

Contents

Part I Structure-Tensor Computation

Structure Tensor Estimation: Introducing Monomial Quadrature Filter Sets	3
Hans Knutsson, Carl-Fredrik Westin, and Mats Andersson	
1 Introduction.....	4
1.1 This Chapter Presents.....	4
2 Monomial Filters.....	5
2.1 Radial Part.....	5
2.2 Directional Matrix.....	6
2.3 Monomial Filter Matrices.....	7
3 Monomials Link Order, Scale and Gradients.....	9
3.1 The Gradient Operator Increases Order and Shifts Scale.....	11
4 Monomial Filter Response Matrices.....	12
4.1 Signal Classes.....	13
5 Monomial Structure Tensors.....	15
5.1 Two Simple Examples.....	15
5.2 General Structure Tensor Construction.....	16
5.3 Monomial Quadrature.....	17
5.4 Phase Invariance.....	17
5.5 Tensor Positivity.....	17
6 Structure Tensor Variations.....	18
6.1 The Structure Tensor, \mathbf{T}_q	18
6.2 The Gradient Tensor, \mathbf{T}_G	19
6.3 The Boundary Tensor, \mathbf{T}_B	19
6.4 The Energy Tensor, \mathbf{T}_E	20
6.5 Gradient Energy Tensor, \mathbf{T}_{GE}	20
6.6 Spatial 2nd Order Polynomial Tensor.....	20
6.7 Spherical Harmonics.....	21
6.8 Sum of Monomial Tensors.....	21
7 Table of Structure Tensor Related Algorithms.....	21

8	Higher Order Structure Tensors	22
8.1	Non-simple Signals	23
9	Conclusion	24
	References	24
	Adaptation of Tensor Voting to Image Structure Estimation	29
	Rodrigo Moreno, Luis Pizarro, Bernhard Burgeth, Joachim Weickert, Miguel Angel Garcia, and Domenec Puig	
1	Introduction	30
2	Tensor Voting	32
2.1	<i>Stick</i> Tensor Voting	33
2.2	The <i>Plate</i> Tensor Voting	34
2.3	The <i>Ball</i> Tensor Voting	35
3	Relationships Between the Structure Tensor and Tensor Voting	35
3.1	Similarities	35
3.2	Differences	36
4	Tensor Voting for Structure Estimation	37
4.1	Gray-Scale Images	37
4.2	Color and Vector-Valued Images	38
4.3	Tensor-Valued Images	39
5	Experimental Results	42
6	Concluding Remarks	47
	References	48
	Edge-Enhancing Diffusion Filtering for Matrix Fields	51
	Bernhard Burgeth, Luis Pizarro, and Stephan Didas	
1	Introduction	52
2	Edge-Enhancing Diffusion	54
3	Basic Differential Calculus for Matrix Fields	55
4	The Generalised Structure Tensor \overline{S}_G for Matrix Fields	57
4.1	A Diffusion Tensor \overline{D} for Matrix Fields	58
5	Edge-Enhancing Diffusion Filtering for Matrix Fields	58
6	Numerical Issues	59
7	Experiments	60
8	Concluding Remarks	66
	References	66
	Part II Tensor-Field Visualization	
	Fabric-Like Visualization of Tensor Field Data on Arbitrary Surfaces in Image Space	71
	Sebastian Eichelbaum, Mario Hlawitschka, Bernd Hamann, and Gerik Scheuermann	
1	Motivation and Related Work	72
2	Method	73
2.1	Initial Noise Texture Generation	74

2.2	Projection Step	75
2.3	Silhouette Detection	77
2.4	Advection.....	77
2.5	Compositing	79
2.6	Postprocessing	79
2.7	Implementation.....	83
3	Results	84
3.1	Artificial Test Data Sets.....	85
3.2	Modification for Medical Data Processing.....	86
3.3	Mechanical Datasets	87
3.4	Performance	88
4	Conclusions and Possible Directions for Future Research	89
	References	90

Beyond Topology: A Lagrangian Metaphor to Visualize the Structure of 3D Tensor Fields..... 93

Xavier Tricoche, Mario Hlawitschka, Samer Barakat, and
Christoph Garth

1	Introduction and Motivation	94
2	Related Work	95
2.1	Topological Methods.....	95
2.2	Ridges and Valleys	96
3	Theory	97
3.1	Tensor Field Topology	97
3.2	Crease Manifolds in Tensor Fields	98
3.3	Lagrangian Coherent Structures.....	99
4	A Lagrangian Model of Structure in Tensor Fields.....	100
4.1	An Extension of LCS to Tensor Fields.....	100
4.2	Computation.....	102
5	Results	103
6	Conclusion and Future Work	107
	References	108

Tensor Field Design: Algorithms and Applications 111

Eugene Zhang

1	Introduction.....	111
2	Applications of Tensor Field Design	112
2.1	Vector Field Design	112
2.2	Second-Order Tensor Field Design	114
2.3	Higher-Order Tensor Field Design	115
2.4	Requirements.....	117
3	First-Order Tensor Field Design.....	117
4	Second-Order Tensor Field Design.....	121
5	Higher-Order Tensor Field Design	124
6	Tensor Field Design on Manifold Surfaces	126
7	Visualization.....	128

8 Conclusion 129

References 131

**Part III Applications of Tensor-Field Analysis
and Visualization**

**Interactive Exploration of Stress Tensors Used in
Computational Turbulent Combustion 137**

Adrian Maries, Md. Abedul Haque, S. Levent Yilmaz, Mehdi B.
Nik, and G. Elisabeta Marai

1 Introduction 138

2 Tensors in Turbulent Combustion 139

2.1 Turbulent Combustion Modeling 139

2.2 Challenges 142

3 Related Work 143

4 Methods 145

4.1 Datasets 145

4.2 Glyph Representation 146

4.3 Volume Rendering and Streamlines 147

4.4 Interactive Filtering 149

5 Results and Discussion 151

6 Conclusion 153

References 154

**Shear Wave Diffusion Observed by Magnetic Resonance
Elastography 157**

Sebastian Papazoglou, Jürgen Braun, Dieter Klatt, and Ingolf Sack

1 Introduction 157

2 Theory 159

3 Methods 161

4 Results 163

5 Discussion and Conclusions 166

References 167

Part IV Diffusion Weighted MRI Visualization

**A Comparative Analysis of Dimension Reduction Techniques
for Representing DTI Fibers as 2D/3D Points 171**

Xiaoyong Yang, Ruiyi Wu, Zíáng Ding, Wei Chen, and Song
Zhang

1 Introduction 171

2 Background 172

3 Dimension Reduction Methods 173

3.1 Multidimensional Scaling (MDS) 174

3.2 Locally Linear Embedding (LLE) 175

3.3	Principal Component Analysis (PCA)	175
3.4	IsoMap (Isometric Feature Mapping)	176
4	Experiment	176
4.1	Data	177
4.2	Method.....	177
4.3	Results	179
4.4	Discussion on Dimension Reduction.....	179
5	User Interface.....	181
6	Conclusions.....	183
7	Implementation	184
	References	184
	Exploring Brain Connectivity with Two-Dimensional Maps	187
	Çağatay Demiralp, Radu Jianu, and David H. Laidlaw	
1	Introduction	187
2	DWI	189
3	Related Work	190
4	Methods.....	191
4.1	Image Acquisition and Fiber Tract Generation	191
4.2	Measuring Similarities Between Fiber Tracts	192
4.3	Clustering.....	192
4.4	Planar Projections of Fiber Tracts	193
4.5	Linked Multi-view Interaction.....	196
4.6	Digital Map Interface	197
4.7	Implementation.....	200
5	User Evaluation	200
5.1	Anecdotal Study: Methods and Results	200
5.2	Quantitative Study	201
6	Discussion	203
7	Conclusions.....	205
	References	206
	Uncertainty Propagation in DT-MRI Anisotropy Isosurface	
	Extraction	209
	Kai Pöthkow and Hans-Christian Hege	
1	Introduction	209
2	Related Work	211
3	Methods.....	212
3.1	Uncertainty Model	212
3.2	Signal to Noise Ratio	212
3.3	Condition Numbers	213
3.4	Uncertainty Propagation	215
3.5	Uncertain Isosurfaces	216
3.6	Visualization.....	216
4	Results	217
4.1	Synthetic DTI Data	217
4.2	Brain DTI Data.....	217

5 Discussion and Conclusions	218
Appendix	222
References	224

Part V Beyond Second-Order Diffusion Tensor MRI

Classification Study of DTI and HARDI Anisotropy Measures for HARDI Data Simplification	229
Vesna Prčkovska, Maxime Descoteaux, Cyril Poupon, Bart M. ter Haar Romeny, and Anna Vilanova	
1 Introduction	230
2 Related Work	233
3 Diffusion Data Acquisition	234
4 Methods	235
4.1 HARDI Measures	235
4.2 DTI Measures	237
4.3 Analysis of Measures	237
4.4 Real Data Analysis	240
5 Results	240
5.1 Phantom Results	240
6 Discussion and Conclusions	246
Appendix	249
References	249

Towards Resolving Fiber Crossings with Higher Order Tensor Inpainting	253
Thomas Schultz	
1 Introduction	253
2 Related Work	254
3 Higher-Order Tensor Voting	255
3.1 Basics of Tensor Voting	255
3.2 Introducing Higher-Order Tensors	256
3.3 Formalizing the Voting Process	257
3.4 Analyzing the Accumulated Votes	258
4 Inpainting as a Preprocess for Tractography	259
5 Results	260
5.1 Results on Synthetic Data	260
5.2 Result on Real Data	261
5.3 Distinguishing Crossings from Junctions	261
6 Conclusion and Future Work	263
References	263

Representation and Estimation of Tensor-Pairs	267
Carl-Fredrik Westin and Hans Knutsson	
1 Introduction	268
2 Tensor-Pair Representation	268
2.1 Representing an Un-Ordered Pair of Vectors	268

2.2	Reconstruction of the Two Vectors from the Representation	269
3	Mean Vector-Pair Estimation from Distributions	271
3.1	Mean Vector-Pair Estimation from a Distribution of Vector-Pairs.....	271
3.2	Mean Vector-Pair Estimation from a Distribution of Vectors	272
3.3	Estimating the Amount of Samples from \mathbf{u} and \mathbf{v}	274
4	Tensor Neighborhoods	274
5	Neighborhood Averaging	275
5.1	Normalized Convolution.....	276
6	Experiments and Results	276
6.1	Two-Tensor Field Neighborhoods.....	277
6.2	Single-Tensor Field Neighborhoods	277
7	Discussion and Conclusion	278
	References	280

Part VI Tensor Metrics

On the Choice of a Tensor Distance for DTI White Matter

Segmentation

283

Rodrigo de Luis-García, Carlos Alberola-López, and Carl-Fredrik Westin

1	Introduction.....	284
2	Tensor Similarity Measures	286
2.1	Frobenius Distance.....	286
2.2	Kullback-Leibler Distance.....	287
2.3	Information Geodesic Distance.....	288
2.4	Log-Euclidean Metrics.....	288
2.5	Hybrid Distance	289
3	Tensor Distances Evaluation	290
3.1	DT-MRI White Matter Atlases	290
3.2	Tensor Distances Evaluation.....	291
4	Results and Discussion.....	293
5	Summary.....	303
	References	304

Divergence Measures and Means of Symmetric Positive-Definite Matrices

307

Maher Moakher

1	Introduction.....	307
2	Bhattacharyya Divergence	310
2.1	Bhattacharyya Divergence-Based Mean	313
3	Modified Bhattacharyya Divergence	315
3.1	Mean Based on the Modified Bhattacharyya Divergence	317
4	Kullback-Leibler Divergence	317
4.1	Kullback-Leibler Divergence-Based Mean	319

5	Conclusion	320
	References	320
	Metric Selection and Diffusion Tensor Swelling	323
	Ofer Pasternak, Nir Sochen, and Peter J. Basser	
1	Introduction	323
2	Riemannian Metrics for Diffusion Tensors	324
2.1	The Euclidean and Geometric Metrics	325
2.2	Metric Selection	326
3	Determinant Versus Trace	326
3.1	Physical Considerations	327
4	Tensor Swelling	329
4.1	Variability Caused by Johnson Noise	329
4.2	The Extent of Tensor Shrinking in the Brain	331
4.3	Why Do Tensors Swell?	333
5	Beyond Riemannian Metrics	334
6	Summary	335
	References	335

Part VII Tensor Analysis

	\mathcal{H}^2-Matrix Compression	339
	Steffen Börm	
1	Overview	339
2	\mathcal{H}^2 -Matrix Representation	341
3	Compression	346
4	Improvements	356
4.1	Data-Sparse Input Matrices	357
4.2	Refined Error Control	358
4.3	Vector- or Matrix-Valued Matrices	359
4.4	Three-Dimensional Data	360
5	Summary	360
	References	361

	Harmonic Field Analysis	363
--	--------------------------------------	------------

Christian Wagner, Christoph Garth, and Hans Hagen

1	Introduction	363
2	Related Work	364
3	Harmonic Analysis	365
3.1	Fourier Decomposition	365
3.2	Spectral Theorem	366
3.3	Discrete Setting	367
3.4	Arbitrary Domain and Field Type	367
3.5	Global Harmonic Analysis	367
4	Local Harmonic Analysis	369
4.1	Locality and Local Feature Definition	369

5 Discretizations and Computational Issues 371

5.1 Finite Element Discretization..... 371

5.2 Discrete Exterior Calculus (DEC) Discretization 372

5.3 Comparison of FEM and DEC Discretizations 374

5.4 Computation of Large Eigenvalue Sets 376

6 Conclusion 377

References 378

Index..... 381

New Developments in the Visualization and Processing
of Tensor Fields

Laidlaw, D.H.; Vilanova, A. (Eds.)

2012, XXI, 384 p. 153 illus., 129 illus. in color.,

Hardcover

ISBN: 978-3-642-27342-1