\frac{k_{s,1,2}}{2}g^2(\mathbf{y},\mathbf{x})(\beta_s(\mathbf{x})\beta_s(\mathbf{y}))\exp(-\frac{|\mathbf{x}-\mathbf{y}|^2}{d^2}),$$

with

$$g(\mathbf{y},\mathbf{x})=2(\mathbf{y}-\mathbf{x})\cdot(\mathbf{u}_2(\mathbf{y})-\mathbf{u}_1(\mathbf{x})).$$

The exponential function with distance d, describes the attenuation of non-local actions with distance $|\mathbf{x}-\mathbf{y}|$ between points $\mathbf{x}$ and $\mathbf{y}$. We assume no dissipation with respect to $\delta E_{s,1,2}(\mathbf{x},\mathbf{y})$ and have the constitutive law

$$-B_{s,1}(\mathbf{x},\mathbf{y}) = \frac{\partial \Psi_{s,1,2}}{\partial \beta_s(\mathbf{x})}(E_{s,1,2}(\mathbf{x},\mathbf{y})) = \frac{k_{s,1,2}}{2} g^2(\mathbf{x},\mathbf{y})\beta_s(\mathbf{y})\exp(-\frac{|\mathbf{x}-\mathbf{y}|^2}{d^2}),$$

$$-B_{s,2}(\mathbf{x},\mathbf{y}) = \frac{\partial \Psi_{s,1,2}}{\partial \beta_s(\mathbf{y})}(E_{s,1,2}(\mathbf{x},\mathbf{y})) = \frac{k_{s,1,2}}{2} g^2(\mathbf{x},\mathbf{y})\beta_s(\mathbf{x})\exp(-\frac{|\mathbf{x}-\mathbf{y}|^2}{d^2}),$$

$$-M(\mathbf{x},\mathbf{y}) = \frac{\partial \Psi_{s,1,2}}{\partial g(\mathbf{y},\mathbf{x})}(E_{s,1,2}(\mathbf{x},\mathbf{y})) = k_{s,1,2} g(\mathbf{x},\mathbf{y})\left(\beta_s(\mathbf{x})\beta_s(\mathbf{y})\right)\exp(-\frac{|\mathbf{x}-\mathbf{y}|^2}{d^2}).$$

Let us note that all the constitutive laws involve the reactions to the internal constraints when needed, which are clearly non linear relationships, and linear relationships between the forces and the state quantities and velocities. Thus they are simple and we think that they have to account for the main physical phenomena: non linear constitutive laws are to be chosen only to make the results more precise and adapted to deal with a particular situation. But the linear relationships have to be sufficient to capture the main properties.

8.4 The Equations

They result from the equations of motion and the constitutive laws. They are:

8.4.1 On the Contact Surface

$$\begin{aligned}
&c_s \frac{\partial \beta_s}{\partial t} - k_s \Delta_s \beta_s + \partial I(\beta_s) + \partial I_-\left(\frac{\partial \beta_s}{\partial t}\right) \ni \\
&w_s - \frac{\hat{k}_s}{2}\left(\mathbf{u}_2 - \mathbf{u}_1\right)^2 + k_{s,1}\left(\beta_1 - \beta_s\right) + k_{s,2}\left(\beta_2 - \beta_s\right) \\
&-\int_{\partial\Omega_1 \cap \partial\Omega_2} \frac{k_{s,1,2}}{2}(g^2(\mathbf{y},\mathbf{x}) + g^2(\mathbf{x},\mathbf{y})\beta_s(\mathbf{y})\exp(-\frac{|\mathbf{x}-\mathbf{y}|^2}{d^2})d\mathbf{y},\ in\ \partial\Omega_1 \cap \partial\Omega_2, \\
&k_s \frac{\partial \beta_s}{\partial n_s} = 0,\ on\ \partial(\partial\Omega_1 \cap \partial\Omega_2).
\end{aligned}$$

The last but one term is not 0 when

$$\mathbf{u}_2 - \mathbf{u}_1 = 0.$$

It is responsible for the damage resulting from elongation. The glue damage source in the right hand side results from the gap between the two solids, from the elongation (the non-local effect) and from the flux of damaging work coming from the concrete. It is proportional to the difference of damage between the concrete and the glue. Thus it is more difficult to damage the glue when the concrete is not damaged. In this case the glue cohesion is $w_s + k_{s,1} + k_{s,2}$ whereas it is w_s when the concrete is completely damaged. The contact boundary conditions on the glued contact surface $\partial\Omega_1 \cap \partial\Omega_2$ are

$$\forall \mathbf{x} \in \partial\Omega_1 \cap \partial\Omega_2,\ \sigma_1 \mathbf{N}_1(\mathbf{x}) = \beta_s \hat{k}_s (\mathbf{u}_2 - \mathbf{u}_1) + \partial I_- ((\mathbf{u}_2 - \mathbf{u}_1) \cdot \mathbf{N}_2) \mathbf{N}_2$$

$$- \int_{\partial\Omega_1 \cap \partial\Omega_2} 2(\mathbf{x} - \mathbf{y}) k_{s,1,2} g(\mathbf{x}, \mathbf{y}) (\beta_s(\mathbf{x}) \beta_s(\mathbf{y})) \exp(-\frac{|\mathbf{x} - \mathbf{y}|^2}{d^2}) d\Gamma(\mathbf{y}),$$

$$\forall \mathbf{y} \in \partial\Omega_1 \cap \partial\Omega_2,\ \sigma_2 \mathbf{N}_2(\mathbf{y}) = -\beta_s \hat{k}_s (\mathbf{u}_2 - \mathbf{u}_1) - \partial I_- ((\mathbf{u}_2 - \mathbf{u}_1) \cdot \mathbf{N}_2) \mathbf{N}_2,$$

$$- \int_{\partial\Omega_1 \cap \partial\Omega_2} 2(\mathbf{y} - \mathbf{x}) k_{s,1,2} g(\mathbf{x}, \mathbf{y}) (\beta_s(\mathbf{x}) \beta_s(\mathbf{y})) \exp(-\frac{|\mathbf{x} - \mathbf{y}|^2}{d^2}) d\Gamma(\mathbf{x}),$$

$$k_1 \frac{\partial \beta_1}{\partial N_1} = k_{s,1} (\beta_s - \beta_1),\ k_1 \frac{\partial \beta_2}{\partial N_2} = k_{s,2} (\beta_s - \beta_2). \tag{8.3}$$

In the following numerical applications, we have neglected the non local mechanical effect on the contact surface stresses because it has not an important overall mechanical effect. The values of parameters $\hat{k}_s \gg k_{s,1,2}$ of the constitutive laws we choose in the sequel agree with this assumption. Let us recall that this non-local effect has been introduced to take into account damage which is produced by the elongation, i.e. by displacements such that

$$\mathbf{u}_2(\mathbf{x}) - \mathbf{u}_1(\mathbf{x}) = 0,$$

with

$$\mathbf{u}_2(\mathbf{x}) - \mathbf{u}_1(\mathbf{y}) \neq 0,\ for\ \mathbf{x} \neq \mathbf{y}.$$

The boundary condition (8.3) means that the damaging work flux in the concrete is proportional to the difference of damage between the glue and the concrete.

8.4.2 In the Domains

As already said, they are identical for the two domain. For domain Ω_1, they are

$$\text{div}(\beta_1 \{\lambda_1 tr\varepsilon_1(\mathbf{u}_1) 1 + 2\mu_1 \varepsilon_1(\mathbf{u}_1)\}) = 0,$$

$$c_1 \frac{\partial \beta_1}{\partial t} - k_1 \Delta \beta_1 + \partial I(\beta_1) + \partial I_- \left(\frac{\partial \beta_1}{\partial t} \right) \ni w_1 - \frac{1}{2} \left\{ \lambda_1 (tr\varepsilon_1)^2 + 2\mu_1 \varepsilon_1 : \varepsilon_1 \right\},$$

with initial conditions

$$\beta_1(\mathbf{x},0) = \beta_1^0(\mathbf{x}),\ in\ \Omega_1,$$
$$\beta_s(\mathbf{x},0) = \beta_1^0(\mathbf{x}),\ on\ \partial\Omega_1 \cap \partial\Omega_2,$$

and boundary conditions

$$\sigma_1 \mathbf{N}_1 = \mathbf{g}_1,\ on\ \partial\Omega_1 \backslash (\partial\Omega_1 \cap \partial\Omega_2),$$
$$k_1 \frac{\partial \beta_1}{\partial N_1} = 0,\ on\ \partial\Omega_1 \backslash (\partial\Omega_1 \cap \partial\Omega_2).$$

8.5 Examples

The following examples show how important are the interaction parameters $k_{s,1}$ and $k_{s,2}$ which couple the damages of solids 1 and 2: when solid 1 damages in the neighbourhood of solid 2, solid 2 damages also. The examples confirm also that it is more difficult to damage the glue when the concrete is not damaged than when the concrete is damaged: the glue cohesion or threshold is $w_s + k_{s,1} + k_{s,2}$ when the concrete is not damaged whereas it is w_s when concrete is completely damaged in the two solids. The examples show also the important effect of the stretching described by the non local interactions. All the computations are due to Francesco Freddi, [10, 27, 28, 99, 100].

8.5.1 Four Points Bending

Some experimental results due to Marie Paule Thaveau, [203], are reported on Fig. 8.1. One piece of concrete or two pieces of concrete glued on one another are tested. Their length is 0.4 m, their width is 0.3 m, their height is either 0.1 m or 2×0.05 m. The Young modulus is 38 GPa, the Poisson modulus is 0.2. The maximum load is 14.3 kN for the one piece specimen and 18.2 kN for the two pieces specimen. Numerical results are shown on Figs. 8.2–8.5. They concern three models:

1. There is no damaging interaction between solids 1, 2 and the glue: $k_{s,1} = k_{s,2} = 0$. Stretching is not taken into account: $k_{s,1,2} = 0$.
2. There is damaging interaction between solids 1, 2 and the glue: $k_{s,1} \neq 0$, $k_{s,2} \neq 0$. Stretching is not taken into account: $k_{s,1,2} = 0$.
3. There is damaging interaction between solids 1, 2 and the glue: $k_{s,1} \neq 0$, $k_{s,2} \neq 0$. Stretching is taken into account: $k_{s,1,2} \neq 0$.

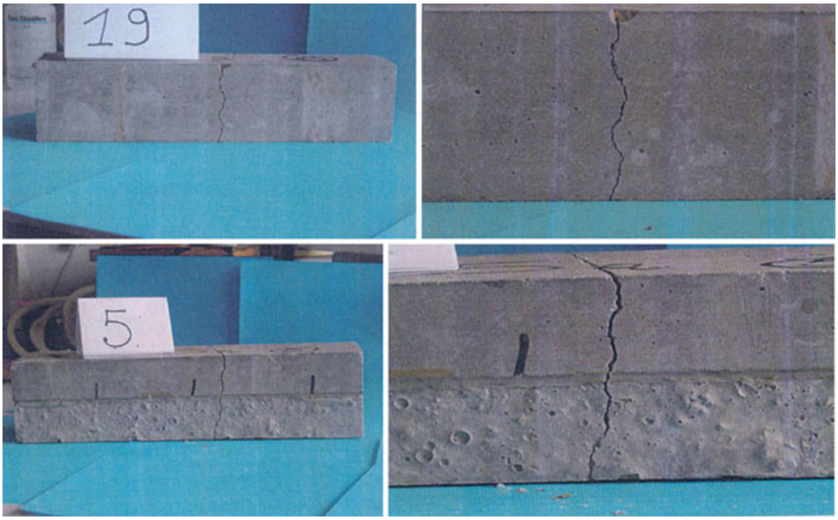


Fig. 8.1 Four points bending, experimental results due to Marie Paule Thaveau, [203]. The experiments are either with two pieces of concrete glued on one another (number 5) or with one piece of concrete (number 19)

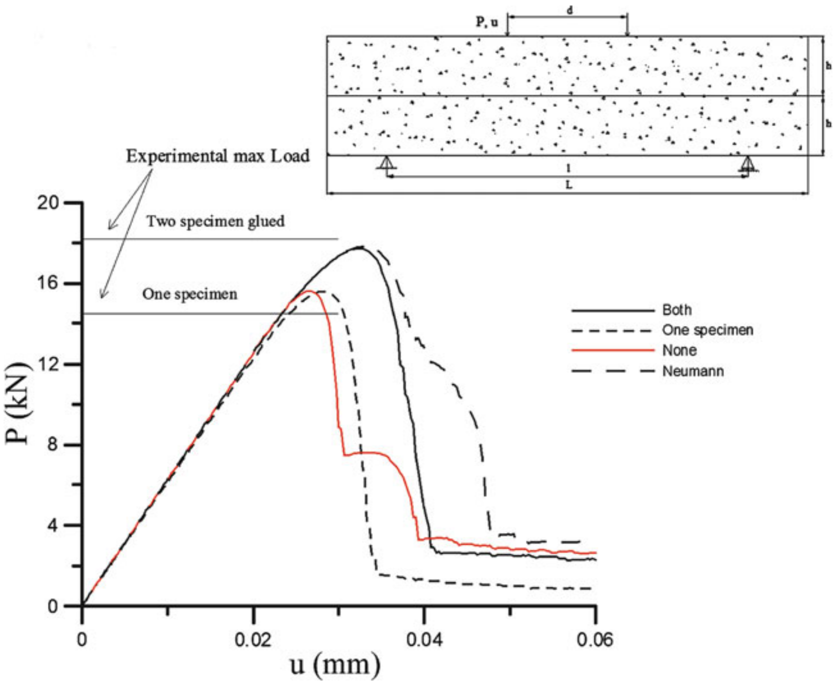


Fig. 8.2 Load versus displacement curves for one piece and two pieces concrete specimens in four point bending. The short dashed line is for the one piece specimen. The red line, long dashed line and continuous line are for the two pieces concrete specimen and models 1, 2 and 3 respectively

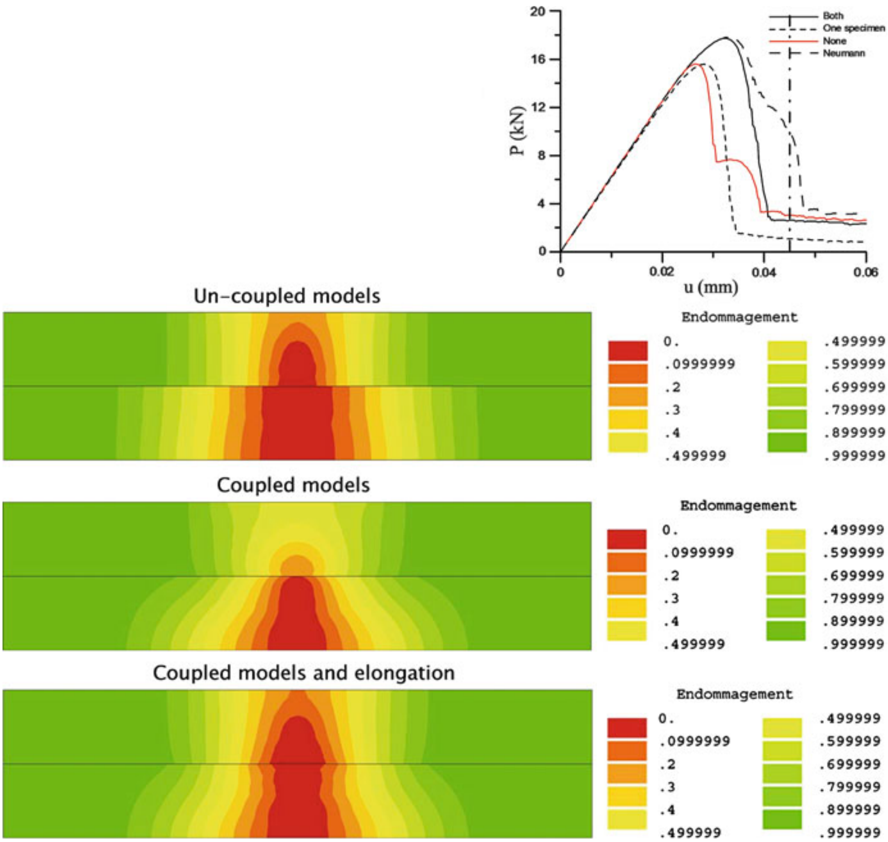


Fig. 8.3 The damage for the two pieces and the three models (model 1 at the top, model 2 in the middle, model 3 at the bottom). The applied displacement is $u = 0.045$ mm. The contact surface of model 1 is a barrier for damage. It is not very good. The model 2 allows interactions of the damages of the two pieces. The model 3 allows interactions of the damages and the effect of stretching. It seems to be the best model. The color scales are different in the pictures (the color scale is given on the right of them)

The first model is unable to account for experimental results: the contact surface is a barrier which stops the damage (see Fig. 8.4). The two other models are good but the best is the one which takes into account the stretching effect.

8.5.2 Pull Test

A vertical force is applied to two glued pieces of concrete (Fig. 8.6). The relative stiffness of the concrete and of the glue governs the behaviour of the structure.

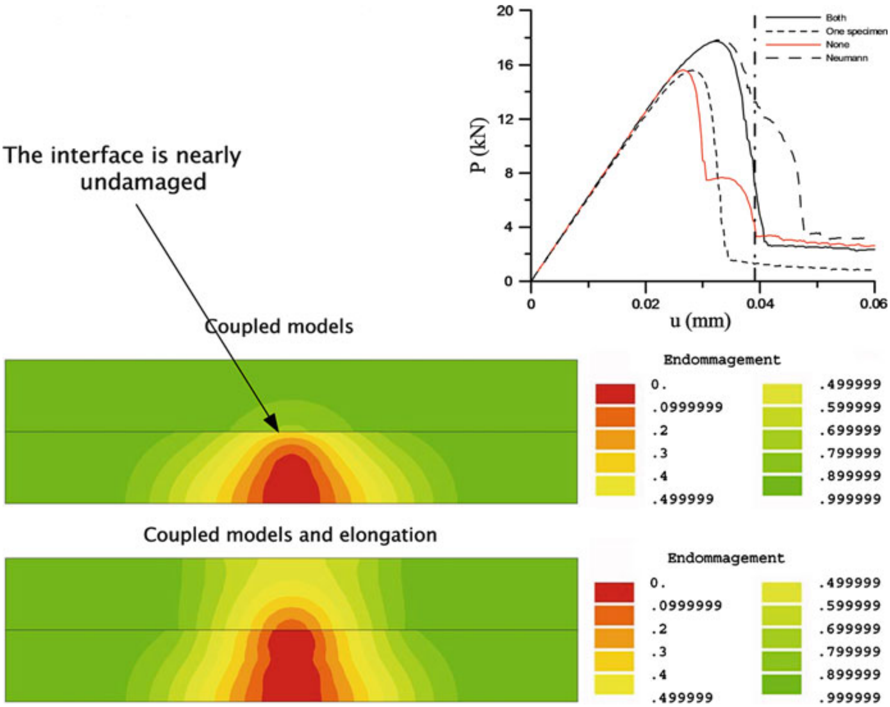


Fig. 8.4 The contact surface is a barrier in model 1 whereas the effect of damage interaction and the effect of stretching are important in the best model 3. The applied displacement is $u = 0.039$ mm

When glue is solid and the concrete is weak, damage occurs in concrete just under the contact surface (Fig. 8.7). If the glue is weak and the concrete is solid, separation of the two pieces occurs on the contact surface and the concrete is not damaged, (Fig. 8.8).

8.5.3 Fibre-Reinforced Polymers-Concrete Delamination

The experiment is described in Fig. 8.9. Experimental and numerical results are shown in Fig. 8.10. It appears a thin damaged zone in the concrete as well as large displacements: they correspond to a layer of concrete which remains glued on the Fibre-reinforced polymers (FRP, [15–17, 59, 103]) in the experiments. The determination of the parameters of the predictive theory from practical engineering knowledge is described in [28] together with the practical computation of the maximal load which a structure can bear.

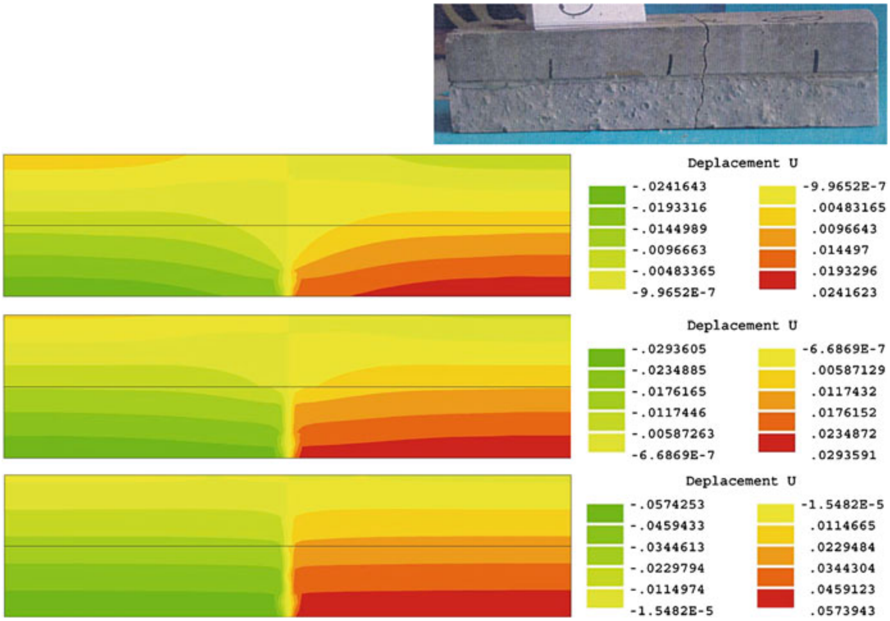


Fig. 8.5 The sharp discontinuity of the horizontal displacement accounts for the fracture. Red is a positive displacement towards the right, green is a negative displacement towards the left

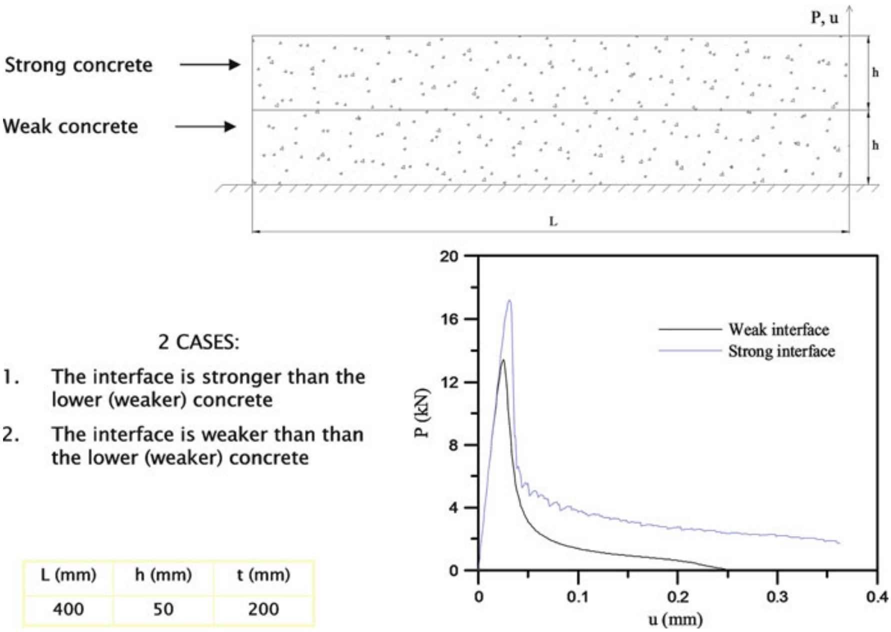


Fig. 8.6 Pull test with important contrasts between the glue and concrete stiffness properties

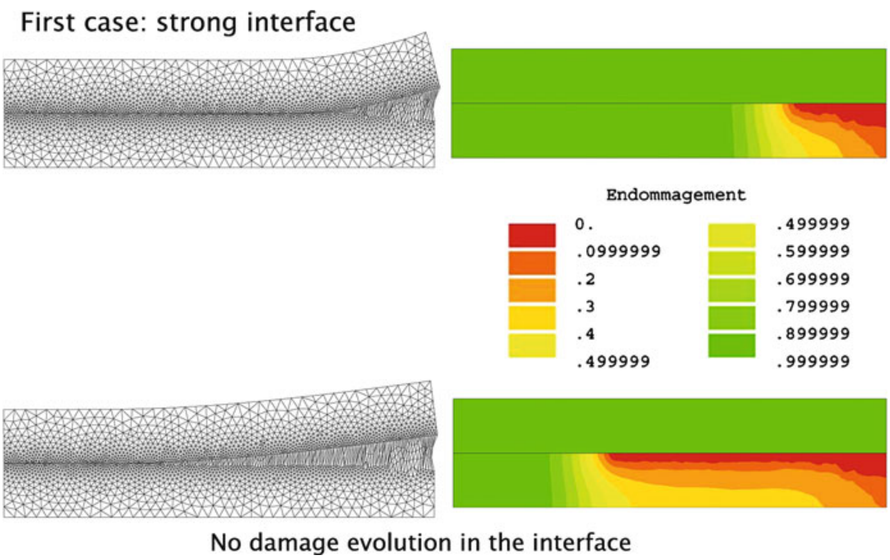


Fig. 8.7 The glue is much more solid than the concrete; The damage occurs within the concrete just under the contact surface whereas the glue does not break

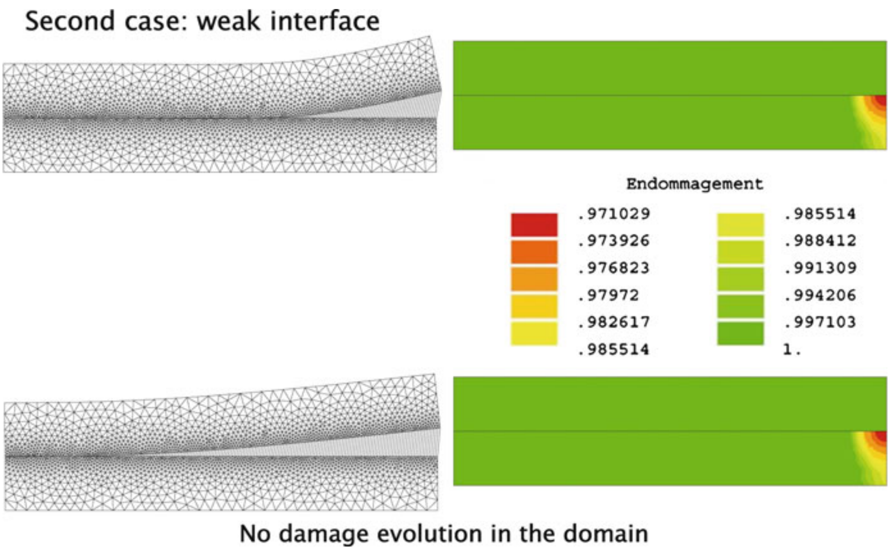
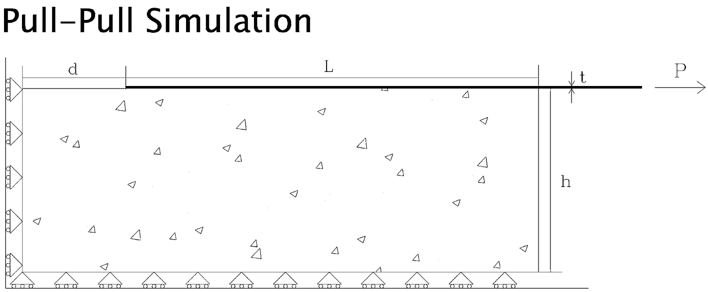


Fig. 8.8 The glue is weaker than the concrete. Damage occurs only in the glue. The concrete even if it is somewhere red is not damaged or almost not damaged. Let recall that the color scales are unique for each picture



t (mm)	h (mm)	E_p (MPa)	E_c (MPa)	f_c (MPa)	L (mm)	d (mm)
1.016	100	230000	33640	36.4	100	50

Fig. 8.9 A pull experiment for a FRP reinforced structure

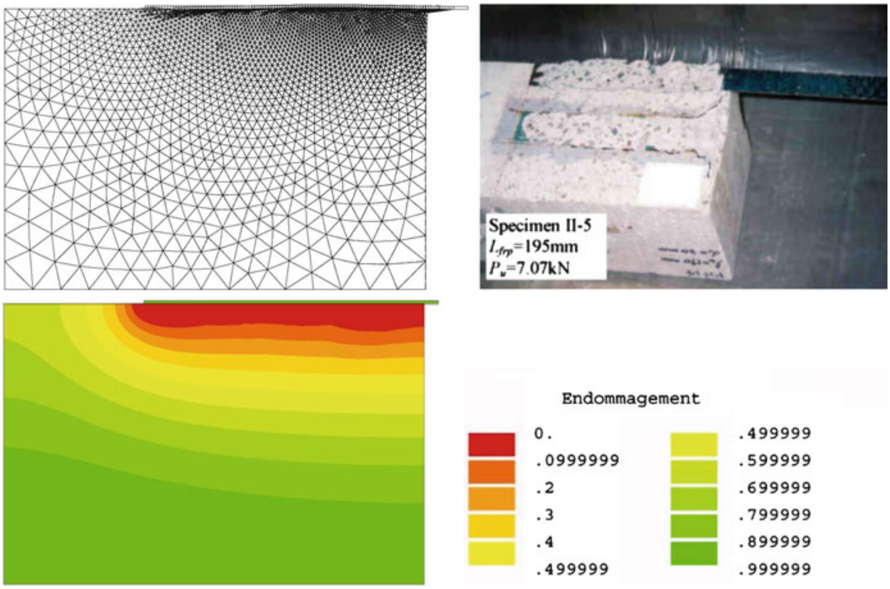


Fig. 8.10 Damage occurs in the concrete and not in the FRP in agreement with experiments. Note the very large displacement and the damaged concrete in the right up corner where the FRP is pulled

Remark 8.2. In collisions which are investigated in following chapters, the damage β may be discontinuous with respect to space jumping from $\beta = 1$, sound material, to $\beta = 0$, completely damaged material. When β becomes 0 on a line whereas it is 1 elsewhere, a fracture is created. Fractures resulting from collisions may be investigated with these tools. This point of view is developed in [100].

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