

Chapter 1

Introduction

Fluids and flows are pervading our daily lives without our conscious perception. The air we breathe, the shower in the bathroom with the shampoo, coffee or tea, the blood in our vascular tree including the heart and the brain, the ingredients of food like mayonnaise, oil, vinegar, yogurt, etc. The design and the engineering of new materials like polymers, plastics, ceramics, foams, etc., have produced complex fluids because the materials processing based on extrusion, molding, blowing, etc. is using them in the fluid state. Note that in polymers the presence of long molecular chains and their entanglement affect the mechanical behavior of the fluid. Therefore fluids are flowing around us in a swirl of natural events and of technological applications. Most flows in nature are complex due to their turbulent character: river and ocean currents, waves on a beach, storms, hurricanes, tsunamis, Many technological applications use turbulence to achieve the goal of the design, for example homogeneous mixing in materials processing. Moreover the understanding of the turbulent phenomena may lead to avoid them or reduce their intensity and thereby their effects, for example, in the design of hydraulic machines. These flows and fluids are reputed as complex. Therefore we need to better understand the meaning of this epithet.

Complexity is of course contrary to simplicity. A flow is simple when the associated geometrical patterns are simple themselves like the parallel streamlines in the so-called parallel flows as the plane Couette and Poiseuille flows. Here the physics is also simple and most of the time, the fluid dynamicist has an intuitive “a priori” feeling about how the solution will look like. In some sense the answer is rather obvious. But simplicity is also connected to linearity which implies that the observed effect is directly proportional to the cause coming either from pressure or shear. The Navier-Stokes equations that govern the dynamics of viscous fluid flows are non-linear from a mathematical standpoint. A linear combination of two (or several) solutions is no longer a solution. This non-linear character impedes our natural way of thinking based on reductionism to obtain a quick correct answer, because non-linearity has a deep (and sometimes, unexpected) influence on the resulting physics. Non-linear problems are extremely difficult to tackle through analytical tools. Numerical methods leading to numerical simulation constitute the state-of-the-art practice in physics and engineering. The demand for general and accurate

models is very high and leads to the elaboration of advanced concepts that will be the major theme of this monograph.

Complexity not only comes from the physics of the flow or the fluid mechanical behavior, it may also be a consequence of the geometrical configuration. This is especially true in engineering where complicated shapes are involved like for example, airfoils, blades, runners, space shuttle, airplane body, wings, etc. The interplay between all parameters: geometry, flow and fluid renders the problems quite intricate and involved. We will address flow and fluid complexity separately leaving for case studies, the full aspect of the three complexity sources.

1.1 Complex Fluids

Complex fluids are designated as non-Newtonian or more generally as viscoelastic because their mechanical behavior does not fit the pattern of the classical Newtonian fluids represented, for example, by water in standard conditions. The Newtonian constitutive equation is given by the relationship

$$\boldsymbol{\Sigma} = -p \mathbf{I} + 2\mu \mathbf{D} , \quad (1.1)$$

where $\boldsymbol{\Sigma}$ denotes the Cauchy stress tensor, p the pressure, $\mathbf{I}$ the identity tensor, μ the dynamic shear viscosity, and $\mathbf{D}$ the rate of deformation tensor defined as the symmetric part of the velocity gradient

$$\mathbf{D} = \frac{1}{2} \left(\nabla \mathbf{v} + (\nabla \mathbf{v})^T \right) , \quad (1.2)$$

where $\mathbf{v}$ is the velocity field, $\nabla \mathbf{v}$ is the velocity gradient, and the superscript T indicates the transpose. It is not surprising that when writing about two somewhat diverse topics that notational ambiguities arise. As just defined, the tensor $\mathbf{D}$ is termed the rate of deformation tensor in the rheology community; whereas, in the fluid mechanics community it is termed the rate of strain or strain rate tensor and denoted by $\mathbf{S}$.

The constitutive equation (1.1) is indeed linear as it is based only on the first power of $\mathbf{D}$. Water is clearly a good real world example of such a fluid responding to this kind of mechanical behavior.

A first class of non-Newtonian fluids is concerned with the generalized Newtonian fluids where the dynamic shear viscosity μ is dependent of the shear rate $\dot{\gamma}$ that is obtained from the second invariant of the tensor $\mathbf{D}$ such that $\dot{\gamma} = \sqrt{2\mathbf{D} : \mathbf{D}}$. (In index notation, $\mathbf{D} : \mathbf{D}$ is given by $D_{ij}D_{ji}$ where the Einstein summation convention of repeated indices is used throughout unless otherwise noted.) This then leads to

$$\boldsymbol{\Sigma} = -p \mathbf{I} + 2\mu(\dot{\gamma}) \mathbf{D} . \quad (1.3)$$

Shear-thinning or pseudo-plastic effects are present in most polymer flows or melts, and this is also the case of blood flow. A very popular relation is the power law

$$\mu(\dot{\gamma}) = K \dot{\gamma}^{n-1} , \quad (1.4)$$

where K is the consistency factor and n the power law index ($n = 1$ for the Newtonian case). For many polymers, the value of n lies between 0.3 and 0.6. Shear-thinning fluids have $n < 1$ while shear-thickening or dilating fluids are those with $n > 1$. When memory effects come into play, non-Newtonian fluids are called viscoelastic fluids that incorporate complicated and rather counter-intuitive physical phenomena.

1.1.1 Physical Considerations

The deviation of the flow field behavior of a non-Newtonian fluid from that of a Newtonian fluid is often quantified by normal stress field differences and characterized by fluid memory effects. The two are related and serve to identify key elements of the dynamic mechanisms responsible for the unique behavior of such fluids in even relatively simple geometric flow fields.

1.1.1.1 Normal Stress Effects

Newtonian fluids do not show imbalance in the first normal stress differences defined by the relations

$$N_1 = \Sigma_{11} - \Sigma_{22}, \quad N_2 = \Sigma_{22} - \Sigma_{33} \quad (1.5)$$

in simple shear flows. On the contrary for non-Newtonian fluids, $N_1 \neq 0$ and $N_2 \neq 0$. A spectacular experiment is the Weissenberg effect occurring in a cylindrical Couette device with a free surface when the inner cylinder rotates and the outer one is fixed. For the Newtonian case, under the centrifugal forces, the surface deforms and takes a paraboloid shape climbing along the exterior cylinder. A small trough is formed close to the inner cylinder. For the non-Newtonian case, the fluid rises up the inner shaft draining in some situations the container, Fig. 1.1. Another interesting effect is the swelling that is characteristic of the jet flowing out of a die. For the Newtonian fluid, the jet shape remains almost constant at the die outflow section. In the non-Newtonian test, the jet swells and can take three to four times the diameter of the orifice, see Fig. 1.2.

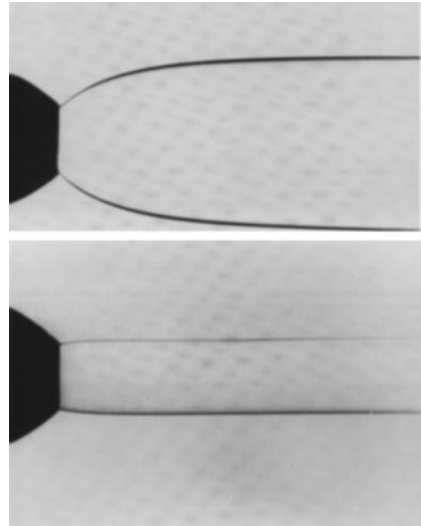
1.1.1.2 Memory Considerations

Memory effects are induced by the fact that the present time stresses depend on the history of deformation of the fluid. When submitted to a brief and intense force, the viscoelastic fluid behaves as an elastic solid while over very long time intervals, its response is that of a viscous fluid. Between these two situations, one gets the viscoelastic behavior. Viscoelastic fluids are characterized by stress relaxation. To maintain a constant deformation in the fluid, the applied force decreases over time in

Fig. 1.1 Example of the Weissenberg effect with fluid rising up the inner cylinder of a Couette device¹



Fig. 1.2 Example of the die swell phenomenon: *top*, non-Newtonian fluid; *bottom*, Newtonian fluid



an exponential fashion. They present also the phenomenon of creep which produces under a constant force a permanent deformation depending on time. For example, it is well known that stained glass windows in cathedrals are thicker at the bottom than at the top because gravity has been acting over the centuries. On the other side if a glass pane is hit by a soft ball, it reacts as an elastic solid. This example shows that the choice of the constitutive relationship depends strongly on the physics to be modeled.

¹Figures 1.1 and 1.2 reprinted with permission from: D.V. Boger, K. Walters (1993) *Rheological Phenomena in Focus*. Elsevier Science Ltd., Amsterdam.

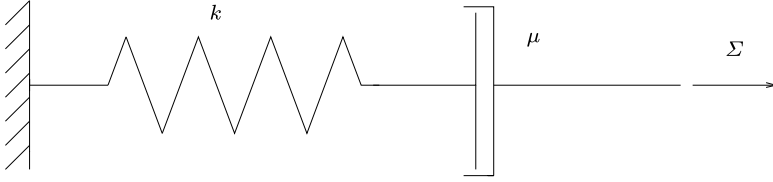


Fig. 1.3 Schematic of Maxwell linear viscoelastic mechanical model

1.1.2 Viscoelastic Fluids

Elementary models for viscoelastic fluids are easily described through mechanical analogs using springs and dashpots. A very simple viscoelastic model is made of a spring with constant stiffness k for the elastic part and a dashpot of viscosity μ (see Fig. 1.3). For example, the Maxwell model connects these two mechanical elements in series with an intrinsic characteristic time of the model being $\lambda = \mu/k$. The one-dimensional (1D) stress-strain relation for the spring is $\Sigma = k \epsilon_1$, and for the dashpot $\Sigma = \mu \dot{\epsilon}_2$. The same stress (force), Σ , applied to the model generates a displacement Σ/k in the spring and a velocity Σ/μ in the dashpot. The total velocity is then given by the sum

$$\dot{\epsilon} = \dot{\epsilon}_1 + \dot{\epsilon}_2 = \frac{\dot{\Sigma}}{k} + \frac{\Sigma}{\mu}, \quad (1.6a)$$

that is,

$$\Sigma + \lambda \dot{\Sigma} = \mu \dot{\epsilon}. \quad (1.6b)$$

With the aim of generalizing this relation to a viscoelastic fluid, the Cauchy stress tensor is split into two parts

$$\boldsymbol{\Sigma} = -p\mathbf{I} + \boldsymbol{\Xi}, \quad (1.7)$$

where the deviatoric tensor $\boldsymbol{\Xi}$ also called the extra-stress tensor is decomposed into a viscoelastic component $\boldsymbol{\Xi}_v$ and a purely viscous component

$$\boldsymbol{\Xi} = \boldsymbol{\Xi}_v + 2\mu_0 \mathbf{D} \quad (1.8)$$

with μ_0 the Newtonian dynamic viscosity (e.g., the viscosity of the Newtonian solvent in which the polymer is diluted).

A straightforward three-dimensional extension of Eq. (1.6b) might be written as

$$\boldsymbol{\Xi}_v + \lambda \frac{d\boldsymbol{\Xi}_v}{dt} = 2\mu_1 \mathbf{D}, \quad (1.9)$$

where μ_1 characterizes the viscoelastic viscosity. Unfortunately, as will be detailed later in Chap. 4, Eq. (1.9) cannot be accepted since the total derivative of the stress tensor is not objective and would violate the Principle of Material Indifference. As will be discussed in detail later, it is necessary to introduce the upper-convective derivative, denoted by $\overset{\nabla}{\boldsymbol{\Xi}}_v$, to resolve this difficulty,

$$\overset{\nabla}{\boldsymbol{\Xi}}_v = \frac{d\boldsymbol{\Xi}_v}{dt} - (\nabla \mathbf{v}) \boldsymbol{\Xi}_v - \boldsymbol{\Xi}_v (\nabla \mathbf{v})^T. \quad (1.10)$$

The upper-convective derivative is one of the objective forms of the temporal derivative. The combination of Eqs. (1.7), (1.8) and (1.10) defines the Oldroyd-B fluid which incorporates many features of the Boger fluid that is used in experiments. For example, in steady state simple shear flow this model yields a constant viscosity, a N_1 that is quadratic in $\dot{\gamma}$ and $N_2 = 0$.

1.1.3 Viscometric Flows

In a viscometric flow, the fluid particle undergoes a history characterized by constant stretching. To investigate those flows it is necessary to introduce mathematical considerations on kinematics and of general principles on constitutive equations. This will be undertaken in later chapters, but the book by Coleman et al. (1966) constitutes a landmark of this theory. For the purposes of this introduction only the following simple nonlinear model will be considered:

$$\boldsymbol{\Sigma} = -p\mathbf{I} + K_1(I_2(\mathbf{D}), I_3(\mathbf{D}))\mathbf{D} + K_2(I_2(\mathbf{D}), I_3(\mathbf{D}))\mathbf{D}^2, \quad (1.11)$$

where K_1 , K_2 are scalar functions of the second, $I_2(\mathbf{D})$, and third, $I_3(\mathbf{D})$, invariants of the tensor $\mathbf{D}$ defined by

$$\begin{aligned} I_2(\mathbf{D}) &= \frac{1}{2} \left((tr \mathbf{D})^2 - tr(\mathbf{D}^2) \right) = -\frac{1}{2} tr(\mathbf{D}^2), \\ I_3(\mathbf{D}) &= \det \mathbf{D}, \end{aligned} \quad (1.12)$$

and where for an incompressible fluid $tr(\mathbf{D})$, which is the first invariant $I_1(\mathbf{D})$, vanishes. The symbol tr represents the trace operator. This model is the Reiner-Rivlin fluid which is a particular case of the Rivlin-Ericksen fluid of second-order. Notice that if $K_2 \equiv 0$ and $K_1 = \mu$, the Newtonian case is recovered.

1.1.3.1 Plane Shear Flow

Let us consider the simple shear flow of the Reiner-Rivlin fluid (see Eq. (1.11)) in a Cartesian orthonormal coordinate system

$$v_1 = \dot{\gamma} x_2, \quad v_2 = v_3 = 0. \quad (1.13)$$

The components D_{ij} of the tensor $\mathbf{D}$ vanish except for $D_{12} = D_{21} = \dot{\gamma}/2$, which then leads to $I_2(\mathbf{D}) = -\dot{\gamma}^2/4$ and $I_3(\mathbf{D}) = 0$. We will denote the matrices associated with tensors by their symbol in between square brackets. Therefore the matrices $[D]$, $[D^2]$ are given by

$$[D] = \begin{pmatrix} 0 & \frac{\dot{\gamma}}{2} & 0 \\ \frac{\dot{\gamma}}{2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad [D^2] = \begin{pmatrix} \frac{\dot{\gamma}^2}{4} & 0 & 0 \\ 0 & \frac{\dot{\gamma}^2}{4} & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (1.14)$$

The corresponding stress components are then

$$\Sigma_{11} = \Sigma_{22} = -p + K_2 \frac{\dot{\gamma}^2}{4}, \quad \Sigma_{33} = -p, \quad (1.15a)$$

$$\Sigma_{12} = \Sigma_{21} = K_1 \frac{\dot{\gamma}}{2} = \tau(\dot{\gamma}), \quad (1.15b)$$

$$\Sigma_{13} = \Sigma_{23} = 0, \quad (1.15c)$$

with the resulting normal stress differences being

$$N_1 = 0 \quad \text{and} \quad N_2 = K_2 \frac{\dot{\gamma}^2}{4}. \quad (1.16)$$

However, this does not correspond to the physical reality as experimental data show $N_1 \neq 0$ and $N_2 \neq 0$. We therefore need a different kind of model to resolve the right normal stress differences. It can be shown that the three functions N_1 , N_2 , τ do depend on the nature of the fluid, and are called the viscometric functions of the material.

1.1.3.2 Circular Couette Flow

The circular Couette flow occurs in the gap between two rotating concentric cylinders. The inner cylinder of radius R_1 has the angular velocity ω_1 while the outer cylinder of radius R_2 spins at ω_2 . The apparatus has a height H which is much larger than either cylinder radii so that the apparatus height is assume infinite. If we refer to a cylindrical coordinates system r, θ, z (with r, θ and z , the radial, azimuthal and axial coordinates, respectively), the steady state velocity field is such that

$$v_r = 0, \quad v_\theta = v_\theta(r), \quad v_z = 0. \quad (1.17)$$

This v_θ velocity field is then determined from the integration of the θ -momentum equation

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \Sigma_{r\theta} \right) = 0. \quad (1.18)$$

It can be shown that the velocity field for the constitutive equation (1.11) is then given by

$$v_\theta = \frac{\omega_2 R_2^2 - \omega_1 R_1^2}{R_2^2 - R_1^2} r - \frac{(\omega_2 - \omega_1) R_1^2 R_2^2}{R_2^2 - R_1^2} \frac{1}{r}. \quad (1.19)$$

In the case of a fixed outer cylinder $\omega_2 = 0$ and the velocity is given by

$$v_\theta = Ar + \frac{B}{r} = \frac{\omega_1 R_1^2}{R_2^2 - R_1^2} \left(\frac{R_2^2}{r} - r \right). \quad (1.20)$$

With the stress component

$$\Sigma_{rr} = -p + \frac{K_2}{4} \left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} \right)^2$$

the r -momentum equation

$$\frac{d \Sigma_{rr}}{dr} + \frac{1}{r} (\Sigma_{rr} - \Sigma_{\theta\theta}) = -\rho \frac{v_\theta^2}{r} \quad (1.21)$$

yields

$$-\frac{\partial p}{\partial r} + K_2 \frac{\partial}{\partial r} \left(\frac{B^2}{r^4} \right) = -\rho \frac{v_\theta^2}{r}. \quad (1.22)$$

As $\Sigma_{zz} = -p$, one obtains

$$-\Sigma_{zz} = p = K_2 \left. \frac{B^2}{r^4} \right|_{R_1}^r + \int_{R_1}^r \rho \frac{v_\theta^2}{r} dr + C \quad (1.23)$$

$$= p(R_1) + K_2 \frac{B^2}{r^4} + \int_{R_1}^r \rho \frac{v_\theta^2}{r} dr. \quad (1.24)$$

If the fluid is Newtonian, $K_2 = 0$ and the pressure increases from the inner to the outer cylinder. The fluid rises along the outer cylinder under centrifugal forces. For the non-Newtonian fluid, if $K_2 > 0$ and if B is sufficiently large under a high shear due to a small gap between the cylinders, the pressure increases when one approaches the inner cylinder and this produces the so-called rod-climbing effect.

1.2 Complex Flows

In order to study flow complexity, we will now choose simple geometries, the fluid being assumed incompressible, viscous and Newtonian. The flow phenomena in classical viscous Newtonian fluid dynamics are globally characterized via the dimensionless Reynolds number, Re , defined by the relation

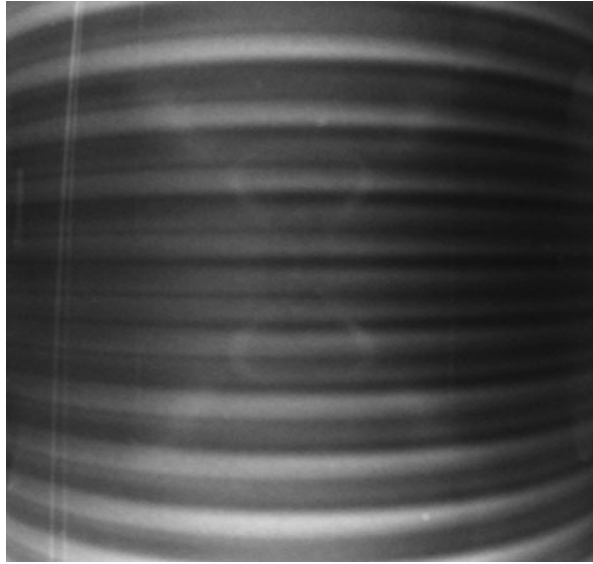
$$Re = \frac{UL}{\nu}, \quad (1.25)$$

where U and L are reference velocity and length, respectively, and $\nu = \mu/\rho$ the kinematic viscosity of the fluid expressed in $\text{m}^2 \text{s}^{-1}$ in the international unit system. The volumetric mass ρ is usually called in English texts density, but this is a misnomer as density is defined as the ratio of the volumetric mass of the considered fluid to that of water. Water density is consequently equal to 1 while its volumetric mass is $\rho = 1000 \text{ kg m}^{-3}$. For water, it is interesting to note that $\nu_{\text{water}} = 10^{-6} \text{ m}^2 \text{s}^{-1}$. If U and L are both of the order of unity, the Reynolds number will be $O(10^6)$. We will observe later that this value is typical of turbulence, one of the major topics that will be developed later.

1.2.1 Physical Considerations: Circular Couette Flow

Let us return now to the circular Couette flow. For the Couette flow it is usual to choose the inner angular velocity $U = \omega_1 R_1$ and the gap $L = R_2 - R_1$ as reference quantities for the Reynolds number definition. For creeping flows, at very low

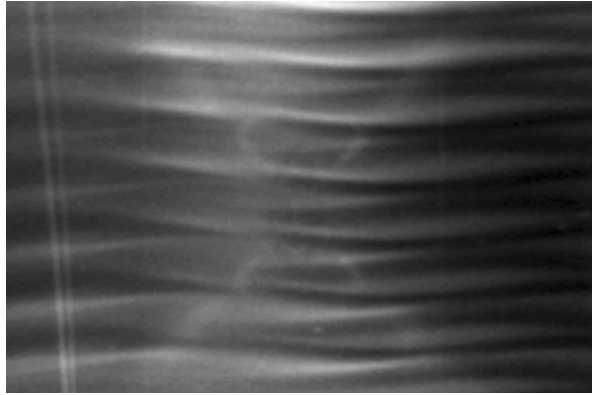
Fig. 1.4 Symmetric Taylor vortices. Courtesy of N. Borhani with permission



or vanishing Reynolds numbers, the flow is laminar. This indicates that the material fluid particles follow closed circles centered on the rotation axis. This situation is indeed simple and experimentally verified. When the Reynolds number increases, a first transition occurs and secondary flows are generated in the meridian r, z planes. They are superposed to the former Couette flow and we arrive at the situation of the Taylor vortices discovered in 1923 (Taylor 1923). Figure 1.4 shows the toroidal Taylor vortices pattern which is axisymmetric and steady-state, i.e. time independent. These vortices appear in counter rotating pairs and they are characterized by k_z , the axial wavenumber. The value at which the transition occurs is called the critical Reynolds number and will be denoted by Re_c . For example, Coles (1965) reports on a Couette flow where $\omega_2 = 0$ and the ratio of the radii $\eta = R_2/R_1 = 0.87$. For $Re < 41.5$ the flow is laminar and given by Eq. (1.19). The first critical Reynolds number $Re_{c,1}$ is reached at 42. Increasing the inner cylinder rotation, for $Re_{c,2} = 66$, a second transition occurs and the Taylor toruses are deformed in the azimuthal direction with k_θ the corresponding wavenumber. Figure 1.5 displays the steady-state wavy Taylor vortices pattern. Still increasing the Reynolds number, the geometrical vortices pattern is affected through changes in k_θ and k_z . At some Reynolds number value later transitions modify the number of vortex pairs and k_z accordingly in the axial direction, and the undulations of the azimuthal deformations, i.e. k_θ . These flows remain steady state.

The aspect ratio $\Gamma = H/(R_2 - R_1)$ has a significant influence on the physics of the Couette flow. Benjamin (1978) treats the case of short Couette cylinders with the end effects modifying the observations made on long devices. Koschmieder (1993) reviews the Taylor vortices state-of-the-art in the early nineties indicating the subsequent route to turbulence.

Fig. 1.5 Wavy Taylor vortices. Courtesy of N. Borhani with permission



1.2.2 Transitional Flows

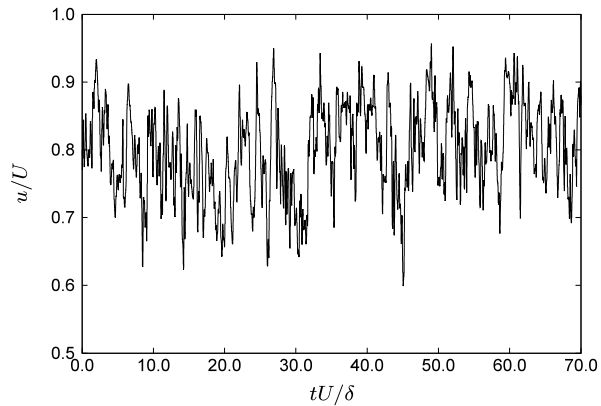
It is difficult to quantify what is meant by transition. Broadly speaking, it is characterized by unsteady motion accompanied by a spectral broadening, in space and time, of the fluid motion. Flows that do transition between the laminar and turbulence state do have identifiable characteristics that mark the process.

For the Taylor-Couette flow, one can adopt the criterion that the flow will be in transition as soon as it becomes unsteady with or without small scale structures. Fenstermacher et al. (1979) carried out an experimental investigation of the time-dependent Taylor vortex and wavy vortex flows. Inspection of the velocity spectra showed that the first transition was a Hopf bifurcation, giving in phase space a limit cycle with a clearly identified period. The second transition was a quasi-periodic flow with two frequencies irrationally related. For higher Reynolds number, the sharp components and their subharmonics disappear and one gets a spectrum with a broad component and a background continuum. This feature is characteristic of weak turbulence also known as chaos, a theory that comes from nonlinear dynamics.

For flat plate boundary layer flows, the downstream transition to the turbulent state involves a sequence of linear to nonlinear instability mechanisms with an ever increasing broadening of the space-time spectral characteristics. Two types of transitions can occur (Schmid and Henningson 2001). One is a natural transition where unsteadiness in the oncoming flow or irregularities in the solid surface initiate this sequence of instabilities through a receptivity process. The other is a by-pass transition where strong disturbances (turbulence) in the oncoming flow void the naturally occurring sequence of instability and cause a sudden transition to the fully turbulent state on the plates. This latter type of transition often occurs in turbomachinery applications. Other factors significantly contribute to the detailed features of transitional flows including effects of pressure gradients, surface temperature, and Mach number.

The analysis and prediction of transitional flows has not developed as in the turbulent flow case. The formalisms have evolved from the study of the dynamics of

Fig. 1.6 Time variation of instantaneous velocity field at a fixed point within a turbulent boundary layer: U mean free-stream velocity and δ boundary layer thickness



deterministic disturbances rather than from the study of the dynamics of statistical quantities with associated probability density functions. In the latter case, constitutive equations are needed for higher-order (unknown) correlations. For this reason, the discussion of transitional flows will be minimal in the remainder of the text.

1.2.3 Turbulent Flows

Turbulence can be generated at (moderately) high value of the Reynolds number such that for many engineering flows, these values can range from $O(10^4 \dots 10^6)$. Turbulence is rotational and three-dimensional, and is characterized by a high level of vorticity fluctuations. This is the reason why vorticity dynamics plays such an essential role in turbulence theory. Its origin comes from the flow instabilities engendered inside the governing equations by the interaction between the viscous and nonlinear terms of the acceleration. This interaction intensifies as the inertial effects dominate, that is for sufficiently high Reynolds numbers.

All turbulent flows have some general characterizing features that have varying degrees of prominence depending on particular conditions relative to each flow. The transient character of turbulent flows are never steady-state. They are irregular in space and time and present a random aspect (see Fig. 1.6) with an associated probability distribution. While all turbulent flows are unsteady, their statistical characteristics are not necessarily so. Such statistically steady, or stationary, flows are not uncommon and occur in many free shear and wall-bounded flows. Another important characteristic of a turbulent flow is the enhanced transport of energy brought on by the fluctuating velocity field. This enhanced transport implies high momentum and heat transfer, and brings a significant increase in passive scalar mixing. Such flow fields are important in materials processing in order to obtain better quality for the end product. A third feature is the dissipative character of the turbulence. Viscous shear stresses produce deformation work that increases the internal energy of the fluid at the expense of the turbulent kinetic energy. The turbulence will decrease

with time if energy is not provided to the fluid in order to counteract the friction losses which are more important than in the laminar case.

It is to be understood that turbulence is not an intrinsic feature of the fluid mechanical behavior, but is linked to the flow properties of the fluid. Thus, while in the description of complex (viscoelastic) fluids constitutive relations are necessary to characterize the fluid behavior, in the description of complex (turbulent) flows constitutive relations are necessary to characterize the flow behavior. This statement is a fundamental point in the developments to be presented and discussed throughout the book.

1.3 Elastic Turbulence

This topic has emerged about a decade and a half ago with the review paper of Shaqfeh (1996) on elastic instabilities in viscoelastic flows. At the beginning of this millennium Groisman and Steinberg (2000) showed experimentally in a plate-plate geometry that the velocity spectra were quite similar to the ones encountered in turbulence. This phenomenon was named elastic turbulence since it displayed characteristics of turbulence without dominant inertia since it occurred at very low Reynolds numbers. Groisman and Steinberg (2004) also confirmed the presence of elastic turbulence in curvilinear flows. In these situations the key parameter is the Weissenberg number, We , defined by the relation

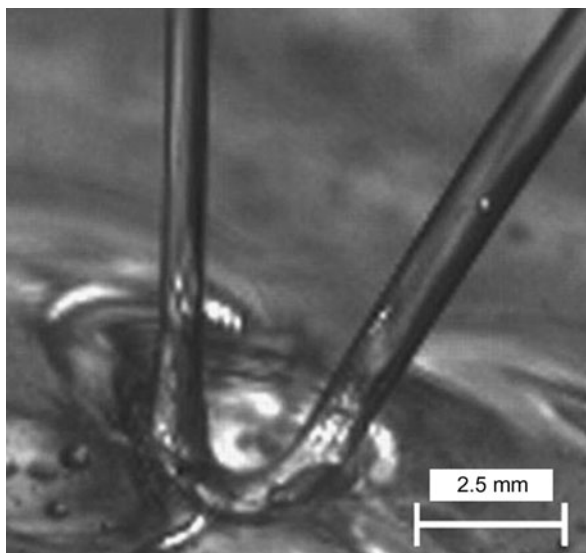
$$We = \frac{\lambda U}{L}, \quad (1.26)$$

which plays a similar role as the Reynolds number for Newtonian fluids. The Weissenberg number measures the elasticity of the fluid as λ is the material relaxation time of the viscoelastic fluid and L/U is the inertial time of the flow ($We = 0$ corresponds to a Newtonian fluid). With increasing We , the flow physics changes drastically and the flow phenomena resembles classical turbulence. In the non-Newtonian case, the turbulence is built up by the elastic stresses that produce small-scale structures even at low Reynolds number. This is a consequence of the presence of the nonlinearities in constitutive equations such as the Oldroyd-B fluid. The elastic stresses induce anisotropy of the flow by the normal stress differences that cause the curvature of the streamlines. This mechanism leads to instability where some regions of high-shear generate and store locally elastic energy. By flow advection, this energy is transported to low shear zones where it enhances turbulence production. Recently Morozov and van Saarloos (2007) have assessed the state-of-the-art in a review (the authors call it an essay) about this complex physical phenomenon.

1.4 Examples of a Complex Fluid and Flow

To briefly illustrate the interesting type of phenomena that can be observed in studying complex fluids and flows, rebounding jets generated by an incoming jet falling

Fig. 1.7 Kaye effect: the geometry of the dimple inside the heap²



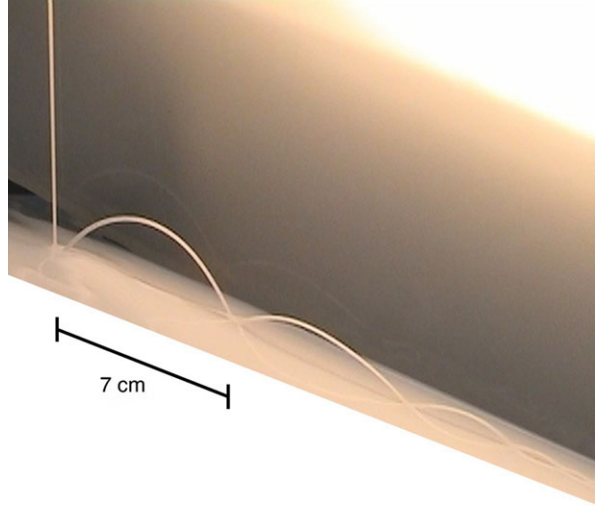
under gravity onto a bath made of the same liquid is shown. Two experiments have been investigated recently that yield similarly observed unique phenomena but with different dynamic mechanisms. The first one is the Kaye effect described in Kaye (1963), and the second one is a bouncing jet using a very common fluid—cooking oil.

1.4.1 The Kaye Effect: Shear-Thinning Evidence

In Versluis et al. (2006), the Kaye effect is revisited and explained. Indeed Kaye noticed that when shampoo is poured down from a height of ~ 20 cm on a thin layer of the same fluid, a viscous heap is formed as it would occur if one were using honey or silicone oil. After a very short time (< 1 s), the incoming jet will slip away from the heap, leading to the formation of a lasso type ring. Because shampoo is a shear-thinning non-Newtonian fluid, the departing jet creates a low viscosity layer that expels the jet at low inclination. At the same time, the falling jet exerts a vertical force that affects the shape of the heap where a dimple is formed. The dimple gets deeper and this geometrical change induces that the lower part of the lasso sticks to the thin bath and the outgoing jet straightens. Figure 1.7 illustrates the formation of the dimple in the heap. Observe in the figure that the lasso disappeared and the radii of the incoming and outgoing jets are clearly different. At some point the outgoing

²Figures 1.7, 1.8, 1.9 reprinted with permission from: M. Versluis, C. Blom, D. van der Meer, K. van der Weele, D. Lohse (2006) Leaping shampoo and the stable Kaye effect. J. Stat. Mech. P07007.

Fig. 1.8 Kaye effect:
influence of a slanted wall



jet will reach the vertical jet in such a way that this disturbance may stop the Kaye effect. (The cited reference contains enticing videos of the phenomena.) Figure 1.8 exhibits the several bounced jets when they hit a thin layer on a slanted wall.

It is interesting to contrast that in Collyer and Fischer (1976), elasticity of the fluid was invoked to explain the physics of this flow; however, much later in Versluis et al. (2006) only shear-thinning viscosity arguments come into play. The experiments nevertheless show that the outgoing jet has a radius such that $R_{out} > R_{in}$. Therefore, the velocity of the outgoing jet is smaller than that of the incoming jet as global mass conservation enforces $V_{in}R_{in}^2 = V_{out}R_{out}^2$. The main reason for this decreasing velocity is the viscous dissipation in the dimple. Inspection of Fig. 1.9 shows that the jet in the dimple is in contact with the resting heap through a very thin shear layer of thickness δ . There the velocity profile may be taken as linear giving a constant shear rate $\dot{\gamma} = V/\delta$. The shear stress in this layer is given by (1.3) such that $\Sigma = \mu\dot{\gamma}$ ($\eta = \mu$ in the figure). The viscosity is chosen to follow Crow's relation

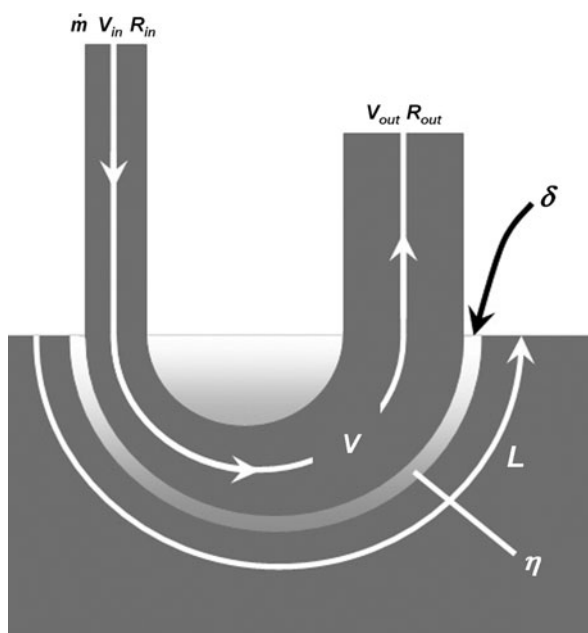
$$\mu(\dot{\gamma}) = \mu_{\infty} + \frac{\mu_0 - \mu_{\infty}}{1 + (\dot{\gamma}/\dot{\gamma}_c)^n}, \quad (1.27)$$

where μ_0 and μ_{∞} are the zero-shear-rate and infinite-shear-rate viscosities, respectively, and $\dot{\gamma}_c$ is the critical shear rate. Versluis et al. (2006) found in their experiments $n = 1$ and $\dot{\gamma} \gg \dot{\gamma}_c$, so that the viscosity relationship could be approximated as

$$\mu(\dot{\gamma}) = \mu_{\infty} + \frac{\mu_0\dot{\gamma}_c\delta}{V}. \quad (1.28)$$

A simple line of reasoning equating dissipation to the rate of change of kinetic energy provides a differential equation for the velocity that can be integrated over the interaction length L between the jet and the heap. From this equation, one obtains the threshold velocity of the incoming jet that ensures stability of the Kaye effect.

Fig. 1.9 Kaye effect: sketch of the flowing jet through the dimple



The developed model takes only shear-thinning into account, and obviously no elasticity is required to explain the observed physical phenomena.

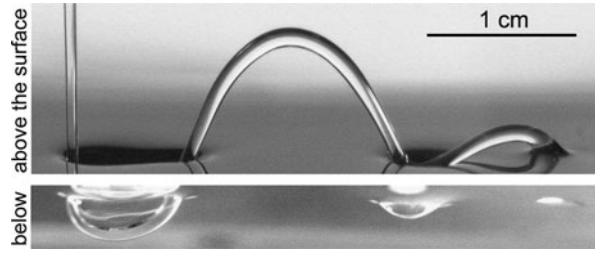
1.4.2 Bouncing Newtonian Jet

Thrasher et al. (2007) have generated Newtonian bouncing jets when the incoming jets impact a thin, moving bath (see Fig. 1.10). The observed bouncing phenomenon is quite similar to the Kaye effect except that the liquids are typical classical Newtonian fluids (cooking oil). The observations show that the bouncing jet is separated from the surrounding bath by a sheath of air, while in the stable Kaye effect a shear-thinning layer played the lubricating role. Some energy arguments based on drag force of the impinging jet and the interfacial forces on the bouncing jet shed some light on the mechanisms involved in the transition between plunging and bouncing jets. Nonetheless, the physical situation is dynamically complex due to the interplay of viscous, inertial, surface and gravitational forces.

1.4.3 Turbulent Drag Reduction

Since the pioneering experiments by Toms (1949), it is known that the addition of minute amounts of long chain polymers to water or organic solvents can lead

Fig. 1.10 Newtonian bouncing jet on a moving bath³



to significant turbulent drag reduction. However, it has only been within the last decade and a half that computational resources have been available to make it possible to perform (direct) numerical simulations of this phenomenon. As Fig. 1.11 illustrates, the fine-scale turbulence affects the polymer molecule and induces an extensive elongation of the molecular chain. The extended molecule then has an impact on the turbulent energy cascade which reduces the drag. The exact details of this interaction is as yet not known in detail.

The first such simulations considered a generalized Newtonian fluid (den Toonder et al. 1995; Orlandi 1995) in pipe and plane channel flow. Sureshkumar et al.

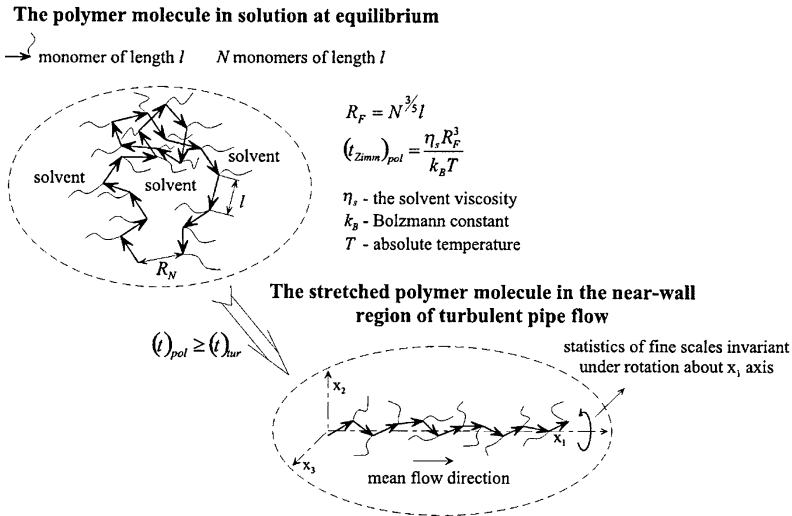


Fig. 1.11 Possible behavior of long chain macromolecule in proximity of wall in turbulent flow⁴

³Figure 1.10 reprinted with permission from: M. Thrasher, S. Jung, Y.K. Pang, C.-P. Chuu, H. Swinney (2007) The bouncing jet: a Newtonian liquid rebounding off a free surface. Phys. Rev. E 76:056319. Copyright 2007 by the American Physical Society.

⁴Figure 1.11 reprinted with permission from: J. Jovanović, M. Pashtapanska, B. Frohnäpfel, F. Durst, J. Koskinen, K. Koskinen (2006) On the mechanism responsible for turbulent drag reduction by dilute addition of high polymers: theory, experiments, simulations and predictions. J. Fluids Eng. 128:118–130.

(1997) were the first to perform direct numerical simulations (DNS) of drag reduction with a viscoelastic fluid and used the FENE-P model (Finitely Extensible Non-linear Elastic in the Peterlin approximation) (see also Sureshkumar and Beris 1995; Dimitropoulos et al. 1998, 2001). More recently, other direct numerical simulations have been carried out (primarily with FENE-P fluid) (Housiadas and Beris 2003; Ptasiński et al. 2003; Dubief et al. 2004; Dimitropoulos et al. 2005, 2006) in order to better understand the drag reduction dynamics.

As with DNS of Newtonian fluids the numerical requirements for viscoelastic DNS makes them only amenable at relatively modest Reynolds numbers on large scale supercomputers. From the engineering point of view, it is necessary to envisage closures for viscoelastic turbulent flows. In this spirit, there has been recent attempts towards RANS (Reynolds Averaged Navier-Stokes) modeling (Cruz et al. 2004; Li et al. 2006; Pinho et al. 2008b). These models were generally low-order models, that is one- or two-equation linear eddy viscosity models. Unfortunately, such low-order models cannot properly capture anisotropy effects that certainly may be prevalent in drag reducing polymer flows due to the inherent viscoelastic effects.

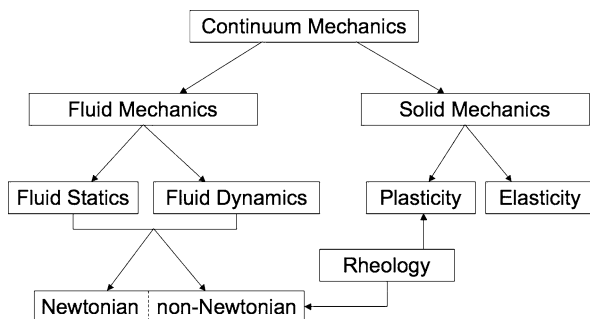
An intermediate approach between direct numerical simulation and the Reynolds-averaged formulation, large eddy simulations (LES), has recently been applied to viscoelastic turbulent flows (Thais et al. 2010). This approach was based on a temporal formulation rather than the traditional spatial filtering approach. Nevertheless, the study showed that such a filtered methodology can be applied to viscoelastic fluids, but new considerations, such as differing filter widths associated with the velocity and extra-stress fields may need to be considered.

These studies have provided additional insight into the polymer-turbulent dynamics that include the effects on vortex stretching due to the high extensional viscosity. In addition, a change in the mean flow distribution across the boundary layer has been observed depending on the extent of drag reduction. At low drag reduction ($DR < 40\%$), the log law region is simply moved away from the wall; however, at high drag reduction, the log region is again moved away from the wall but its slope is increased with respect to the Newtonian flow.

1.5 The Modeling Map

This introductory chapter was intended to provide the reader with some context and physical examples of what was meant by complexity. To achieve our goals of modeling those complicated situations, it is necessary to build up in an axiomatic way a methodology capable of embracing all those physical phenomena. The concepts of continuum mechanics developed over the last half-century will help in that goal. From continuum mechanics (cf. Fig. 1.12), we derive fluid mechanics that covers both statics and dynamics of the fluids. The mechanical behavior of the fluids may be described as Newtonian or non-Newtonian. In this last case the modeler resorts also to rheology which is the science of “everything that flows”, even mountains according to the prophet(ess) Deborah (see Reiner 1964 and Chap. 5). The complex

Fig. 1.12 Filiation of continuum mechanics

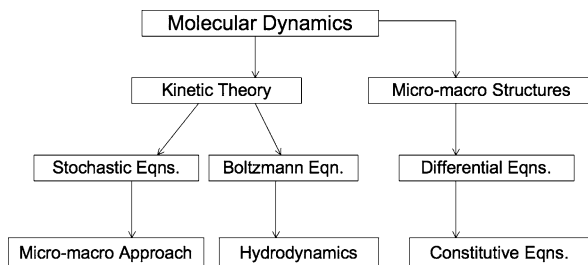


flows to be discussed are turbulent. Even with just the turbulent flow of a Newtonian fluid, the subject is not easily managed since external forces (e.g. buoyancy, magnetic fields, etc.) can be applied that make the complexity even greater. However, strides in the last decade have been made where numerical simulations of the fluid motion conservation equations coupled with viscoelastic constitutive equations have been made. This inherent link then between the complex fluid and complex flow can now be further explored.

Another route to generate the models for complex fluids starts from molecular dynamics (see Fig. 1.13) and goes through two possible branches. The left branch in Fig. 1.13 is based on kinetic theory (that can also be produced from continuum mechanics concepts). Kinetic theory mainly considers particles colliding and streaming and is primarily applied to gases. The random character of the particle motions is described by stochastic equations like the Fokker-Planck equations. These equations engender the micro-macro approach where the basic conservation laws of continuum mechanics are used, and the stress tensor is obtained from an ensemble average of solutions of the stochastic equations. An alternate choice consists in deriving the Boltzmann equation from kinetic theory and from there it is possible by the Chapman-Enskog analysis to build the classical hydrodynamic system within the framework of the lattice Boltzmann method (LBM). The right branch examines in detail the micro-macro structures of the fluid and rests upon the concept of dumbbells. Differential equations are then deduced and lead to the writing of constitutive equations involving partial differential operators.

Chapter 2 provides the reader with the tensorial tools in exploring the various models. Invariant theory and integrity bases will play an essential role in the design of the representation of tensor-valued functions. Chapter 3 will develop the basics of kinematics and dynamics of continua with conservation laws of mass, momentum, moment of momentum obtained. Chapter 4 yields the general principles to write the constitutive equations that relate the stress tensor to the other relevant variables as pressure, velocity gradients, etc. In addition, a section will be devoted to the principle of objectivity and its consequences. In Chap. 5 we start by the simple fluids model. Then we address the nonlinear constitutive equations: Oldroyd-B, Maxwell, and Phan-Thien Tanner. The popular Finite Extensible Nonlinear Elastic (FENE) models that are based on the representation of long polymer chains by dumbbells is also discussed. These developments will be linked to the so-called micro-macro ap-

Fig. 1.13 Filiation of molecular dynamics



proach. Polymer melts are treated by the Doi-Edwards model, the (extended) pom-pom approach and finally the Rolie-Poly equation. For the turbulent case, the closure problem is addressed with Chap. 6 then devoted to averaged and filtered conservation laws. The Reynolds-averaged Navier-Stokes equations, the large eddy simulation, the various explicit algebraic models will be discussed. The final chapter will bring another view of the Navier-Stokes equations by the lattice Boltzmann method (LBM) where virtual particles collide and stream. The Chapman-Enskog multi-scale analysis leads to the compressible Navier-Stokes equations at low Mach number. This LBM will be put in perspective with the continuum mechanics methodology to emphasize the broad scope of the latter.

At the end of this intellectual trip through the mathematical models aimed at the understanding and prediction of the complicated physics of fluid behavior and intricate flows, the major challenge and task that remain are to solve them. As non-linearity dominates these models, there is little hope to build up analytical solutions in closed forms. This imposes linearization procedures that are severely restricted by convergence assumptions. The resort to numerical simulation is therefore unavoidable and the numericist faces the choice of the discretization method. There is no doubt that the former century brought to the scientific computing community the weak formulation of the problem as the essential tool for high performance computing. The availability of huge parallel machines and the speed-up provided by the use of graphics processing units opens the door to unknown territory of simulations that practitioners could not dream of until recently.

The production of this myriad of numerical databases calls for the development of new tools to accurately and deeply analyze the results. Plots of vector fields are not enough and new routes based on sound models are needed. Furthermore, the comparison of theory and computations with experimental data is also a source of concern. This triad forms the tripod of modern science and engineering in a feedback loop that endlessly improves each discipline.

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