Chapter 2
Magnetotellurics: Basic Theoretical Concepts

2.1 Introduction

The magnetotelluric method or magnetotellurics (MT) is an electromagnetic geophysical exploration technique that images the electrical properties (distribution) of the earth at subsurface depths. The energy for the magnetotelluric technique is from natural source of external origin. When this external energy, known as the primary electromagnetic field, reaches the earth's surface, part of it is reflected back and remaining part penetrates into the earth. Earth acts as a good conductor, thus electric currents (known as telluric currents) are induced in turn produce a secondary magnetic field.

Magnetotellurics is based on the simultaneous measurement of total electromagnetic field, i.e. time variation of both magnetic field B(t) and induced electric field E(t). The electrical properties (e.g. electrical conductivity) of the underlying material can be determined from the relationship between the components of the measured electric (E) and magnetic field (B) variations, or transfer functions: The horizontal electric (Ex and Ey) and horizontal (Bx and By) and vertical (Bz) magnetic field components. According to the property of electromagnetic waves in the conductors, the penetration of electromagnetic wave depends on the oscillation frequency. The frequency of the electromagnetic fields development of the theory determines the depth of penetration.

The basis for MT method is found by Tikhonov and Cagniard [1, 2]. In half a century since its inception, important developments in formulation, instrumentation and interpretation techniques have yielded MT as a competitive geophysical method, suitable to image broad range of geological targets.

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Fig. 2.1 Distortion of the magnetosphere due to interaction of the solar wind

2.2 Source Field of MT Signals

The MT signals are generated from two sources:

1. At the lower frequencies, generally less than 1 Hz, or more than 1 cycle per second, the source of the signal is originated from the interaction of the solar wind with the earth's magnetic field. As solar wind emits streams of ions, it travels into space and disturbs the earth's ambient magnetic field and produces low-frequency electromagnetic energy that penetrates the earth (Fig. 2.1).
2. The high frequency signal is greater than 1 Hz or less than 1 cycle per second is created by world-wide thunderstorm activity, usually near the equator. The energy created by these storms travels around the earth in a wave guide between the earth's surface and the ionosphere, with part of the energy penetrates into the earth.

Both of these signal sources create time-varying electromagnetic waves.

Although the variations of electric and magnetic fields are small, they are measurable. Since these signals vary in strength over hours, days, weeks and even over the sunspot cycle (which is about 11 years and creates an increase in the number of solar storms). Geophysicists measuring MT for greater depths have to measure for long hours at each station in order to get good signal to ensure high-quality data. This is especially true when measurements are required for low frequencies (about 0.001 Hz, or 1 cycle per 1,000 s). At these low frequencies, we need to record for 16 min (1,000 s) to get one sample of data! That means we really need to record for several hours just to get many samples (25–50) for meaningful statistical average of the data [3].

2.3 Principles of MT

2.3.1 Maxwell's Equations

The electromagnetic fields within a material of a non-accelerated reference frame can be described by Maxwell's equations. These can be expressed in differential form with the International system of Units (SI) as:

$$\nabla \times \mathrm{E} = -\frac{(\partial B)}{(\partial t)} \quad \text{Faraday's law} \tag{2.1}$$

$$\nabla \times \mathrm{H} = \mathrm{J} + \frac{(\partial D)}{(\partial t)} \quad \text{Ampere's law} \tag{2.2}$$

$$\nabla \cdot \mathrm{D} = \rho_{\mathrm{v}} \quad \text{Gauss's law} \tag{2.3}$$

$$\nabla \cdot \mathrm{B} = 0 \quad \text{Gauss's lawfor magnetism} \tag{2.4}$$

where E (V/m) and H (A/m) are the electric and magnetic fields, B is the magnetic induction. D (C/m^2) is the displacement current and ρ (C/m^3) is the electric charge density owing to free charges. J and ∂D/∂t (A/m^2) are the current density and the varying displacement current respectively.

Maxwell's equations can also be related through their constitutive relationship:

$$\mathrm{J} = \sigma \mathrm{E}, \tag{2.5}$$

$$\mathrm{D} = \varepsilon \mathrm{E}, \tag{2.6}$$

$$\mathrm{B} = \mu \mathrm{H}, \tag{2.7}$$

σ, ε and μ describe intrinsic properties of the materials through which the electromagnetic fields propagate. σ (S/m) is the electrical conductivity (its reciprocal being the electrical resistivity $\rho = 1/\sigma$ (Ω-m)), ε (F/m) is the dielectric permittivity and μ (H/m) is the magnetic permeability. These magnitudes are scalar quantities in isotropic media. In anisotropic materials they must be expressed in a tensorial. In this work, it will be assumed that the properties of the materials are isotropic.

The electrical conductivity of the Earth materials varies and has a wide spectrum up to several orders of magnitude and is sensitive to small changes in minor constituents of the rock. Since conductivity of most rock materials is very low (10^{-5} S/m), the conductivity of the rock unit depends in general on the interconnectivity of minor constituents (by way of fluids or partial melting) or the presence of highly conducting materials such as graphite.

In a vacuum, the dielectric permittivity is $\varepsilon = \varepsilon_0 = 8.85 \times 10^{-12}$ F/m. Within the Earth, this value ranges from ε_0 (vacuum and air) to 80 ε_0 (water). It can also vary depending on the frequency of the electromagnetic fields [3].

For most of the Earth materials and for the air, the magnetic permeability "μ" can be approximated to its value in a vacuum, $\mu_0 = 4\pi \times 10^{-7}$ H/m. However, in highly magnetized materials this value can be greater, for example, due to an increase in the magnetic susceptibility below the Curie point temperature (Hopkinson effect, e.g. [4]).

Across a discontinuity between two materials, named 1 and 2, the boundary conditions to be applied to the electromagnetic fields and currents described by Maxwell's equations are:

$$\mathrm{n} \times (\mathrm{E}_2 - \mathrm{E}_1) = 0, \tag{2.8}$$

$$\mathrm{n} \times (\mathrm{H}_2 - \mathrm{H}_1) = \mathrm{Js}, \tag{2.9}$$

$$\mathrm{n} \times (\mathrm{D}_2 - \mathrm{D}_1) = \rho_s, \tag{2.10}$$

$$\mathrm{n} \times (\mathrm{B}_2 - \mathrm{B}_1) = 0, \tag{2.11}$$

$$\mathrm{n} \times (\mathrm{J}_2 - \mathrm{J}_1) = 0, \tag{2.12}$$

where n is the unit vector normal to the discontinuity boundary, Js (A/m^2) is the current density along the boundary surface and ρ_s (C/m^2) is the surface charge density. In the absence of surface currents, and considering constant values of ε and μ, the tangential components of E and the normal components of J are continuous, where as the both tangential and normal components of B are continuous across the discontinuity.

Due to the nature of the electromagnetic sources used in MT, the properties of the Earth materials and the depth of investigations considered, two hypotheses are applicable:

(a) Quasi-stationary approximation: Displacement currents (∂D/∂t) can be neglected relative to conductivity currents (J) for the period range 10^{-5} to 10^5 s and for not extremely low conductivity values. Therefore, the propagation of the electromagnetic fields through the Earth can be explained as a diffusive process, which makes it possible to obtain responses that are volumetric averages of the measured Earth conductivities.
(b) Plane wave hypothesis: The primary electromagnetic field is a plane wave that propagates vertically down towards the Earth surface (z direction) [5].

The searched solutions of the electromagnetic fields from Maxwell's equation can be expressed through a linear combination of harmonic wave:

$$\mathrm{E} = \mathrm{E}_0 \cdot e^{i(wt+kr)} \tag{2.13}$$

$$\mathrm{B} = \mathrm{B}_0 \cdot e^{i(wt+kr)} \tag{2.14}$$

where ω (rad/s) is the angular frequency of the electromagnetic oscillations, t(s) is the time; k(m^{-1}) and r(m) are the wave and position vectors respectively. In both expressions, the first term in the exponent corresponds to wave oscillations and the second term represents wave propagation.

Using the harmonic expressions of the electromagnetic fields (Eqs. 2.13 and 2.14) and their constitutive relationships (Eqs. 2.5–2.7), Maxwell's equations in frequency domain for MT hypothesis (a quasi-stationary approximation) are described as follows:

$$\nabla \times \mathrm{E} = -i\omega \mathrm{B} \tag{2.15}$$

$$\nabla \times \mathrm{H} = \mu_0 \sigma \mathrm{E} \tag{2.16}$$

$$\nabla \cdot \mathrm{E} = \frac{(\rho\varepsilon)}{\in} \tag{2.17}$$

$$\nabla \cdot \mathrm{B} = 0 \tag{2.18}$$

where the value of the magnetic permeability (μ) is considered equal to the value in a vaccum (μ_0).

In the absence of charges, the right term of Eq. 2.17 vanishes, and the electric and magnetic field solutions depend solely upon angular frequency (ω) and conductivity (σ).

Finally using the hypothesis (b) (plane wave) and applying the boundary conditions (Eqs. 2.8–2.12) across discontinuities, the solutions of Maxwell's equations can be obtained.

In the case of an homogeneous structure, the components of the electric and magnetic fields take the form:

$$\mathrm{Ak} = \mathrm{A_{k0}}.\ e^{iwt}.\ e^{-i\alpha z}.\ e^{-\alpha z} \tag{2.19}$$

with $\alpha = \sqrt{\mu\sigma\omega/2}\,(\mathrm{m}^{-1})$ The first factor of the equation is the wave amplitude, the second and third factor (imaginary exponentials) is sinusoidal time and depth variations respectively and the fourth is exponential decay. This decay can be quantified by the skin depth, δ, and the value of z for which this term decays to 1/e [6]:

$$\delta = \sqrt{\frac{2}{\mu 0}\sigma\omega} \approx 500\sqrt{\rho T}\,(\mathrm{m}). \tag{2.20}$$

The skin depth permits the characterization of the investigation depth, which, as can be seen, increases according to the square root of the product of medium resistivity and period. Although it has been defined for homogeneous media, its use can be extended to heterogeneous cases as well (e.g. geological structures). The above text has been taken from the Telford et al. [7].

2.3.2 Assumptions of Magnetotellurics

The following are the considerable assumptions applicable in electromagnetic induction in the earth (e.g., [2, 8]):

- The Earth does not generate electromagnetic (EM) energy, but only dissipates or absorbs it.
- Maxwell's electromagnetic (EM) equations are obeyed.
- All electromagnetic fields are treated as conservative and analytic away from their sources.

- The passive electromagnetic source fields, being generated by large-scale ionospheric current systems that are relatively far away from the Earth surface, may be treated as uniform, plane-polarized electromagnetic waves impinging on the Earth at near vertical incidence.
- Accumulation of free charges can't be expected to be sustained within a layered Earth. However, in multi-dimensional earth, charges can accumulate along discontinuities. Earth behaves as an Ohmic conductor and obeying the equation: $J = \sigma E$, Where, J is total electric current density (in Am^{-2}), σ is the conductivity of the medium (in Sm^{-1}), and E is the electric field (in Vm^{-1}).
- The time varying displacement currents (arising from polarization effects) are negligible compared with time varying conduction currents and promotes the treatment of electromagnetic induction in the Earth purely as a diffusion process.
- The variations in the magnetic permeabilities and electrical permitivities of rocks are assumed negligible.

2.3.3 Skin Depth

The diffusion factor describes the penetration depths of the fields, the "skin depth" ($\delta\ (\omega)\ m$) in a homogeneous earth is defined as:

$$\delta(\omega) = \sqrt{}2/|\mathrm{k}^2| = \sqrt{}2/\omega\mu\sigma, \tag{2.21}$$

This represents the exponential decay of the EM-field amplitude with depth. At depth $\delta\ (\omega)$, the amplitude of the EM-field drops by $1/e$ with respect to its value at the surface. The skin depth is proportional to the square root of T ($T = 2\pi/\omega$, infers that the skin depth increases with the period T. For a 1-D stratified Earth of N layers the penetration depth of the EM-fields measured at the surface ($C1\ (\omega)$) is solved iteratively, with a recursive formula described by the EM-response function Ci (ω) [9]. The index i refer to the EM-response measured at the top of the layer i [10]:

$$Ci(\omega) = [1 - ri\exp(-2kidi)]\,/ki[1 + ri\exp(-2kidi)] \tag{2.22}$$

where $i = N - 1, N - 2, , \ldots 1$ and $ri = 1 - [kiCi + 1(\omega)]/[1 + kiCi + 1(\omega)].d_i$ is the thickness of the layer i and $ki = \sqrt{i\omega\mu\sigma i}$ the diffusion factor in the layer (of conductivity $\sigma\ i$).

2.3.4 Uniform Half Space

In this case Earth is treated as a conducting half space with a plane surface. The assumptions usually made about the source field [2] are that it is homogeneous, infinite in dimension and is located effectively at infinity so that plane EM waves impinging on the Earth surface. Under these conditions, there are no horizontal variations of the EM field, i.e. $\partial E/\partial x = \partial H/\partial x = \partial E/\partial y = \partial H/\partial y = 0$. Hence $Hz = 0 = Ez$ for the X component, Eq. 2.7 reduces to

$$\frac{\partial^2 E_x}{\partial Z^2} = K^2 E_x \tag{2.23}$$

where $K^2 = i\mu\omega\sigma$. From Maxwell's equation,

$$H_y = (-i/\omega\mu)\frac{\partial E_x}{\partial Z} \tag{2.24}$$

Since the fields originate from a source above the earth, all the field quantities must remain finite. At $Z = \infty$. Hence the solution of Eq. 2.23 is

$$E_x = Q\, e^{-KZ} \tag{2.25}$$

where Q is a constant.

As seen from the foregoing an electromagnetic wave propagating into the earth (linear, homogeneous and isotropic) has its electric and magnetic field wave vectors orthogonal to each other, and the ratio of electric and magnetic field intensity (E/H) termed as the impedance (Z) is a characteristic measure of the EM properties of the sub surface medium, and constitutes the basic MT response function.

For a plane wave, we have

$$Z = \frac{Ex}{Hy} = \frac{i\omega\mu}{k} \tag{2.26}$$

where Z is the characteristic impedance, Ex the electric field intensity (north) in mv/km and Hy the magnetic field intensity (east) in γ (10^{-5} Oe)

$$Z = \sqrt{(i\omega\mu/\sigma)} \tag{2.27}$$

From the above equation it may be deduced that in a homogeneous and isotropic half-space, the magnetic field lags behind the electric field by $\pi/4$ rad.

The true resistivity of the half-space is

$$\rho = \frac{1}{\sigma} = \frac{|Z|^2}{\mu\omega}$$

$$\rho = \frac{T}{2\pi\mu}|Z|^2 \quad \text{where T is the period.} \tag{2.28}$$

with the EM system of units, [2] has obtained the following equation as

$$\rho = 0.2\, T\frac{|Ex|^2}{|Hy|^2} \tag{2.28a}$$

where
ρ = resistivity in Ω-m
E = the horizontal electric field in mv/km
H = the orthogonal horizontal magnetic field in gamma and
T = period in seconds

When the earth resistivity is non-uniform, the right hand sides of Eq. 2.28a provide apparent resistivities (instead of true resistivity), ρ_a, which are frequency (period) dependent, as is the case with 1-D, 2-D, or 3-D situations.

In a homogeneous and isotropic earth, the true resistivity of the earth is related to the characteristic impedance "Z" through the relation:

$$\rho_a = 0.2\,T\,|Z|^2 = 0.2\,T\frac{|E|^2}{|H|^2} \tag{2.29}$$

Where $Z = E/H$ **Note:** $Z = E/H$
$Zxy = Ex/Hy$
$Zyx = Ey/Hx$

where ρ is the resistivity in Ω-m and T is the period in sec.

$$\text{And phase of } Z_{xy},\ \varphi = \tan^{-1}\frac{(\text{imag.}[\frac{Ex}{Hy}])}{(\text{Real}[\frac{Ex}{Hy}])} \tag{2.30}$$

2.4 Dimensionality Models

The MT transfer functions, and particularly the relationship between their components, are reduced to specific expressions depending on the spatial distribution of the electrical conductivity being imaged. These spatial distributions, known as geo-electric dimensionality, can be classified as 1-D, 2-D or 3-D. Other particular expressions of the transfer functions can be obtained when data are affected by galvanic distortion, a phenomenon caused by minor scale (local) inhomogeneities near Earth's surface.

This section presents a summary of the characteristics of the different types of geo-electric dimensionality, regarding its geometry, the behavior of the electromagnetic fields through them and expressions of the related transfer functions. Galvanic distortion is also explained along with the type of transfer functions associated with this phenomenon.

2.4.1 1-D Earth

In this case the conductivity distribution is depth dependent only ($\sigma = \sigma$ (z) $= 1/\rho$ (z)) and Maxwell's equations can be analytically solved by properly applying the boundary conditions. The solutions are electromagnetic waves, with the electromagnetic field always orthogonal to the magnetic field, that travel perpendicular to the surface of the Earth in a constant oscillation direction. They attenuate with depending on their period and conductivity values. As a result, the MT transfer functions are independent of the orientation of the measured axes and are a function only of the frequency.

In the case of horizontally layered earth (1-D earth), the true resistivity $'\rho'$ in Eq. 2.29 becomes an apparent resistivity (ρ_a), and is given by

$$\rho_a = 0.2T\frac{|Ex|^2}{|Hy|^2} \tag{2.31}$$

because of the symmetry of the problem, estimates of characteristic impedance for either a homogeneous or a layered earth do not depend on orientation of measuring axes in the horizontal plane, so that the north and east electric field components are related to the orthogonal magnetic field components through the following linear equations:

$$E_x = ZH_y \text{ and } E_y = -ZH_x \tag{2.32}$$

Thus in this case at any particular period, an electric field component is linearly related to its orthogonal magnetic field component through a single valued complex scalar transfer functions. Equation 2.31 was formulated for the first time by Cagniard and is known as the Cagniard relation [2]. The conditions under which Eq. 2.31 is valid are called the Cagniard conditions; viz., the incidence electromagnetic fields are plane waves at the earth's surface and that the earth consists of parallel layers.

With regard to the tipper, there is no net component of the vertical magnetic field, Bz, due to the assumption that the incidence of the electromagnetic fields is perpendicular to the Earth's surface, and fact that in a 1-D models these fields do not change direction with depth. Therefore, the two components of the tipper, Tx and Ty are zero.

2.4.2 2-D Earth

In a two dimensional Earth the conductivity is constant along one horizontal direction while changing both along the vertical and the other horizontal direction along which the conductivity is constant is known as the geo-electric strike or strike. Considering a right handed Cartesian coordinate system (X, Y, Z), in the

2-D case, the conductivity (or it's reciprocal, the resistivity) varies along two directions- one horizontal direction say Y and the other along the vertical direction (depth). Along the other horizontal direction (X-direction) the resistivity does not change and this direction is called the strike direction. Unlike in the 1-D case, analytical solutions for 2-D structures are cumbersome, owing to coupling between the field components.

The two equations for E and H fields are,

$$\nabla^2 H = i\omega\mu\sigma E \tag{2.33}$$

And

$$\nabla^2 E = i\omega\mu\sigma^2 H - \nabla\sigma \times (\nabla \times H) \tag{2.34}$$

where '×' denotes multiplication sign.

In the case of 2-D structures, a general 2-D field satisfying Eqs. 2.18 and 2.19 can be separated into two distinct modes, and these are generally referred to as E and H polarizations. The two modes corresponding to E and H polarizations have their E and H fields polarized parallel to the strike direction respectively. The impedances corresponding to these polarizations are not only different from each other, but also depend on the location of measurement sites.
For the E-polarization case

$$\mathrm{E} = \mathrm{Ex}; \quad \mathrm{H} = \frac{\mathrm{i}}{\omega\mu}\left[\mathrm{Y}\frac{\partial \mathrm{E}}{\partial \mathrm{Z}} - \mathrm{Z}\frac{\partial \mathrm{E}}{\partial \mathrm{Y}}\right]$$

$$\text{with}\,\frac{\partial \mathrm{Hz}}{\partial \mathrm{Y}} - \frac{\partial \mathrm{Hy}}{\partial \mathrm{Z}} = \sigma \mathrm{Ex} \tag{2.35}$$

For H polarization

$$\mathrm{H} = \mathrm{Hx}; \quad \mathrm{E} = \frac{1}{\sigma}\left[\mathrm{Y}\frac{\partial \mathrm{H}}{\partial \mathrm{Z}} - \mathrm{Z}\frac{\partial \mathrm{H}}{\partial \mathrm{Y}}\right]$$

$$\text{with}\,\frac{\partial \mathrm{Ex}}{\partial \mathrm{Y}} - \frac{\partial \mathrm{Ey}}{\partial \mathrm{y}} = \mathrm{i}\omega\mu \mathrm{H_x} \tag{2.36}$$

where x, y and z are unit vectors along the coordinate axes.

In a more complicated structure, the coupling between electric and magnetic fields is more complex. The electric fields are strongly distorted near a lateral inhomogenity whereas magnetic fields may be relatively less distorted. The electric field is then locally polarized at some angle other than 90° to the regional magnetic field. At each point in the vicinity of the lateral discontinuity, this result in the linear coupling of each of the electric field component and the relationship is expressed in the form:

$$\mathrm{E_x} = \mathrm{aH_x} + \mathrm{bH_y} \tag{2.37}$$

where a and b are called coupling coefficients which depend upon position, coordinate direction, period, geometry and the electric properties of the lateral inhomogenity. In such case an impedance tensor [Z] can be defined as:

$$[\mathrm{Z}] = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \tag{2.38}$$

where Z_{xx}, Z_{xy}, Z_{yx} and Z_{yy} are the impedance tensor elements. And we may write for general case

$$\left.\begin{aligned} &\mathrm{E} = [\mathrm{Z}][\mathrm{H}] \text{ or} \\ &\mathrm{Ex} = \mathrm{ZxxHx} + \mathrm{ZxyHy} \quad \text{and} \\ &\mathrm{Ey} = \mathrm{ZyxHx} + \mathrm{ZyyHy} \end{aligned}\right\} \tag{2.39}$$

2.5 MT Response Functions

2.5.1 Impedance Tensor

The electrical impedance Z [mV/T] is the ratio between the electric and magnetic field components, which comes from the matrix form relation: E = ZB. In a homogeneous media, the ratio of the orthogonal components is

$$\mathrm{Z} = \mathrm{i}\omega/\mathrm{k} \tag{2.40}$$

In a general 3-D earth, the impedance is expressed in matrix form in Cartesian coordinates (*x, y* horizontal and *z* positive downwards):

$$\begin{bmatrix} Ex \\ Ey \end{bmatrix} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \begin{bmatrix} Bx \\ By \end{bmatrix} \tag{2.41}$$

Thus each tensor element is $Zij = Ei/Bj$ ($i, j = x, y$). In a 2-D earth the diagonal elements of Z vanish: $Zxx = Zyy = 0$. For a 2-D Earth, the conductivity varies along one horizontal direction as well as with depth, Zxx and Zyy are equal in magnitude, but have opposite sign, whilst Zxy and Zyx differ, i.e.:

$$\left.\begin{aligned} Zxx &= -Zyy \\ ZxyB &= -Zyx \end{aligned}\right\} \quad \text{2-D} \tag{2.42}$$

For a 2-D Earth with the x- or y-direction aligned along electromagnetic strike, Z_{yy} and Z_{yy} are again zero. Mathematically, a 1-D anisotropic Earth is equivalent to a 2-D Earth.

Being a tensor, Z also contains information about dimensionality and direction. For a 1-D Earth, wherein conductivity varies only with depth, the diagonal elements of the impedance tensor, Z_{yy} and Z_{yy} (which couple parallel electric and magnetic field components) are zero, while the off-diagonal components (which couple orthogonal electric and magnetic field components) are equal in magnitude, but have opposite signs, i.e.,: in 1-D situation,

$$\left.\begin{aligned} Zxx = Zyy = 0 \\ Zxy = -Zyx \end{aligned}\right\} \quad \text{1-D} \tag{2.43}$$

The tensor Z can be rotated to any other coordinate system by an angle θ with the rotationmatrix R:

$$\mathrm{Zm} = RZRT \quad \text{where R} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{2.44}$$

with positive θ describing a c.w. rotation from the coordinate system of Zm.

With measured data, it is often not possible to find a direction in which the condition that Zxx = Zyy = 0 is satisfied. This may be due to distortion (or) 3-D induction (or) both. Generally, the dimensionality evinced by data is scale dependent. Consider any generalized, homogeneous, 3-D conductive anomaly embedded in an otherwise uniform Earth. For short MT sounding periods, corresponding to electromagnetic skin depths that are small compared to the shortest dimensions of the anomaly, the transfer function should appear as 1-D. As the sounding period increases further, the inductive scale length will eventually extend sufficiently to encompass at least one edge of the anomaly, and the transfer functions appear 2-D. As the sounding period increases further, edge effects from all sides of the anomaly will eventually be imposed on the transfer functions, resulting in transfer functions that are evidently 3-D. For sufficiently long periods, such that the electromagnetic skin depth is very much greater than the dimensions of the anomaly, the inductive response of the anomaly become weak, but a non-inductive response persists. The non-inductive response of the anomaly creates a frequency-independent distortion of MT transfer functions that can be stripped away.

Impedance phase

The phase of the impedance element describes the phase shift between the electric and magnetic field components:

$$\mathrm{Z} \;= \left|\frac{E_i}{B_j}\right| e^{i\emptyset}$$

$$\text{where } \emptyset = \Psi Ei - \Psi Bi = \arctan\frac{(imag.(Zij)}{(Real(Zij)}$$

where i, j = x, y and Ei, Bj is the phase of the electric and magnetic field, respectively. In a **homogeneous earth,** the impedance phase (Eq. 2.40) is:

$$Z = \frac{i\omega}{k} = \sqrt{\frac{\omega}{(\mu\sigma)}}\sqrt{i} \to \pi/4$$

This means that the electric field precedes the magnetic field by 45°, given by the diffusive process of the EM plane wave's propagation.

In a 1-D layered earth, the phase increases over 45° when the EM-response penetrates into a higher conductivity media. By analogy, the phase decays below 45°. For the EM-response penetrating into a less conductive media. This means that by the diffusive process the phase shift between the orthogonal electric and magnetic field components attenuates when the fields penetrate into a less conductive media:

In the 1-D/2-D case the phases lie in I or III quadrant ([0, $\pi/2$] or [π, $3\pi/2$]), which means that the real and imaginary parts of *Zxy* (or *Zyx*) have equal sign. This is due to the principle of causality of the interaction between electric and magnetic fields induced in the earth; i.e., any secondary field induced due to a conductivity contrast should necessarily postdate the primary incident field (the initial source).

By convention, the element *Zxy* is defined as positive and therefore *Zyx* is negative, implying an impedance phase in I and III quadrant, respectively.

The principle of causality should be generally satisfied in a 3-D earth. There can be particular conductivity structures, however, which can violate this principle, as was discussed for the first time by Egbert [11].

2.5.2 Directionality Parameter: Strike

The direction in which the conductivity of a 2-D structure does not vary is termed the strike direction (principle conductivity axis). The angle between the principle conductivity axis and the x-axis is called the angle of strike. The axis parallel and perpendicular to the strike are the principle (preferred directions). With reference to the later axis, the impedance tensor is given by

$$Z = \begin{bmatrix} 0 & Z_1 \\ Z_2 & 0 \end{bmatrix} \tag{2.45}$$

where the Z_1 and Z_2 are the impedances parallel and perpendicular to the strike direction respectively.

The angle of strike θ_0 is obtained from the measured impedances by maximizing some suitable functions of Zxy and Zyx under rotation of the axis. Two main functions in use are $\left|Z'_{xy}\right|^2$ [12] and $\left|Z'_{xy}\right|^2 + \left|Z'_{yx}\right|^2$ [13].

$$\theta_0 = {}^1\!/\!_4 \arctan\frac{(\mathrm{Zxx}-\mathrm{Zyy})(\mathrm{Zxy}+\mathrm{Zyx})^* + (\mathrm{Zxx}-\mathrm{Zyy})^*(\mathrm{Zxy}+\mathrm{Zyx})}{|\mathrm{Zxx}-\mathrm{Zyy}|^2 - |\mathrm{Zxy}-\mathrm{Zyx}|^2} \tag{2.46}$$

Where * denotes the complex conjugate.

For the above angle, $Z'xx$ are $Z'yy$ are zero for a 2-D structure. However, owing to the ever present noise in the measured data, Zxx and Zyy never reduce to zero on rotation of axis, but only become very small compared with Zxy and Zyx (1-D and 2-D cases). In a 3-D structure, Zxx and Zyy may still be quite appreciable after axis rotation, but the Eq. 2.35 can still be use to obtained the gross 2-D angle of strike of the 3-D structure [14].

2.5.3 Dimensionality Indicators

Apart from the ratio $\rho_{\max}$ to $\rho_{\min}$, other parameters used in determining the dimensionality of the Earth structure under investigation are Skew and Tipper.

Skew

Skew defined as the ratio of the magnitude of the second invariant to that of the third invariant

$$\mathrm{S} = \frac{|\mathrm{Zxx} + \mathrm{Zyy}|}{|\mathrm{Zxy} - \mathrm{Zyx}|} \tag{2.47}$$

Skew is the measure of the EM coupling between the measured electric and magnetic field variations in the same direction. There is no coupling for the case of a 1-D structure and when measurements are made parallel and perpendicular to the strike of a 2-D structure, but there is always coupling over a 3-D structure except at a point of radial symmetry. Thus, for 1-D and 2-D structures, S should be zero. This is rarely the case in practice as a result of the ever present noise in the data. In a 2-D case where the resistivity contrast across the structure is low, i.e., $|\mathrm{Zxy} - \mathrm{Zyx}| \cong 0$, S is large. For 3-D structures S is generally large.

Tipper

The tipper coefficients (the single station vertical magnetic field transfer functions) A and B by expressing the vertical magnetic component Hz as a linear combination of the horizontal magnetic field components (Hx, Hy), [15, 16] have defined i.e.,

$$\mathrm{Hz} = \mathrm{AHx} + \mathrm{BHy} \tag{2.48}$$

These complex coefficients can be visualized as operating on the horizontal magnetic field and tipping part of it into the vertical. The magnitude of the tipper is given by

$$\mathrm{T} = \sqrt{\left(\mathrm{A}^2 + \mathrm{B}^2\right)} \tag{2.49}$$

The expressing for calculating the tipper coefficients are

$$A = \frac{<HzHx^{*}> <HyHy^{*}> - <HzHy^{*}> <HyHx^{*}>}{<HxHx^{*}> <HyHy^{*}> - <HxHy^{*}> <HyHx^{*}>} \tag{2.50}$$

$$B = \frac{<HzHy^{*}> <HxHx^{*}> - <HzHx^{*}> <HxHy^{*}>}{<HyHy^{*}> <HxHx^{*}> - <HyHx^{*}> <HxHy^{*}>} \tag{2.51}$$

Where * denotes the complex conjugate. For a 2-D structure with strike in the X-direction, A = 0. Tipper thus can be used to identify the presence of 2-D effects in the analyzed data. Information from the vertical magnetic field transfer functions is helpful in determining the structural strike direction [5].

The angle which maximizes the coherency between the horizontal and vertical magnetic fields [17] is given by

$$\begin{aligned}\varphi = \; & T^{-2}[(Re^2A + Re^2B)]\arctan(ReB + ReA) \\ & + \left(Im^2A + Im^2B\right)\arctan(ImB + ImA)\end{aligned} \tag{2.52}$$

The phase difference between Hz and Hφ gives the bearing of some structures. They have also shown that the tipper skew γ is given by

$$\gamma = 2T^{-2}(ReA\ ImB - ImA\ ReB) \tag{2.53}$$

For 2-D structures γ is zero. All three quantities T, φ and γ are independent of axis rotation and provide some information about the subsurface structure.

The reliability of the calculated Hz (Hz^{c}) is estimated from the coherence between it and the measured Hz (Hz^{m}), i.e.

$$\begin{aligned}& Coh[Hz^{m}Hz^{c}] \\ & = \frac{|A^{*} <Hz^{m}Hx^{*}> + B^{*} <Hz^{m}Hy^{*}>|}{\left\{<Hz^{m}Hz^{m*}>^{1/2}[AA^{*}HxHx^{*}> + BB^{*} <Hy\ Hy^{*}> + 2Re(AB^{*}<HxHy^{*}>)]^{1/2}\right\}}\end{aligned} \tag{2.54}$$

With * denoting the complex conjugate.

2.5.4 Induction Arrows

Induction arrows are vector representations of the complex (i.e., containing real and imaginary parts) ratios of vertical to horizontal magnetic field components. Since vertical magnetic fields are generated by lateral conductivity gradients, induction arrows can be used to infer the presence or absence of lateral variations in conductivity [18]. The vectors point towards the anomalous internal concentrations of current [19] called the Parkinson convention whereas the vectors points away from internal current concentrations are called the wise convention [20]. The vectors are also sometimes called tipper vectors as they transform horizontal magnetic fields into the vertical plane according to the relationship:

$$\mathrm{Hz}(\omega) = (\mathrm{Tx}(\omega) \quad \mathrm{Ty}(\omega))\begin{pmatrix} \mathrm{B_x}/\mu_0 \\ \mathrm{B_y}/\mu_0 \end{pmatrix} \tag{2.55}$$

In a 2-D Earth, induction arrows are associated only with the E-polarization. Thus, insulator conductor boundaries extending through a 2-D Earth gives rise to induction arrows that orient perpendicular to them, and have magnitude that are proportional to the intensities of anomalous current concentrations [21], which are intern determined by the magnitude of the conductivity gradient or discontinuity. However, an absence of induction arrows at a single site does not necessarily confirm an absence of laterally displaced conductivity boundaries [18].

2.5.5 Concept of Static Shift

Static shift causes a frequency-independent off set in apparent resistivity curves so that they plot parallel to their true level, but are scaled by real factors. The scaling factor(s) cannot be determined directly from MT data recorded at a single site. A parallel shift between two polarizations of the apparent resistivity curves is a reliable indicator that static shift is present in the data. However, a lack of shift between two apparent resistivity curves does not necessarily guaranty polarizations an absence of static shift, since both curves might be shifted by the same amount. The correct level of the apparent resistivity curves may lie above, below or between their measured levels. If MT data are interpreted via 1-D modeling without correcting for static shift, the depth to a conductive body will be shifted by the square root of the factor by which the apparent resistivities are shifted ($\sqrt{s}$), and the modeled resistivity will be shifted by S. 2-D and 3-D models may contain extraneous structure if static shifts are ignored. Therefore, additional data or assumptions are often required.

Static shift corrections may be classified into three broad methods.

- Short-period corrections relying on active near surface measurements (e.g., TEM, DC).
- Averaging (statistical) techniques.
- Long period corrections relying on assumed deep structure (e.g., a resistivity drop at the mid-mantle transition zones) or long period magnetic transfer functions.

The concept of static shift is caused by multi-dimensional conductivity contrasts having depths and dimensions less than the true penetration depth of electromagnetic fields [18]. As a result of conservation of electric charge, conductivity discontinuities cause local distortion of the amplitudes of electric fields and hence causing impedance magnitudes to be enhanced or diminished by real scaling factors. When current crosses a discontinuity, charges build up along a discontinuity. The resulting shift in apparent resistivity curves is referred as 'static'. Indeed, the presence of static shift is most easily identifiable in measured data in which apparent resistivities are shifted relative to each other, but impedance phases lie together. As a result of static shift, apparent resistivity curves are shifted by a constant, real scaling factor, and therefore preserve the same shape as in shifted apparent resistivity curves in the period range where static shift occurs. The small-scale conductivity heterogeneities have more significant effects on electric fields and this is more common in highly resistive environments, where problem of Static shifts occur.

2.6 3-D Galvanic Distortion and Decomposition of MT Impedance Tensor

The electromagnetic signal becomes weak when the electromagnetic skin depth becomes greater than the dimensions of the anomaly but it continues to have a non-inductive response termed as galvanic [22].

Electromagnetic data containing galvanic effects can often be described by superimposition or decomposition model in which the data are decomposed into a non-inductive response owing to multi-dimensional heterogeneities with dimensions significantly less than the inductive scale length of the data (often described as local), and a response owing to an underlying 1-D or 2-D structure (often described as regional). In such cases, determining the electromagnetic strike involves decomposing the measured impedance tensor into matrices representing the inductive and non-inductive parts. The inductive part is contained in a tensor composed of components that have both magnitude and phase (i.e. its components are complex), where as the non-inductive part exhibits DC behavior only, and is described by a distortion tensor, the components of which must be real and therefore frequency independent [18].

Interpretation of experimental magnetotelluric results is easiest in those cases where the surveyed structure is one dimensional (1-D) or 2-D. However, experimentally determined magnetotelluric impedance tensors rarely conform to the ideal 2-D impedance tensor. That is there is no rotation of the co-ordinate axis such that the diagonal elements of the impedance tensor are both exactly zero. This may occur either (1) because of data errors in the case of 1-D or 2-D induction, (2) because of 3-D induction, or (3) because of 1-D or 2-D induction coupled with the effects of galvanic(frequency independent) telluric distortion. For historical reasons connected with the ease of calculating inductive responses for 2-D structures and the difficulty of doing the same for 3-D structures, it has been customary to assume the first of the above possibilities in presenting data and to rotate the coordinate axes so as to make the measured tensor as close as possible to an ideal 2-D tensor (one with zero diagonal elements) in some sense [usually a least square sense e.g. [13].

Improvements in data quality in recent years have made it obvious that the third possibility (1-D or 2-D induction coupled with 3-D telluric distortion) is important in practice. The measured impedance tensor, if such distortion is present, need not be close to a true 2-D impedance tensor, and rotation or decomposition methods based on this assumption make no sense in this situation. A number of alternative decomposition methods have been proposed e.g. [23–26] which do not make any simplifying assumptions about the physical model and use as many parameters to represent the tensor as there are data (eight real parameters in contrast to the five kept by rotation to an idealized 2-D tensor). In the case of induction in one or two dimensions coupled with 3-D galvanic scattering, then these general decompositions may not be optimal since they fail to take advantage of the simplicities of underlying the model.

Galvanic distortion or current channeling does not destroy most of the information present about an underlying 2-D inductive process [27]. Bhar demonstrates possible ways in which this information can be recovered and shows an application to a field situation. Therefore the physical approach we take to the decomposition problem is to make the specific assumption that a measured impedance tensor is produced by local galvanic distortion, by arbitrary 3-D structures, of the electric currents induced on the large scale in a regionally 1-D or 2-D structure. Even when this model is not true for all frequencies of the data set, it may still be true over limited frequency ranges since the definition of a "regional" scale can be different for different frequency ranges.

In summary, the purpose of our decomposition is to separate local and regional parameters as much as possible under the assumption that the regional structure is at most 2-D and the local structure causes only galvanic scattering of the electric fields, and to do so in the form of a product factorization.

The decomposition technique, which is used to determine the electromagnetic strike using the decomposition hypothesis is an inverse technique proposed by Groom and Bailey [28]. In Groom Bailey's decomposition technique, separation of the "localized" effects of 3-D current channeling from the 'regional' 2-D inductive behavior is achieved by factorizing the impedance tensor problem in terms of a rotation matrix, β, and a distortion tensor, C, which is itself the product of three tensor sub operators (twist, T, Shear, S, local anisotropy, A, and a scalar, g, [18]**:**

$$\mathrm{Z} = \beta\, \mathrm{C}\, \mathrm{Z}\, \beta^{\mathrm{T}} \quad \text{where } \mathrm{C} = \mathrm{g\, T\, S\, A}$$

2.7 Rotating the Impedance Tensor

In a layered Earth, the Impedance in all directions can be calculated (from the biavariate regression) simply by measuring the electric field variation in the perpendicular direction. We could test the hypothesis of a layered Earth either (1) by performing the measurements in different co-ordinate frames—(x,y) and (x′,y′)- and comparing the elements of the Impedance tensors Z and Z′, respectively, or (2) by applying a mathematical rotation to the impedance tensor estimated from data measured in a fixed co-ordinate frame. Theoretically, we can simulate a field site setup with sensors oriented in any direction, α, via a mathematical rotation involving matrix multiplication of the measured fields (or impedance tensor) with a rotation matrix, β:

$$\left.\begin{array}{l} E' = \beta E \\ B' = \beta B \end{array}\right\} \quad \beta E = Z'\beta(B/\mu_0) \Rightarrow Z' = \beta Z \beta^T \tag{2.56}$$

where

$$\beta = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \tag{2.57}$$

And superscript T denotes the transpose of β,

$$\beta^T = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \tag{2.58}$$

Equation 2.1 can be expanded as,

$$\left.\begin{array}{l} Z'xx = Zxx\ \mathrm{Cos}^2\alpha + (Zxy + Zyx)\ \mathrm{Sin}\ \alpha\ \mathrm{Cos}\ \alpha + Zyy\mathrm{Sin}^2\alpha \\ Z'xy = Zxy\ \mathrm{Cos}^2\alpha + (Zxx + Zyy)\ \mathrm{Sin}\ \alpha\ \mathrm{Cos}\ \alpha - Zyx\mathrm{Sin}^2\alpha \\ Z'yx = Zyx\ \mathrm{Cos}^2\alpha + (Zyy - Zxx)\ \mathrm{Sin}\ \alpha\ \mathrm{Cos}\ \alpha - Zxy\mathrm{Sin}^2\alpha \\ Z'yy = Zyy\ \mathrm{Cos}^2\alpha + (Zyx - Zyx)\ \mathrm{Sin}\ \alpha\ \mathrm{Cos}\ \alpha - Zxx\mathrm{Sin}^2\alpha \end{array}\right\} \tag{2.59}$$

or more elegantly,

$$\left.\begin{array}{r} Z'xx = S1 + S2\mathrm{Sin}\alpha\mathrm{Cos}\ \alpha \\ Z'xy = D1 + S1\mathrm{Sin}\alpha\mathrm{Cos}\ \alpha \\ Z'yx = D2 - D1\mathrm{Sin}\alpha\mathrm{Cos}\ \alpha \\ Z'yy = -D1 - D2\mathrm{Sin}\alpha\mathrm{Cos}\ \alpha \end{array}\right\}, \tag{2.60}$$

where S1, S2, D1 and D2 are the modified impedances [5]:

$$\begin{array}{ll} S1 = Zxx + Zyy & S2 = Zxy + Zyx \\ D1 = Zxx - Zyy & D2 = Zxy - Zyx \end{array}$$

The rotated modified impedances are simplify

$$\begin{aligned} S_1' &= Z'xx + Z'yy = Zxx + Zyy = S_1 \\ D'1 &= Z'xx - Z'yy \\ &= \left(\mathrm{Cos}^2\alpha - \mathrm{Sin}^2\alpha\right)(Zxx - Zyy) + 2\mathrm{Cos}\alpha\mathrm{Sin}\alpha(Zxy - Zyx) \\ &= \mathrm{Cos}^2\alpha D_1 + \mathrm{Sin}^2\alpha S_2 \end{aligned} \tag{2.61}$$

And similarly

$$S'_{2=}\mathrm{Cos}^2\alpha\ S_2 - \ \mathrm{Sin}^2\alpha\ D_1)\ \text{and}\ D'2 = \ D_2 \qquad (2.62)$$

Hence, S1 and D2 are rotationally invariant [18].

2.8 Inversion Schemes

Geophysical data is modeled and interpreted in terms of subsurface geology in two ways: a direct way, known as forward modeling, and indirect way, known as inverse modeling. In the forward method, the model parameters of the subsurface geology are estimated from geophysical observations and response functions. On the other hand, in the inverse method a model of the subsurface is assumed and a theoretical geophysical response is computed for the assumed model and compared with the observed data. This process is repeated for various models through an iterative process until a minimum difference between the computed and observed responses is achieved.

In the past, modeling of the data was carried out by master curve matching, through trial and error processes. Two-layer and three master curves were published by Cagniard and Yungul [2, 29]. The trial and error method is generally very painstaking, especially when a large number of parameters are involved, and the curve matching technique is very limited in resolution. With the general availability of electronic computers and better-matched recording equipment, it is now possible to invert data automatically to save time and to attain maximum use of the data.

With the advancement in the computational facility inversion schemes gained widespread use and many of these deploy, a combination of least-squares method and an iteration scheme for achieving the fit between the observed data and model response.The least-squares condition to be satisfied is:

$$\phi = \Sigma(\rho_{\mathrm{ci}} - \rho_{\sigma\mathrm{i}})^2 = \mathrm{a\,minimum.}$$

where ρ_{ci} → calculated apparent resistivity at a period T_i
And $\rho_{\sigma\mathrm{i}}$ → observed resistivity at a period T_i

Several iterations are generally required for convergence of ϕ to a limit. The step size and the direction of the correlation vector is determined simultaneously, to insure proper convergence [30]. Uniqueness however, is not generated in this method.

The traditional multidimensional MT inversion procedure is as follows: the conductivity of the earth is parameterized by assigning its values for different layers in the case of 1-D, or at a number of nodes or in a number of predefined elements in the case of 2-D/3-D. A starting model is guessed and a matrix, F, of partial derivatives of the data with respect to small changes in the parameters is calculated. This involves solutions of multiple forward problems. Singular value decomposition (SVD) or some other damped generalized inverse of FFT is then used to predict the conductivity perturbations that should improve the fit to the data/these perturbations are added to the initial guess to produce a new starting model. The forward problem is then solved for one more time to calculate new data residuals. And the whole process is repeated until a satisfactory fit to the data has been obtained.

In solving any inverse problem, one seeks not merely a model which fits a given set of data, but also a knowledge of what features in that model are required by the data and more not merely incidental to the manner in which the model was obtained as starting points. For 2-D or 3-D models, since unconstrained details may persist in later iterations and be mistakenly interpreted as significant structure.

Evaluating what features are resolved has been well studied for the linear inverse problem. Backus and Gilbert [31] have shown to construct average of models that are uniquely determined by the data. Knowledge of the resolution functions and the variance of the average allow critical evaluation of details in the structure. With the uncertainties and the nonlinear effects in MT inversion, one should seek models that have the minimum structure for some tolerable level of misfit to the data. If a minimum structure mode exhibits a particular feature, the confidence limit regarding that feature improves. Conversely, if a minimum-structure model does not exhibit a particular feature, then that feature is certainly not required by the data.

All the real data have measurement errors, so that it is generally neither possible not desirable to fit the data exactly. The chi squared statistic is given by:

$$\chi^2 = \Sigma \nabla \gamma \mathrm{i} / \varepsilon \mathrm{i}$$

where $\nabla\gamma$i are data residuals and εi are data standard errors

NLCG inversion

A new algorithm for computing regularized solutions of the 2-D magnetotelluric inverse problem is a nonlinear conjugate gradients (NLCG) scheme to minimize an objective function that penalizes data residuals and second spatial derivatives of resistivity [32]. This algorithm is compared theoretically and numerically to two previous algorithms for constructing such “minimum-structure” models: the Gauss–Newton method, which solves a sequence of linearized inverse problems and has been the standard approach to nonlinear inversion in geophysics, and an algorithm due to Mackie and Madden, which solves a sequence of linearized inverse problems incompletely using a (linear) conjugate gradients technique. Numerical experiments involving synthetic and field data indicate that the two algorithms based on conjugate gradients (NLCG and Mackie-Madden).

References

1. Tikhnov AN (1950) The determination of the electrical properties of deep layers of the earth's crust. Dokl Acad Nauk SSR 73:295–297 (in Russian)
2. Cagniard L (1953) Basic theory of the magnetotelluric method of geophysical prospecting. Geophysics 18:605–635
3. Christopherson KR (1998) MT gauges earth's electric fields. AAPG explorer, pp 22–31
4. Keller GV (1987) Resistivity characteristics of geological targets. In: Nabighian M (ed) Electromagnetic methods in applied geophysics. Society of Exploration Geophysicists, Tulsa Oklahoma, pp 13–51
5. Radhakrishnamurthy C, Likhite SD (1970) Hopkinson effect, blocking temperature and Curie point in basalts. Earth Planet Sci Lett 7:389–396
6. Vozoff K (1972) The magnetotelluric method in the exploration of sedimentary basins. Geophysics 37:98–141
7. Vozoff K (1991) The magnetotelluric method. In: Nabighian M (ed) Electromagnetic methods in applied geophysics II. Society of Exploration Geophysicists, Tulsa Oklahoma
8. Telford WM, Geldart LP, Sheriff RE (1990) Applied geophysics. Cambridge University Press, Cambridge, pp 306–309
9. Keller GV, Frischknecht FC (1966) Electrical methods in geophysical prospecting. Pergamon, New York, p 517
10. Wait JR (1970) Electromagnetic waves in stratified media: a classic reissue. IEEE Press, 1996
11. Weaver JT (1994) Mathematical methods for Geo-electromagnetic induction. Research studies press Ltd., Taunton
12. Egbert G (1990) Comments on "concerning dispersion relations for the magnetotelluric impedance tensor". In: Yee E, Paulson K (eds) Geophys J Int 102:1–8
13. Everett JE, Hyndman RD (1967) Magnetotelluric investigations in southwestern Australia. Phys Earth Planet Inter 1:49–54
14. Swift CM Jr, (1967) A magnetotelluric investigation of an electrical conductivity anomaly in the southern United states. Ph.D. thesis, Department of Geology and Geophysics, M.I.T, MA
15. Jones FW, Vozoff K (1978) The calculation of magnetotelluric quantities for three-dimensional inhomogenities. Geophysics 43:1167–1175
16. Everett JE, Hyndman RD (1967) Geomagnetic variations and electrical conductivity structure in southwestern Australia. Phys Earth Planet Inter 1:24–34
17. Madden TR, Swift CM Jr (1969) Magnetotelluric studies of the electrical conductivity structure of the crust and upper mantle: in the earth's crust and upper mantle. In: PJ Hart (ed) Am Geophys Un Monogr 469–479
18. Jupp DLB, Vozoff K (1976) Stable iterative methods for the inversion of geophysical data. Geophy J Ro Astr Soc 42:957–976
19. Simpson F, Bahr K (2005) Practical magnetotellurics. Cambridge University Press, Cambridge
20. Parkinson W (1959) Directions of rapid geomagnetic variations. Geophys J R Astr Soc 2:1–14
21. Wiese H (1962) Geomagnetische tiefensondierung. Teil II: Die streichrichtung der Untergrundstrukturen des elektrischen Widerstandes, erschlossen aus geomagnetischen variationen. Geofis Pura et Appl 52:83–103
22. Jones FW, Price AT (1970) The perturbations of alternating geomagnetic fields by conductivity anomalies. Geophys J R astr Soc 20:317–334
23. Wannamaker PE, Hohmann GW, Ward SH (1984) Magnetotelluric responses of three-dimensional bodies in layered earths. Geophysics 49:1517–1533
24. Eggers DE (1982) An eigenstate formulation of the magnetotelluric impedance tensor. Geophysics 47:1204–1214

25. Spitz S (1985) The magnetotelluric impedance tensor properties with respect to rotations. Geophysics 50:1610–1617
26. Latorraca GA, Madden TR, Korringa J (1986) An analysis of the magnetotelluric impedance for three-dimensional conductivity structures. Geophysics 51:1819–1829
27. Yee S, Paulson KV (1987) The canonical decomposition and its relationship to other forms of magnetotelluric impedance tensor analysis. J Geophys 61:173–189
28. Bahr K (1988) Interpretation of the magnetotelluric impedance tensor: regional induction and local distortion. J Geophys 62:119–127
29. Groom RW, Baily RC (1989) Decomposition of magnetotellurics impedance tensor in the presence of local three dimensional galvanic distortions. J Geophys Res 94:1913–1925
30. Yungul SH (1961) Magnetotelluric sounding three-layer interpretation curves. Geophysics 26:465–473
31. Marquardt DW (1963) An algorithm for least-square estimation of non linear parameters. J SIMA 11:431–441
32. Backus GE, Gillbert JF (1968) The resolving power of gross earth data. Geophys J R Astr Soc 16:169–205
33. Rodi W, Mackie RL (2001) Nonlinear conjugate gradients algorithm for 2-D magnetotelluric inversions. Geophysics 66:174–187

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