# Preface

Impulsive differential equations are suitable for the mathematical simulation of evolutionary processes in which the parameters undergo relatively long periods of smooth variation followed by a short-term rapid changes (i.e., jumps) in their values. Processes of this type are often investigated in various fields of science and technology.

The question of the existence and uniqueness of almost periodic solutions of differential equations is an age-old problem of great importance. The concept of almost periodicity was introduced by the Danish mathematician Harald Bohr. In his papers during the period 1923–1925, the fundamentals of the theory of almost periodic functions can be found. Nevertheless, almost periodic functions are very much a topic of research in the theory of differential equations. The interplay between the two theories has enriched both. On one hand, it is now well known that certain problems in celestial mechanics have their natural setting in questions about almost periodic solutions. On the other hand, certain problems in differential equations have led to new definitions and results in almost periodic functions theory. Bohr's theory quickly attracted the attention of very famous researchers, among them V.V. Stepanov, S. Bochner, H. Weyl, N. Wiener, A.S. Besicovitch, A. Markoff, J. von. Neumann, etc. Indeed, a bibliography of papers on almost periodic solutions of ordinary differential equations contains over 400 items. It is still a very active area of research.

At the present time, the qualitative theory of impulsive differential equations has developed rapidly in relation to the investigation of various processes which are subject to impacts during their evolution. Many results on the existence and uniqueness of almost periodic solutions of these equations are obtained.

In this book, a systematic exposition of the results related to almost periodic solutions of impulsive differential equations is given and the potential for their application is illustrated.

Some important features of the monograph are as follows:

1. It is the first book that is dedicated to a systematic development of *almost periodic theory* for *impulsive differential equations*.
2. It fills a void by making available a book which describes existing literature and authors results on the *relations* between the *almost periodicity* and *stability* of the solutions.
3. It shows the manifestations of *direct constructive methods*, where one constructs a uniformly convergent series of almost periodic functions for the solution, as well as of *indirect methods* of showing that certain bounded solutions are almost periodic, by demonstrating how these effective techniques can be applied to investigate almost periodic solutions of impulsive differential equations and provides interesting *applications* of many practical problems of diverse interest.

The book consists of four chapters.

Chapter 1 has an introductory character. In this chapter a description of the systems of impulsive differential equations is made and the main results on the fundamental theory are given: conditions for absence of the phenomenon "beating," theorems for existence, uniqueness, continuability of the solutions. The class of piecewise continuous Lyapunov functions, which are an apparatus in the almost periodic theory, is introduced. Some comparison lemmas and auxiliary assertions, which are used in the remaining three chapters, are exposed. The main definitions and properties of almost periodic sequences and almost periodic piecewise continuous functions are considered.

In Chap. 2, some basic existence and uniqueness results for almost periodic solutions of different classes of impulsive differential equations are given. The hyperbolic impulsive differential equations, impulsive integro-differential equations, forced perturbed impulsive differential equations, impulsive differential equations with perturbations in the linear part, dichotomous impulsive differential systems, impulsive differential equations with variable impulsive perturbations, and impulsive abstract differential equations in Banach space are investigated. The relations between the strong stability and almost periodicity of solutions of impulsive differential equations with fixed moments of impulse effect are considered. Many examples are considered to illustrate the feasibility of the results.

Chapter 3 is dedicated to the existence and uniqueness of almost periodic solutions of impulsive differential equations by Lyapunov method. Almost periodic Lyapunov functions are offered. The existence theorems of almost periodic solutions for impulsive ordinary differential equations, impulsive integro-differential equations, impulsive differential equations with time-varying delays, and nonlinear impulsive functional differential equations are stated. By using the concepts of uniformly positive definite matrix functions and Hamilton–Jacobi–Riccati inequalities, the existence theorems for almost periodic solutions of uncertain impulsive dynamical systems are proved.

Finally, in Chap. 4, the applications of the theory of almost periodicity to impulsive biological models, Lotka–Volterra models, and neural networks are presented. The impulses are considered either as means of perturbations or as control.

The book is addressed to a wide audience of professionals such as mathematicians, applied researches, and practitioners.

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