Chapter 2
With Additive Gain Variations

Keywords Continuous-time T–S fuzzy systems · Discrete-time T–S fuzzy systems · Additive gain variations · H_∞ filter · Linear matrix inequalities (LMIs)

2.1 Problem Formulation

In this chapter, the additive gain variations will be considered which are independent on filter gain matrices [4]. A block diagram for representation of the additive uncertainty is given in Fig. 2.1.

In this case, we consider the following non-fragile fuzzy filter to estimate $z(t)$:

$$\begin{aligned}
&R^j : \text{if } \xi_1(t) \text{ is } M_{1j} \text{ and } \ldots\ \xi_p(t) \text{ is } M_{pj},\\
&\text{then}\quad \dot{x}_F(t) = (A_{Fj} + \Delta A_{Fj}(t))x_F(t) + (B_{Fj} + \Delta B_{Fj}(t))y(t),\\
&\qquad\quad\ z_F(t) = (C_{Fj} + \Delta C_{Fj}(t))x_F(t) + (D_{Fj} + \Delta D_{Fj}(t))y(t),
\end{aligned} \tag{2.1}$$

where $x_F(t) \in \mathscr{R}^n$ and $z_F(t) \in \mathscr{R}^q$ are the state and output of the filter, respectively. $A_{Fj} \in \mathscr{R}^{n\times n}$, $B_{Fj} \in \mathscr{R}^{n\times f}$, $C_{Fj} \in \mathscr{R}^{q\times n}$, and $D_{Fj} \in \mathscr{R}^{q\times f}$ for $j = 1, 2, \ldots, r$ are to be determined filter gain matrices. $\Delta A_{Fj}(t) \in \mathscr{R}^{n\times n}$, $\Delta B_{Fj}(t) \in \mathscr{R}^{n\times f}$, $\Delta C_{Fj}(t) \in \mathscr{R}^{q\times n}$, and $\Delta D_{Fj}(t) \in \mathscr{R}^{q\times f}$ are uncertainties defined as follows:

$$\begin{aligned}
\Delta A_{Fj}(t) &= H_{Aj}\Delta_A(t)E_{Aj}, \quad \Delta B_{Fj}(t) = H_{Bj}\Delta_B(t)E_{Bj},\\
\Delta C_{Fj}(t) &= H_{Cj}\Delta_C(t)E_{Cj}, \quad \Delta D_{Fj}(t) = H_{Dj}\Delta_D(t)E_{Dj},
\end{aligned}$$

for $j = 1, 2, \ldots, r$, where $H_{\alpha j}, E_{\alpha j}, \alpha = A, B, C, D$ are constant matrices with appropriate dimensions and $\Delta_\alpha(t), \alpha = A, B, C, D$ are uncertain matrices satisfying $\Delta_\alpha^T(t)\Delta_\alpha(t) \le I$.

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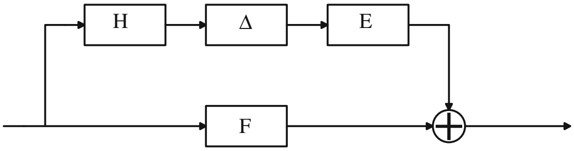

Fig. 2.1 Additive uncertainty

The defuzzified output of (2.1) can be inferred by

$$\begin{aligned}\dot{x}_F(t) &= (A_F(h) + \Delta A_F(h))x_F(t) + (B_F(h) + \Delta B_F(h))y(t),\\ z_F(t) &= (C_F(h) + \Delta C_F(h))x_F(t) + (D_F(h) + \Delta D_F(h))y(t),\end{aligned} \tag{2.2}$$

where

$$\begin{aligned}A_F(h) &= \sum_{j=1}^{r} h_j(\xi(t))A_{Fj}, \quad B_F(h) = \sum_{j=1}^{r} h_j(\xi(t))B_{Fj},\\ C_F(h) &= \sum_{j=1}^{r} h_j(\xi(t))C_{Fj}, \quad D_F(h) = \sum_{j=1}^{r} h_j(\xi(t))D_{Fj},\\ \Delta A_F(h) &= \sum_{j=1}^{r} h_j(\xi(t))\Delta A_{Fj}(t), \quad \Delta B_F(h) = \sum_{j=1}^{r} h_j(\xi(t))\Delta B_{Fj}(t),\\ \Delta C_F(h) &= \sum_{j=1}^{r} h_j(\xi(t))\Delta C_{Fj}(t), \quad \Delta D_F(h) = \sum_{j=1}^{r} h_j(\xi(t))\Delta D_{Fj}(t).\end{aligned}$$

Combining (1.6) and (2.2) leads to the following filtering error system:

$$\begin{aligned}\dot{\psi}(t) &= \tilde{A}(h)\psi(t) + \tilde{B}(h)w(t),\\ e(t) &= \tilde{C}(h)\psi(t) + \tilde{D}(h)w(t),\end{aligned} \tag{2.3}$$

where

$$\begin{aligned}\psi(t) &= \left[x^T(t)\ x_F^T(t)\right]^T, \quad e(t) = z(t) - z_F(t),\\ \tilde{A}(h) &= \begin{bmatrix} A(h) & 0 \\ B_F(h)C(h) + \Delta B_F(h)C(h) & A_F(h) + \Delta A_F(h) \end{bmatrix},\\ \tilde{B}(h) &= \begin{bmatrix} B(h) \\ B_F(h)D(h) + \Delta B_F(h)D(h) \end{bmatrix},\\ \tilde{C}(h) &= [L(h) - D_F(h)C(h) - \Delta D_F(h)C(h) - C_F(h) - \Delta C_F(h)],\\ \tilde{D}(h) &= -D_F(h)D(h) - \Delta D_F(h)D(h).\end{aligned}$$

2.2 Non-Fragile H_∞ Filter Design

In the section, we will present sufficient conditions for designing a non-fragile H_∞ filter in the form of (2.2), that is, to determine the filter matrices in (2.2) such that the filtering error system (2.3) is asymptotically stable with H_∞ performance γ.

2.2.1 Continuous-Time Case

Theorem 2.1 *Consider the filtering error system* (2.3)*. For a given scalar $\gamma > 0$, if there exist matrices P_1, P_2, P_3, G, M_{1ij}, M_{2ij}, M_{3ij}, G_{1ij}, G_{2ij}, $\mathscr{A}_{Fj}$, $\mathscr{B}_{Fj}$, $\mathscr{C}_{Fj}$, and $\mathscr{D}_{Fj}$, scalars ε_{ij} and σ_{ij}, for $i, j = 1, 2, \ldots, r$ such that following inequalities hold:*

$$\begin{bmatrix} P_1 & * \\ P_2 & P_3 \end{bmatrix} > 0, \tag{2.4}$$

$$\left[\begin{array}{cccccccccccccc}
\theta_{11ii} & * & * & * & * & * & * & * & * & * & * & * & * & * \\
\theta_{21ii} & \theta_{22ii} & * & * & * & * & * & * & * & * & * & * & * & * \\
\theta_{31ii} & \theta_{32ii} & \theta_{33ii} & * & * & * & * & * & * & * & * & * & * & * \\
\theta_{41ii} & \theta_{42ii} & \theta_{43ii} & \theta_{44ii} & * & * & * & * & * & * & * & * & * & * \\
\theta_{51ii} & \theta_{52ii} & \theta_{53ii} & \theta_{54ii} & \theta_{55} & * & * & * & * & * & * & * & * & * \\
H_{Bi}^T P_2 & H_{Bi}^T P_3 & 0 & 0 & 0 & -\varepsilon_{ii} I & * & * & * & * & * & * & * & * \\
H_{Ai}^T P_2 & H_{Ai}^T P_3 & 0 & 0 & 0 & 0 & -\varepsilon_{ii} I & * & * & * & * & * & * & * \\
H_{Bi}^T P_2 & H_{Bi}^T P_3 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{ii} I & * & * & * & * & * & * \\
H_{Ai}^T P_2 & H_{Ai}^T P_3 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{ii} I & * & * & * & * & * \\
\theta_{101ii} & \theta_{102ii} & \theta_{103ii} & 0 & 0 & 0 & 0 & 0 & 0 & -2I & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -H_{Di}^T & -\sigma_{ii} I & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -H_{Ci}^T & 0 & -\sigma_{ii} I & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -H_{Di}^T & 0 & 0 & -\sigma_{ii} I & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -H_{Ci}^T & 0 & 0 & 0 & -\sigma_{ii} I
\end{array}\right] < 0, \quad i = 1, 2, \ldots, r, \tag{2.5}$$

$$
\left[\begin{array}{cccccccccccccc}
\theta_{11ij} & * & * & * & * & * & * & * & * & * & * & * & * & * \\
\theta_{21ij} & \theta_{22ij} & * & * & * & * & * & * & * & * & * & * & * & * \\
\theta_{31ij} & \theta_{32ij} & \theta_{33ij} & * & * & * & * & * & * & * & * & * & * & * \\
\theta_{41ij} & \theta_{42ij} & \theta_{43ij} & \theta_{44ij} & * & * & * & * & * & * & * & * & * & * \\
\theta_{51ij} & \theta_{52ij} & \theta_{53ij} & \theta_{54ij} & \theta_{55} & * & * & * & * & * & * & * & * & * \\
H_{Bj}^T P_2 & H_{Bj}^T P_3 & 0 & 0 & 0 & -\varepsilon_{ij} I & * & * & * & * & * & * & * & * \\
H_{Aj}^T P_2 & H_{Aj}^T P_3 & 0 & 0 & 0 & 0 & -\varepsilon_{ij} I & * & * & * & * & * & * & * \\
H_{Bi}^T P_2 & H_{Bi}^T P_3 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{ij} I & * & * & * & * & * & * \\
H_{Ai}^T P_2 & H_{Ai}^T P_3 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{ij} I & * & * & * & * & * \\
\theta_{101ij} & \theta_{102ij} & \theta_{103ij} & 0 & 0 & 0 & 0 & 0 & 0 & -2I & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -H_{Dj}^T & -\sigma_{ij} I & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -H_{Cj}^T & 0 & -\sigma_{ij} I & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -H_{Di}^T & 0 & 0 & -\sigma_{ij} I & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -H_{Ci}^T & 0 & 0 & 0 & -\sigma_{ij} I
\end{array}\right] < 0,
$$

$$
i, j = 1, 2, \ldots, r, \quad i < j, \tag{2.6}
$$

where

$$
\begin{aligned}
\theta_{11ij} &= M_{1ij}(A_i + A_j) + \mathscr{B}_{Fj} C_i + \mathscr{B}_{Fi} C_j + (A_i + A_j)^T M_{1ij}^T + C_i^T \mathscr{B}_{Fj}^T \\
&\quad + C_j^T \mathscr{B}_{Fi}^T \varepsilon_{ij} \left(C_i^T E_{Bj}^T E_{Bj} C_i + C_j^T E_{Bi}^T E_{Bi} C_j \right) \\
&\quad + \sigma_{ij} \left(C_i^T E_{Dj}^T E_{Dj} C_i + C_j^T E_{Di}^T E_{Di} C_j \right), \\
\theta_{21ij} &= M_{2ij}(A_i + A_j) + \mathscr{B}_{Fj} C_i + \mathscr{B}_{Fi} C_j + \mathscr{A}_{Fj}^T + \mathscr{A}_{Fi}^T, \\
\theta_{22ij} &= \mathscr{A}_{Fj} + \mathscr{A}_{Fj}^T + \mathscr{A}_{Fi} + \mathscr{A}_{Fi}^T + \varepsilon_{ij} \left(E_{Aj}^T E_{Aj} + E_{Ai}^T E_{Ai} \right)
\end{aligned}
$$

$$
\begin{aligned}
&+\sigma_{ij}\left(E_{Cj}^T E_{Cj} + E_{Ci}^T E_{Ci}\right),\\
\theta_{31ij} &= M_{3ij}(A_i + A_j) + (B_i + B_j)^T M_{1ij}^T + D_i^T \mathscr{B}_{Fj}^T\\
&+ D_j^T \mathscr{B}_{Fi}^T \varepsilon_{ij}\left(D_i^T E_{Bj}^T E_{Bj} C_i + D_j^T E_{Bi}^T E_{Bi} C_j\right)\\
&+\sigma_{ij}\left(D_i^T E_{Dj}^T E_{Dj} C_i + D_j^T E_{Di}^T E_{Di} C_j\right),\\
\theta_{32ij} &= (B_i + B_j)^T M_{2ij}^T + D_i^T \mathscr{B}_{Fj}^T + D_j^T \mathscr{B}_{Fi}^T,\\
\theta_{33ij} &= M_{3ij}(B_i + B_j) + (B_i + B_j)^T M_{3ij}^T - 2\gamma^2 I,\\
&\varepsilon_{ij}\left(D_i^T E_{Bj}^T E_{Bj} D_i + D_j^T E_{Bi}^T E_{Bi} D_j\right)\\
&+\sigma_{ij}\left(D_i^T E_{Dj}^T E_{Dj} D_i + D_j^T E_{Di}^T E_{Di} D_j\right),\\
\theta_{41ij} &= P_1 - M_{1ij}^T + G_{1ij}(A_i + A_j) + \mathscr{B}_{Fj} C_i + \mathscr{B}_{Fi} C_j,\\
\theta_{42ij} &= P_2^T - M_{2ij}^T + \mathscr{A}_{Fj} + \mathscr{A}_{Fi},\\
\theta_{43ij} &= -M_{3ij}^T + G_{1ij}(B_i + B_j) + \mathscr{B}_{Fj} D_i + \mathscr{B}_{Fi} D_j,\\
\theta_{44ij} &= -G_{1ij} - G_{1ij}^T,\\
\theta_{51ij} &= P_2 - G^T + G_{2ij}(A_i + A_j) + \mathscr{B}_{Fj} C_i + \mathscr{B}_{Fi} C_j,\\
\theta_{52ij} &= P_3 - G^T + \mathscr{A}_{Fj} + \mathscr{A}_{Fi},\\
\theta_{53ij} &= G_{2ij}(B_i + B_j) + \mathscr{B}_{Fj} D_i + \mathscr{B}_{Fi} D_j,\\
\theta_{54ij} &= -G_{2ij} - G^T,\\
\theta_{55} &= -G - G^T,\\
\theta_{101ij} &= L_i - \mathscr{D}_{Fj} C_i + L_j - \mathscr{D}_{Fi} C_j,\\
\theta_{102ij} &= -\mathscr{C}_{Fj} - \mathscr{C}_{Fi},\\
\theta_{103ij} &= -\mathscr{D}_{Fj} D_i - \mathscr{D}_{Fi} D_j,
\end{aligned}
$$

for $i, j = 1, 2, \ldots, r$, then the prescribed H_∞ performance $\gamma > 0$ is guaranteed.

The matrices for an H_∞ filter in the form of (2.2) are given by

$$
A_{Fi} = G^{-1}\mathscr{A}_{Fi}, \quad B_{Fi} = G^{-1}\mathscr{B}_{Fi}, \quad C_{Fi} = \mathscr{C}_{Fi}, \quad D_{Fi} = \mathscr{D}_{Fi}. \tag{2.7}
$$

Proof Consider the following Lyapunov function:

$$
V(\psi(t)) = \psi^T(t) P \psi(t), \quad P > 0. \tag{2.8}
$$

Then, the time derivative of $V(\psi(t))$ is

$$
\dot{V}(\psi(t)) = \dot{\psi}^T(t) P \psi(t) + \psi^T(t) P \dot{\psi}(t). \tag{2.9}
$$

From (2.3), we have

$$
\begin{aligned}
&\dot{V}(\psi(t)) + e^T(t)e(t) - \gamma^2 w^T(t)w(t) \\
&= (\tilde{A}(h)\psi(t) + \tilde{B}(h)w(t))^T P\psi(t) + \psi^T(t)P(\tilde{A}(h)\psi(t) + \tilde{B}(h)w(t)) \\
&\quad + (\tilde{C}(h)\psi(t) + \tilde{D}(h)w(t))^T(\tilde{C}(h)\psi(t) + \tilde{D}(h)w(t)) - \gamma^2 w^T(t)w(t) \\
&= \begin{bmatrix} \psi(t) \\ w(t) \end{bmatrix}^T \left([\tilde{A}(h)\tilde{B}(h)]^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} [\tilde{A}(h)\tilde{B}(h)] \right. \\
&\quad \left. + [\tilde{C}(h)\tilde{D}(h)]^T[\tilde{C}(h)\tilde{D}(h)] + \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \right) \begin{bmatrix} \psi(t) \\ w(t) \end{bmatrix} \\
&= \begin{bmatrix} \psi(t) \\ w(t) \end{bmatrix}^T \left(\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t)) \left(A_{ij}^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} A_{ij} \right.\right. \\
&\quad \left. + \Delta A_{ij}^T(t) \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} \Delta A_{ij}(t) + \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \right) \\
&\quad + \left(\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))(C_{ij} + \Delta C_{ij}(t)) \right)^T \\
&\quad \times \left.\left(\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))(C_{ij} + \Delta C_{ij}(t)) \right) \right) \begin{bmatrix} \psi(t) \\ w(t) \end{bmatrix} \\
&= \begin{bmatrix} \psi(t) \\ w(t) \end{bmatrix}^T \left(\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))\, h_j(\xi(t)) \left(A_{ij}^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} A_{ij} \right.\right. \\
&\quad \left. + \Delta A_{ij}^T(t) \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} \Delta A_{ij}(t) + \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \right) \\
&\quad + \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{f=1}^{r}\sum_{s=1}^{r} h_i(\xi(t))h_j(\xi(t))h_f(\xi(t))h_s(\xi(t)) \\
&\quad \left. \times (C_{ij} + \Delta C_{ij}(t))^T(C_{fs} + \Delta C_{fs}(t)) \right) \begin{bmatrix} \psi(t) \\ w(t) \end{bmatrix} \\
&= \begin{bmatrix} \psi(t) \\ w(t) \end{bmatrix}^T \left(\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t)) \left(A_{ij}^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} A_{ij} \right.\right. \\
&\quad \left. + \Delta A_{ij}^T(t) \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} \Delta A_{ij}(t) + \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \right) \\
&\quad + \frac{1}{8} \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{f=1}^{r}\sum_{s=1}^{r} h_i(\xi(t))h_j(\xi(t))h_f(\xi(t))h_s(\xi(t))
\end{aligned}
$$

$$
\begin{aligned}
&\times ((C_{ij} + \Delta C_{ij}(t) + C_{ji} + \Delta C_{ji}(t))^T \\
&\times (C_{fs} + \Delta C_{fs}(t) + C_{sf} + \Delta C_{sf}(t)) \\
&+ (C_{fs} + \Delta C_{fs}(t) + C_{sf} + \Delta C_{sf}(t))^T \\
&\times (C_{ij} + \Delta C_{ij}(t) + C_{ji} + \Delta C_{ji}(t)))\Bigg) \begin{bmatrix} \psi(t) \\ w(t) \end{bmatrix},
\end{aligned}
$$

where

$$
\begin{aligned}
A_{ij} &= \begin{bmatrix} A_i & 0 & B_i \\ B_{Fj}C_i & A_{Fj} & B_{Fj}D_i \end{bmatrix}, \\
\Delta A_{ij}(t) &= \begin{bmatrix} 0 & 0 & 0 \\ H_{Bj}\Delta_B(t)E_{Bj}C_i & H_{Aj}\Delta_A(t)E_{Aj} & H_{Bj}\Delta_B(t)E_{Bj}D_i \end{bmatrix}, \\
C_{ij} &= [L_i - D_{Fj}C_i \ \ -C_{Fj} \ \ -D_{Fj}D_i], \\
\Delta C_{ij}(t) &= [-H_{Dj}\Delta_D(t)E_{Dj}C_i \ \ -H_{Cj}\Delta_C(t)E_{Cj} \ \ -H_{Dj}\Delta_D(t)E_{Dj}D_i].
\end{aligned}
$$

Using Lemma 1.2 with $\varepsilon = 1$, we have

$$
\begin{aligned}
&\dot{V}(\psi(t)) + e^T(t)e(t) - \gamma^2 w^T(t)w(t) \\
&\leq \begin{bmatrix} \psi(t) \\ w(t) \end{bmatrix}^T \Bigg(\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t)) \\
&\quad \times \left(A_{ij}^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} A_{ij} + \Delta A_{ij}^T(t) \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} \Delta A_{ij}(t) + \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix}\right) \\
&\quad + \frac{1}{8}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{f=1}^{r}\sum_{s=1}^{r} h_i(\xi(t))h_j(\xi(t))h_f(\xi(t))h_s(\xi(t)) \\
&\quad \times ((C_{ij} + \Delta C_{ij}(t) + C_{ji} + \Delta C_{ji}(t))^T (C_{ij} + \Delta C_{ij}(t) + C_{ji} + \Delta C_{ji}(t)) \\
&\quad + (C_{fs} + \Delta C_{fs}(t) + C_{sf} + \Delta C_{sf}(t))^T (C_{fs} + \Delta C_{fs}(t) + C_{sf} + \Delta C_{sf}(t)))\Bigg) \\
&\quad \times \begin{bmatrix} \psi(t) \\ w(t) \end{bmatrix} \\
&= \begin{bmatrix} \psi(t) \\ w(t) \end{bmatrix}^T \Bigg(\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t)) \\
&\quad \times \Bigg(A_{ij}^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} A_{ij} + \Delta A_{ij}^T(t) \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} \Delta A_{ij}(t) + \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \\
&\quad + (C_{ij} + C_{ji} + \Delta C_{ij}(t) + \Delta C_{ji}(t))^T \\
&\quad \times \frac{1}{4}\ (C_{ij} + C_{ji} + \Delta C_{ij}(t) + \Delta C_{ji}(t))\Bigg)\Bigg) \begin{bmatrix} \psi(t) \\ w(t) \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&= \begin{bmatrix} \psi(t) \\ w(t) \end{bmatrix}^T \Bigg(\sum_{i=1}^{r} h_i^2(\xi(t)) \\
&\quad \times \Bigg(A_{ii}^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} A_{ii} + \Delta A_{ii}^T(t) \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} \Delta A_{ii}(t) + \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \\
&\quad + (C_{ii} + C_{ii} + \Delta C_{ii}(t) + \Delta C_{ii}(t))^T \frac{1}{4} (C_{ii} + C_{ii} + \Delta C_{ii}(t) + \Delta C_{ii}(t)) \Bigg) \\
&\quad + \sum_{i=1}^{r} \sum_{i<j}^{r} h_i(\xi(t)) h_j(\xi(t)) \\
&\quad \times \Bigg(A_{ij}^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} A_{ij} + \Delta A_{ij}^T(t) \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} \Delta A_{ij}(t) + \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \\
&\quad + (C_{ij} + C_{ji} + \Delta C_{ij}(t) + \Delta C_{ji}(t))^T \frac{1}{4} (C_{ij} + C_{ji} + \Delta C_{ij}(t) + \Delta C_{ji}(t)) \\
&\quad + A_{ji}^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} A_{ji} + \Delta A_{ji}^T(t) \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} \Delta A_{ji}(t) + \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \\
&\quad + (C_{ji} + C_{ij} + \Delta C_{ji}(t) + \Delta C_{ij}(t))^T \\
&\quad \times \frac{1}{4} (C_{ji} + C_{ij} + \Delta C_{ji}(t) + \Delta C_{ij}(t)) \Bigg) \Bigg) \begin{bmatrix} \psi(t) \\ w(t) \end{bmatrix}.
\end{aligned} \tag{2.10}
$$

Then, $\dot{V}(\psi(t)) + e^T(t)e(t) - \gamma^2 w^T(t)w(t) < 0$ for any $\begin{bmatrix} \psi(t) \\ w(t) \end{bmatrix} \neq 0$ if the following inequalities are satisfied:

$$
\begin{aligned}
&(A_{ii} + A_{ii})^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} (A_{ii} + A_{ii}) \\
&\quad + (\Delta A_{ii}(t) + \Delta A_{ii}(t))^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} (\Delta A_{ii}(t) + \Delta A_{ii}(t)) + 2 \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \\
&\quad + (C_{ii} + C_{ii} + \Delta C_{ii}(t) + \Delta C_{ii}(t))^T \frac{1}{2} (C_{ii} + C_{ii} + \Delta C_{ii}(t) + \Delta C_{ii}(t)) < 0, \\
&\qquad i = 1, 2, \ldots, r,
\end{aligned} \tag{2.11}
$$

and

$$
\begin{aligned}
&(A_{ij} + A_{ji})^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} (A_{ij} + A_{ji}) + (\Delta A_{ij}(t) + \Delta A_{ji}(t))^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T \\
&\quad + \begin{bmatrix} P \\ 0 \end{bmatrix} (\Delta A_{ij}(t) + \Delta A_{ji}(t)) + 2 \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix}
\end{aligned}
$$

$$+ (C_{ij} + C_{ji} + \Delta C_{ij}(t) + \Delta C_{ji}(t))^T \frac{1}{2}(C_{ij} + C_{ji} + \Delta C_{ij}(t) + \Delta C_{ji}(t)) < 0,$$
$$i, j = 1, 2, \ldots, r, \quad i < j. \tag{2.12}$$

Define

$$\begin{aligned}
X_{Aij} &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ H_{Bj} & H_{Aj} & H_{Bi} & H_{Ai} \end{bmatrix}, \quad Y_{Aij} = \begin{bmatrix} E_{Bj}C_i & 0 & E_{Bj}D_i \\ 0 & E_{Aj} & 0 \\ E_{Bi}C_j & 0 & E_{Bi}D_j \\ 0 & E_{Ai} & 0 \end{bmatrix}, \\
\tilde{\Delta}A(t) &= \mathrm{diag}\{\Delta_B(t), \Delta_A(t), \Delta_B(t), \Delta_A(t)\}, \\
X_{Cij} &= [\, -H_{Dj} \;\; -H_{Cj} \;\; -H_{Di} \;\; -H_{Ci} \,], \\
Y_{Cij} &= \begin{bmatrix} E_{Dj}C_i & 0 & E_{Dj}D_i \\ 0 & E_{Cj} & 0 \\ E_{Di}C_j & 0 & E_{Di}D_j \\ 0 & E_{Ci} & 0 \end{bmatrix}, \\
\tilde{\Delta}C(t) &= \mathrm{diag}\{\Delta_D(t),\; \Delta_C(t),\; \Delta_D(t),\; \Delta_C(t)\},
\end{aligned} \tag{2.13}$$

for $i, j = 1, 2, \ldots, r$.

Then, we have

$$\begin{aligned}
&(A_{ii} + A_{ii})^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} (A_{ii} + A_{ii}) \\
&+ (\Delta A_{ii}(t) + \Delta A_{ii}(t))^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} (\Delta A_{ii}(t) + \Delta A_{ii}(t)) + 2\begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \\
&+ (C_{ii} + C_{ii} + \Delta C_{ii}(t) + \Delta C_{ii}(t))^T \frac{1}{2}(C_{ii} + C_{ii} + \Delta C_{ii}(t) + \Delta C_{ii}(t)) \\
=\,&(A_{ii} + A_{ii})^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} (A_{ii} + A_{ii}) \\
&+ \begin{bmatrix} P \\ 0 \end{bmatrix} X_{Aii} \tilde{\Delta}_A(t) Y_{Aii} + Y_{Aii}^T \tilde{\Delta}_A^T(t) X_{Aii}^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + 2\begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \\
&+ (C_{ii} + C_{ii} + X_{Cii}\tilde{\Delta}_C(t)Y_{Cii})^T \frac{1}{2}(C_{ii} + C_{ii} + X_{Cii}\tilde{\Delta}_C(t)Y_{Cii}),
\end{aligned}$$
$$i = 1, 2, \ldots, r, \tag{2.14}$$

and

$$(A_{ij} + A_{ji})^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} (A_{ij} + A_{ji})$$

$$
\begin{aligned}
&+ (\Delta A_{ij}(t) + \Delta A_{ji}(t))^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} (\Delta A_{ij}(t) + \Delta A_{ji}(t)) + 2 \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \\
&+ (C_{ij} + C_{ji} + \Delta C_{ij}(t) + \Delta C_{ji}(t))^T \frac{1}{2} (C_{ij} + C_{ji} + \Delta C_{ij}(t) + \Delta C_{ji}(t)) \\
= &(A_{ij} + A_{ji})^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} (A_{ij} + A_{ji}) \\
&+ \begin{bmatrix} P \\ 0 \end{bmatrix} X_{Aij} \tilde{\Delta}_A(t) Y_{Aij} + Y_{Aij}^T \tilde{\Delta}_A^T(t) X_{Aij}^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + 2 \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \\
&+ (C_{ij} + C_{ji} + X_{Cij} \tilde{\Delta}_C(t) Y_{Cij})^T \frac{1}{2} (C_{ij} + C_{ji} + X_{Cij} \tilde{\Delta}_C(t) Y_{Cij}), \\
&\qquad i, j = 1, 2, \ldots, r, \quad i < j, \qquad (2.15)
\end{aligned}
$$

with $\tilde{\Delta}_A^T(t)\tilde{\Delta}_A(t) \leq I$ and $\tilde{\Delta}_C^T(t)\tilde{\Delta}_C(t) \leq I$.

Using Lemmas 1.2 and 1.4, (2.11) and (2.12) hold if the following inequalities (2.16) and (2.17) hold:

$$
\begin{aligned}
&(A_{ii} + A_{ii})^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} (A_{ii} + A_{ii}) \\
&+ \varepsilon_{ii}^{-1} \begin{bmatrix} P \\ 0 \end{bmatrix} X_{Aii} X_{Aii}^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \varepsilon_{ii} Y_{Aii}^T Y_{Aii} + 2 \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \\
&+ (C_{ii} + C_{ii})^T \left(2I - \sigma_{ii}^{-1} X_{Cii} X_{Cii}^T\right)^{-1} (C_{ii} + C_{ii}) + \sigma_{ii} Y_{Cii}^T Y_{Cii} < 0, \\
&\qquad i = 1, 2, \ldots, r, \qquad (2.16)
\end{aligned}
$$

and

$$
\begin{aligned}
&(A_{ij} + A_{ji})^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} (A_{ij} + A_{ji}) \\
&+ \varepsilon_{ij}^{-1} \begin{bmatrix} P \\ 0 \end{bmatrix} X_{Aij} X_{Aij}^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \varepsilon_{ij} Y_{Aij}^T Y_{Aij} + 2 \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \\
&+ (C_{ij} + C_{ji})^T \left(2I - \sigma_{ij}^{-1} X_{Cij} X_{Cij}^T\right)^{-1} (C_{ij} + C_{ji}) + \sigma_{ij} Y_{Cij}^T Y_{Cij} < 0, \\
&\qquad i, j = 1, 2, \ldots, r, \quad i < j. \qquad (2.17)
\end{aligned}
$$

Using Lemma 1.5 to (2.16) and (2.17), respectively, then (2.16) and (2.17) can be verified by

$$
\begin{bmatrix} \beta_{ii} & * \\ \begin{bmatrix} P \\ 0 \end{bmatrix}^T - M_{ii}^T + G_{ii}(A_{ii} + A_{ii}) & -G_{ii} - G_{ii}^T \end{bmatrix} < 0, \quad i = 1, 2, \ldots, r, \qquad (2.18)
$$

and

$$\begin{bmatrix} \beta_{ij} & * \\ \begin{bmatrix} P \\ 0 \end{bmatrix}^T - M_{ij}^T + G_{ij}(A_{ij} + A_{ji}) & -G_{ij} - G_{ij}^T \end{bmatrix} < 0, \\ i, j = 1, 2, \ldots, r, \quad i < j, \tag{2.19}$$

where

$$\begin{aligned} \beta_{ii} &= 2\begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + (A_{ii} + A_{ii})^T M_{ii}^T + M_{ii}(A_{ii} + A_{ii}) \\ &\quad + \varepsilon_{ii}^{-1} \begin{bmatrix} P \\ 0 \end{bmatrix} X_{Aii} X_{Aii}^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \varepsilon_{ii} Y_{Aii}^T Y_{Aii} \\ &\quad + (C_{ii} + C_{ii})^T \left(2I - \sigma_{ii}^{-1} X_{Cii} X_{Cii}^T\right)^{-1} (C_{ii} + C_{ii}) + \sigma_{ii} Y_{Cii}^T Y_{Cii}, \\ \beta_{ij} &= 2\begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + (A_{ij} + A_{ji})^T M_{ij}^T + M_{ij}(A_{ij} + A_{ji}) \\ &\quad + \varepsilon_{ij}^{-1} \begin{bmatrix} P \\ 0 \end{bmatrix} X_{Aij} X_{Aij}^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \varepsilon_{ij} Y_{Aij}^T Y_{Aij} \\ &\quad + (C_{ij} + C_{ji})^T \left(2I - \sigma_{ij}^{-1} X_{Cij} X_{Cij}^T\right)^{-1} (C_{ij} + C_{ji}) + \sigma_{ij} Y_{Cij}^T Y_{Cij}. \end{aligned}$$

By using the Schur complement to (2.18) and (2.19), respectively, we have

$$\begin{bmatrix} \delta_{ii} & * & * & * & * \\ \begin{bmatrix} P \\ 0 \end{bmatrix}^T - M_{ii}^T + G_{ii}(A_{ii} + A_{ii}) & -G_{ii} - G_{ii}^T & * & * & * \\ X_{Aii}^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T & 0 & -\varepsilon_{ii} I & * & * \\ C_{ii} + C_{ii} & 0 & 0 & -2I & * \\ 0 & 0 & 0 & X_{Cii}^T & -\sigma_{ii} I \end{bmatrix} < 0, \\ i = 1, 2, \ldots, r, \tag{2.20}$$

and

$$\begin{bmatrix} \delta_{ij} & * & * & * & * \\ \begin{bmatrix} P \\ 0 \end{bmatrix}^T - M_{ij}^T + G_{ij}(A_{ij} + A_{ji}) & -G_{ij} - G_{ij}^T & * & * & * \\ X_{Aij}^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T & 0 & -\varepsilon_{ij} I & * & * \\ C_{ij} + C_{ji} & 0 & 0 & -2I & * \\ 0 & 0 & 0 & X_{Cij}^T & -\sigma_{ij} I \end{bmatrix} < 0, \\ i, j = 1, 2, \ldots, r, \quad i < j, \tag{2.21}$$

where

$$\begin{aligned}\delta_{ii} &= 2\begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + (A_{ii} + A_{ii})^T M_{ii}^T + M_{ii}(A_{ii} + A_{ii}) \\ &\quad + \varepsilon_{ii} Y_{Aii}^T Y_{Aii} + \sigma_{ii} Y_{Cii}^T Y_{Cii}, \\ \delta_{ij} &= 2\begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + (A_{ij} + A_{ji})^T M_{ij}^T + M_{ij}(A_{ij} + A_{ji}) \\ &\quad + \varepsilon_{ij} Y_{Aij}^T Y_{Aij} + \sigma_{ij} Y_{Cij}^T Y_{Cij}.\end{aligned}$$

Now, we assume that matrices P, M_{ii}, G_{ii}, M_{ij}, and G_{ij} are of the following form:

$$\begin{aligned} P &= \begin{bmatrix} P_1 & * \\ P_2 & P_3 \end{bmatrix} > 0, \quad M_{ii} = \begin{bmatrix} M_{1ii} & G \\ M_{2ii} & G \\ M_{3ii} & 0 \end{bmatrix}, \quad M_{ij} = \begin{bmatrix} M_{1ij} & G \\ M_{2ij} & G \\ M_{3ij} & 0 \end{bmatrix}, \\ G_{ii} &= \begin{bmatrix} G_{1ii} & G \\ G_{2ii} & G \end{bmatrix}, \quad G_{ij} = \begin{bmatrix} G_{1ij} & G \\ G_{2ij} & G \end{bmatrix}. \end{aligned} \tag{2.22}$$

With $\mathscr{A}_{Fi} = GA_{Fi}$, $\mathscr{B}_{Fi} = GB_{Fi}$, $\mathscr{C}_{Fi} = C_{Fi}$, and $\mathscr{D}_{Fi} = D_{Fi}$, we can obtain (2.5) and (2.6) from (2.20) to (2.22).

If (2.4)–(2.6) are satisfied, we have $\dot{V}(\psi(t)) + e^T(t)e(t) - \gamma^2 w^T(t)w(t) < 0$ for any $\begin{bmatrix} \psi(t) \\ w(t) \end{bmatrix} \neq 0$, which implies that

$$V(\psi(\infty)) - V(\psi(0)) + \int_0^\infty e^T(t)e(t)\mathrm{d}t - \gamma^2 \int_0^\infty w^T(t)w(t)\mathrm{d}t < 0.$$

With zero initial condition $\psi(0) = 0$ and $V(\psi(\infty)) > 0$, we obtain $\int_0^\infty e^T(t)e(t)\mathrm{d}t \leq \gamma^2 \int_0^\infty w^T(t)w(t)\mathrm{d}t$ for any nonzero $w(t) \in L_2[0, \infty)$. Thus, the proof is completed. □

Remark 2.1 Compared with [1], Theorem 2.1 gives two new improvements as follows:

1. The LMI conditions in Theorem 2.1 contain less designed variables. This is because the equation $\begin{bmatrix} P \\ 0 \end{bmatrix} [\tilde{A}(h) \tilde{B}(h)]$ in Theorem 2.1 is equivalent to $\begin{bmatrix} P & H_1 \\ 0 & H_2 \end{bmatrix} \times \begin{bmatrix} \tilde{A}(h) & \tilde{B}(h) \\ 0 & 0 \end{bmatrix}$ in [1]. It implies that the designed variables H_1 and H_2 vanish in Theorem 2.1.
2. The matrix dimensions in Theorem 2.1 are largely reduced. In more detail, the matrix dimension of X_{Aij} in (2.13) is smaller than $\mathscr{H}_{Aij}$ in [1]. So are the other matrices.

In the following, we apply Lemmas 1.10 and 1.11, respectively, by combining Theorem 2.1 to design the non-fragile H_∞ filter in the form (2.2). The resulting design conditions are given by the two following Theorems.

Theorem 2.2 *Consider the filtering error system* (2.3)*. For a given scalar $\gamma > 0$, if there exist matrices P_1, P_2, P_3, G, M_{1ij}, M_{2ij}, M_{3ij}, G_{1ij}, G_{2ij}, $\mathscr{A}_{Fj}$, $\mathscr{B}_{Fj}$, $\mathscr{C}_{Fj}$, and $\mathscr{D}_{Fj}$, scalars ε_{ij} and σ_{ij}, for $i, j = 1, 2, \ldots, r$ such that* (2.4)*,* (2.5)*, and the following inequality hold:*

$$\left[\begin{array}{ccccccccccc}
\vartheta_{11ij} & * & * & * & * & * & * & * & * & * & * \\
\vartheta_{21ij} & \vartheta_{22ij} & * & * & * & * & * & * & * & * & * \\
\vartheta_{31ij} & \vartheta_{32ij} & \vartheta_{33ij} & * & * & * & * & * & * & * & * \\
\vartheta_{41ij} & \vartheta_{42ij} & \vartheta_{43ij} & \vartheta_{44ij} & * & * & * & * & * & * & * \\
\vartheta_{51ij} & \vartheta_{52ij} & \vartheta_{53ij} & \vartheta_{54ij} & \vartheta_{55} & * & * & * & * & * & * \\
\frac{1}{2(r-1)} H_{Bi}^T P_2 & \frac{1}{2(r-1)} H_{Bi}^T P_3 & 0 & 0 & 0 & -\varepsilon & * & * & * & * & * \\
\frac{1}{2(r-1)} H_{Ai}^T P_2 & \frac{1}{2(r-1)} H_{Ai}^T P_3 & 0 & 0 & 0 & 0 & -\varepsilon & * & * & * & * \\
\frac{1}{2(r-1)} H_{Bi}^T P_2 & \frac{1}{2(r-1)} H_{Bi}^T P_3 & 0 & 0 & 0 & 0 & 0 & -\varepsilon & * & * & * \\
\frac{1}{2(r-1)} H_{Ai}^T P_2 & \frac{1}{2(r-1)} H_{Ai}^T P_3 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon & * & * \\
\frac{1}{2} H_{Bj}^T P_2 & \frac{1}{2} H_{Bj}^T P_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon & * \\
\frac{1}{2} H_{Aj}^T P_2 & \frac{1}{2} H_{Aj}^T P_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon \\
\frac{1}{2} H_{Bi}^T P_2 & \frac{1}{2} H_{Bi}^T P_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{2} H_{Ai}^T P_2 & \frac{1}{2} H_{Ai}^T P_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vartheta_{141ij} & \vartheta_{142ij} & \vartheta_{143ij} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vartheta_{151ij} & \vartheta_{152ij} & \vartheta_{153ij} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right.$$

$$
\left.\begin{array}{cccccccccccc}
* & * & * & * & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * & * & * & * \\
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* & * & * & * & * & * & * & * & * & * & * & * \\
-\varepsilon & * & * & * & * & * & * & * & * & * & * & * \\
0 & -\varepsilon & * & * & * & * & * & * & * & * & * & * \\
0 & 0 & -I & * & * & * & * & * & * & * & * & * \\
0 & 0 & 0 & -I & * & * & * & * & * & * & * & * \\
0 & 0 & -\frac{1}{\sqrt{4(r-1)}}H_{Di}^T & 0 & -\sigma & * & * & * & * & * & * & * \\
0 & 0 & -\frac{1}{\sqrt{4(r-1)}}H_{Ci}^T & 0 & 0 & -\sigma & * & * & * & * & * & * \\
0 & 0 & -\frac{1}{\sqrt{4(r-1)}}H_{Di}^T & 0 & 0 & 0 & -\sigma & * & * & * & * & * \\
0 & 0 & -\frac{1}{\sqrt{4(r-1)}}H_{Ci}^T & 0 & 0 & 0 & 0 & -\sigma & * & * & * & * \\
0 & 0 & 0 & -\frac{1}{2}H_{Dj}^T & 0 & 0 & 0 & 0 & -\sigma & * & * & * \\
0 & 0 & 0 & -\frac{1}{2}H_{Cj}^T & 0 & 0 & 0 & 0 & 0 & -\sigma & * & * \\
0 & 0 & 0 & -\frac{1}{2}H_{Di}^T & 0 & 0 & 0 & 0 & 0 & 0 & -\sigma & * \\
0 & 0 & 0 & -\frac{1}{2}H_{Ci}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sigma
\end{array}\right] < 0,
$$

$$
i, j = 1, 2, \ldots, r, \quad i \neq j, \tag{2.23}
$$

where $\varepsilon = \varepsilon_{ij} I$, $\sigma = \sigma_{ij} I$ *and,*

$$
\begin{aligned}
\vartheta_{11ij} =& M_{1ij}\left(\frac{1}{2(r-1)}(A_i + A_i) + \frac{1}{2}(A_i + A_j)\right) \\
&+ \frac{1}{2(r-1)}(\mathscr{B}_{Fi}C_i + \mathscr{B}_{Fi}C_i) + \frac{1}{2}(\mathscr{B}_{Fj}C_i + \mathscr{B}_{Fi}C_j) \\
&+ \left(\frac{1}{2(r-1)}(A_i + A_i) + \frac{1}{2}(A_i + A_j)\right)^T M_{1ij}^T \\
&+ \frac{1}{2(r-1)}(\mathscr{B}_{Fi}C_i + \mathscr{B}_{Fi}C_i)^T + \frac{1}{2}(\mathscr{B}_{Fj}C_i + \mathscr{B}_{Fi}C_j)^T \\
&+ \varepsilon_{ij}\left(C_i^T E_{Bi}^T E_{Bi} C_i + C_i^T E_{Bi}^T E_{Bi} C_i + C_i^T E_{Bj}^T E_{Bj} C_i + C_j^T E_{Bi}^T E_{Bi} C_j\right) \\
&+ \sigma_{ij}\left(C_i^T E_{Di}^T E_{Di} C_i + C_i^T E_{Di}^T E_{Di} C_i + C_i^T E_{Dj}^T E_{Dj} C_i + C_j^T E_{Di}^T E_{Di} C_j\right), \\
\vartheta_{21ij} =& M_{2ij}\left(\frac{1}{2(r-1)}(A_i + A_i) + \frac{1}{2}(A_i + A_j)\right) \\
&+ \frac{1}{2(r-1)}(\mathscr{B}_{Fi}C_i + \mathscr{B}_{Fi}C_i) + \frac{1}{2}(\mathscr{B}_{Fj}C_i + \mathscr{B}_{Fi}C_j)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{2(r-1)}(\mathscr{A}_{Fi}+\mathscr{A}_{Fi})^T+\frac{1}{2}(\mathscr{A}_{Fj}+\mathscr{A}_{Fi})^T,\\
\vartheta_{22ij}=&\frac{1}{2(r-1)}(\mathscr{A}_{Fi}+\mathscr{A}_{Fi})+\frac{1}{2}(\mathscr{A}_{Fj}+\mathscr{A}_{Fi})\\
&+\frac{1}{2(r-1)}(\mathscr{A}_{Fi}+\mathscr{A}_{Fi})^T+\frac{1}{2}(\mathscr{A}_{Fj}+\mathscr{A}_{Fi})^T\\
&+\varepsilon_{ij}\left(E_{Ai}^TE_{Ai}+E_{Ai}^TE_{Ai}+E_{Aj}^TE_{Aj}+E_{Ai}^TE_{Ai}\right)\\
&+\sigma_{ij}\left(E_{Ci}^TE_{Ci}+E_{Ci}^TE_{Ci}+E_{Cj}^TE_{Cj}+E_{Ci}^TE_{Ci}\right),\\
\vartheta_{31ij}=&M_{3ij}\left(\frac{1}{2(r-1)}(A_i+A_i)+\frac{1}{2}(A_i+A_j)\right)\\
&+\left(\frac{1}{2(r-1)}(B_i+B_i)+\frac{1}{2}(B_i+B_j)\right)^T M_{1ij}^T\\
&+\frac{1}{2(r-1)}(\mathscr{B}_{Fi}D_i+\mathscr{B}_{Fi}D_i)^T+\frac{1}{2}(\mathscr{B}_{Fj}D_i+\mathscr{B}_{Fi}D_j)^T\\
&\varepsilon_{ij}\left(D_i^TE_{Bi}^TE_{Bi}C_i+D_i^TE_{Bi}^TE_{Bi}C_i+D_i^TE_{Bj}^TE_{Bj}C_i+D_j^TE_{Bi}^TE_{Bi}C_j\right)\\
&+\sigma_{ij}\left(D_i^TE_{Di}^TE_{Di}C_i+D_i^TE_{Di}^TE_{Di}C_i+D_i^TE_{Dj}^TE_{Dj}C_i+D_j^TE_{Di}^TE_{Di}C_j\right),\\
\vartheta_{32ij}=&\left(\frac{1}{2(r-1)}(B_i+B_i)+\frac{1}{2}(B_i+B_j)\right)^T M_{2ij}^T\\
&+\frac{1}{2(r-1)}(\mathscr{B}_{Fi}D_i+\mathscr{B}_{Fi}D_i)^T+\frac{1}{2}(\mathscr{B}_{Fj}D_i+\mathscr{B}_{Fi}D_j)^T,\\
\vartheta_{33ij}=&M_{3ij}\left(\frac{1}{2(r-1)}(B_i+B_i)+\frac{1}{2}(B_i+B_j)\right)\\
&+\left(\frac{1}{2(r-1)}(B_i+B_i)+\frac{1}{2}(B_i+B_j)\right)^T M_{3ij}^T-\frac{r}{r-1}\gamma^2 I\\
&+\varepsilon_{ij}\left(D_i^TE_{Bi}^TE_{Bi}D_i+D_i^TE_{Bi}^TE_{Bi}D_i+D_i^TE_{Bj}^TE_{Bj}D_i+D_j^TE_{Bi}^TE_{Bi}D_j\right)\\
&+\sigma_{ij}\left(D_i^TE_{Di}^TE_{Di}D_i+D_i^TE_{Di}^TE_{Di}D_i+D_i^TE_{Dj}^TE_{Dj}D_i+D_j^TE_{Di}^TE_{Di}D_j\right),\\
\vartheta_{41ij}=&P_1-M_{1ij}^T+G_{1ij}\left(\frac{1}{2(r-1)}(A_i+A_i)+\frac{1}{2}(A_i+A_j)\right)\\
&+\frac{1}{2(r-1)}(\mathscr{B}_{Fi}C_i+\mathscr{B}_{Fi}C_i)+\frac{1}{2}(\mathscr{B}_{Fj}C_i+\mathscr{B}_{Fi}C_j),\\
\vartheta_{42ij}=&P_2^T-M_{2ij}^T+\frac{1}{2(r-1)}(\mathscr{A}_{Fi}+\mathscr{A}_{Fi})+\frac{1}{2}(\mathscr{A}_{Fj}+\mathscr{A}_{Fi}),\\
\vartheta_{43ij}=&-M_{3ij}^T+G_{1ij}\left(\frac{1}{2(r-1)}(B_i+B_i)+\frac{1}{2}(B_i+B_j)\right)\\
&+\frac{1}{2(r-1)}(\mathscr{B}_{Fi}D_i+\mathscr{B}_{Fi}D_i)+\frac{1}{2}(\mathscr{B}_{Fj}D_i+\mathscr{B}_{Fi}D_j),\\
\vartheta_{44ij}=&-G_{1ij}-G_{1ij}^T,
\end{aligned}
$$

$$
\begin{aligned}
\vartheta_{51ij} =& P_2 - G^T + G_{2ij}\left(\frac{1}{2(r-1)}(A_i + A_i) + \frac{1}{2}(A_i + A_j)\right) \\
&+ \frac{1}{2(r-1)}(\mathscr{B}_{Fi}C_i + \mathscr{B}_{Fi}C_i) + \frac{1}{2}(\mathscr{B}_{Fj}C_i + \mathscr{B}_{Fi}C_j), \\
\vartheta_{52ij} =& P_3 - G^T + \frac{1}{2(r-1)}(\mathscr{A}_{Fi} + \mathscr{A}_{Fi}) + \frac{1}{2}(\mathscr{A}_{Fj} + \mathscr{A}_{Fi}), \\
\vartheta_{53ij} =& G_{2ij}\left(\frac{1}{2(r-1)}(B_i + B_i) + \frac{1}{2}(B_i + B_j)\right) \\
&+ \frac{1}{2(r-1)}(\mathscr{B}_{Fi}D_i + \mathscr{B}_{Fi}D_i) + \frac{1}{2}(\mathscr{B}_{Fj}D_i + \mathscr{B}_{Fi}D_j), \\
\vartheta_{54ij} =& -G_{2ij} - G^T, \\
\vartheta_{55} =& -G - G^T, \\
\vartheta_{141ij} =& \frac{1}{\sqrt{4(r-1)}}(L_i - \mathscr{D}_{Fi}C_i + L_i - \mathscr{D}_{Fi}C_i), \\
\vartheta_{142ij} =& \frac{1}{\sqrt{4(r-1)}}(-\mathscr{C}_{Fi} - \mathscr{C}_{Fi}), \\
\vartheta_{143ij} =& \frac{1}{\sqrt{4(r-1)}}(-\mathscr{D}_{Fi}D_i - \mathscr{D}_{Fi}D_i), \\
\vartheta_{151ij} =& \frac{1}{2(r-1)}(L_i - \mathscr{D}_{Fj}C_i + L_j - \mathscr{D}_{Fi}C_j), \\
\vartheta_{152ij} =& \frac{1}{2(r-1)}(-\mathscr{C}_{Fj} - \mathscr{C}_{Fi}), \\
\vartheta_{153ij} =& \frac{1}{2(r-1)}(-\mathscr{D}_{Fj}D_i - \mathscr{D}_{Fi}D_j),
\end{aligned}
$$

for $i, j = 1, 2, \ldots, r, i \neq j$, then the prescribed H_∞ performance $\gamma > 0$ is guaranteed. And the matrices for an H_∞ filter in the form of (2.2) *are given by* (2.7).

Proof Consider the following two inequalities:

$$
\begin{aligned}
&A_{ii}^T\begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} A_{ii} + \Delta A_{ii}^T(t)\begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix}\Delta A_{ii}(t) + \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \\
&\quad + (C_{ii} + C_{ii} + \Delta C_{ii}(t) + \Delta C_{ii}(t))^T \frac{1}{4}(C_{ii} + C_{ii} + \Delta C_{ii}(t) + \Delta C_{ii}(t)) < 0, \\
&\qquad i = 1, 2, \ldots, r,
\end{aligned}
\tag{2.24}
$$

and

$$
\begin{aligned}
&\frac{1}{r-1}\left(A_{ii}^T\begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} A_{ii} + \Delta A_{ii}^T(t)\begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix}\Delta A_{ii}(t) + \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix}\right. \\
&\quad \left. + (C_{ii} + C_{ii} + \Delta C_{ii}(t) + \Delta C_{ii}(t))^T \frac{1}{4}(C_{ii} + C_{ii} + \Delta C_{ii}(t) + \Delta C_{ii}(t))\right)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{2}\Bigg(A_{ij}^T\begin{bmatrix}P\\0\end{bmatrix}^T+\begin{bmatrix}P\\0\end{bmatrix}A_{ij}+\Delta A_{ij}^T(t)\begin{bmatrix}P\\0\end{bmatrix}^T+\begin{bmatrix}P\\0\end{bmatrix}\Delta A_{ij}(t)+\begin{bmatrix}0&0\\0&-\gamma^2 I\end{bmatrix}\\
&+(C_{ij}+C_{ji}+\Delta C_{ij}(t)+\Delta C_{ji}(t))^T\frac{1}{4}(C_{ij}+C_{ji}+\Delta C_{ij}(t)+\Delta C_{ji}(t))\\
&+A_{ji}^T\begin{bmatrix}P\\0\end{bmatrix}^T+\begin{bmatrix}P\\0\end{bmatrix}A_{ji}+\Delta A_{ji}^T(t)\begin{bmatrix}P\\0\end{bmatrix}^T+\begin{bmatrix}P\\0\end{bmatrix}\Delta A_{ji}(t)+\begin{bmatrix}0&0\\0&-\gamma^2 I\end{bmatrix}\\
&+(C_{ji}+C_{ij}+\Delta C_{ji}(t)+\Delta C_{ij}(t))^T\frac{1}{4}(C_{ji}+C_{ij}+\Delta C_{ji}(t)+\Delta C_{ij}(t))\Bigg)<0,\\
&\qquad\qquad i,j=1,2,\ldots,r,\quad i\neq j.
\end{aligned}
\tag{2.25}
$$

From the proof of Theorem 2.1 we know that the inequality (2.24) is verified if (2.5) holds.

The left part of (2.25) is equivalent to

$$
\begin{aligned}
&\frac{r}{r-1}\begin{bmatrix}0&0\\0&-\gamma^2 I\end{bmatrix}+\left(\frac{1}{2(r-1)}(A_{ii}+A_{ii})+\frac{1}{2}(A_{ij}+A_{ji})\right)^T\begin{bmatrix}P\\0\end{bmatrix}^T\\
&+\begin{bmatrix}P\\0\end{bmatrix}\left(\frac{1}{2(r-1)}(A_{ii}+A_{ii})+\frac{1}{2}(A_{ij}+A_{ji})\right)\\
&+\left(\frac{1}{2(r-1)}(\Delta A_{ii}(t)+\Delta A_{ii}(t))+\frac{1}{2}(\Delta A_{ij}(t)+\Delta A_{ji}(t))\right)^T\begin{bmatrix}P\\0\end{bmatrix}^T\\
&+\begin{bmatrix}P\\0\end{bmatrix}\left(\frac{1}{2(r-1)}(\Delta A_{ii}(t)+\Delta A_{ii}(t))+\frac{1}{2}(\Delta A_{ij}(t)+\Delta A_{ji}(t))\right)\\
&+\frac{1}{4(r-1)}(C_{ii}+C_{ii}+\Delta C_{ii}(t)+\Delta C_{ii}(t))^T(C_{ii}+C_{ii}+\Delta C_{ii}(t)+\Delta C_{ii}(t))\\
&+\frac{1}{4}(C_{ij}+C_{ji}+\Delta C_{ij}(t)+\Delta C_{ji}(t))^T(C_{ij}+C_{ji}+\Delta C_{ij}(t)+\Delta C_{ji}(t)).
\end{aligned}
\tag{2.26}
$$

Considering the definition in (2.22), we have

$$
\begin{aligned}
&\frac{1}{2(r-1)}(\Delta A_{ii}(t)+\Delta A_{ii}(t))+\frac{1}{2}(\Delta A_{ij}(t)+\Delta A_{ji}(t))\\
&=\frac{1}{2(r-1)}X_{Aii}\tilde{\Delta}_A(t)Y_{Aii}+\frac{1}{2}X_{Aij}\tilde{\Delta}_A(t)Y_{Aij}\\
&=\begin{bmatrix}\frac{1}{2(r-1)}X_{Aii}&\frac{1}{2}X_{Aij}\end{bmatrix}\begin{bmatrix}\tilde{\Delta}_A(t)&0\\0&\tilde{\Delta}_A(t)\end{bmatrix}\begin{bmatrix}Y_{Aii}\\Y_{Aij}\end{bmatrix},
\end{aligned}
\tag{2.27}
$$

and

$$
\begin{aligned}
&\frac{1}{4(r-1)}(C_{ii}+C_{ii}+\Delta C_{ii}(t)+\Delta C_{ii}(t))^T(C_{ii}+C_{ii}+\Delta C_{ii}(t)+\Delta C_{ii}(t))\\
&\quad+\frac{1}{4}(C_{ij}+C_{ji}+\Delta C_{ij}(t)+\Delta C_{ji}(t))^T(C_{ij}+C_{ji}+\Delta C_{ij}(t)+\Delta C_{ji}(t))\\
&=\left(\frac{1}{\sqrt{4(r-1)}}(C_{ii}+C_{ii})+\frac{1}{\sqrt{4(r-1)}}(\Delta C_{ii}(t)+\Delta C_{ii}(t))\right)^T\\
&\quad\times\left(\frac{1}{\sqrt{4(r-1)}}(C_{ii}+C_{ii})+\frac{1}{\sqrt{4(r-1)}}(\Delta C_{ii}(t)+\Delta C_{ii}(t))\right)\\
&\quad+\left(\frac{1}{2}(C_{ij}+C_{ji})+\frac{1}{2}(\Delta C_{ij}(t)+\Delta C_{ji}(t))\right)^T\\
&\quad\times\left(\frac{1}{2}(C_{ij}+C_{ji})+\frac{1}{2}(\Delta C_{ij}(t)+\Delta C_{ji}(t))\right)\\
&=\Bigg[\left(\frac{1}{\sqrt{4(r-1)}}(C_{ii}+C_{ii})+\frac{1}{\sqrt{4(r-1)}}(\Delta C_{ii}(t)+\Delta C_{ii}(t))\right)^T\\
&\qquad\left(\frac{1}{2}(C_{ij}+C_{ji})+\frac{1}{2}(\Delta C_{ij}(t)+\Delta C_{ji}(t))\right)^T\Bigg]\\
&\quad\times\begin{bmatrix}\frac{1}{\sqrt{4(r-1)}}(C_{ii}+C_{ii})+\frac{1}{\sqrt{4(r-1)}}(\Delta C_{ii}(t)+\Delta C_{ii}(t))\\ \frac{1}{2}(C_{ij}+C_{ji})+\frac{1}{2}(\Delta C_{ij}(t)+\Delta C_{ji}(t))\end{bmatrix}\\
&=\left(\begin{bmatrix}\frac{1}{\sqrt{4(r-1)}}(C_{ii}+C_{ii})\\ \frac{1}{2}(C_{ij}+C_{ji})\end{bmatrix}+\begin{bmatrix}\frac{1}{\sqrt{4(r-1)}}(\Delta C_{ii}(t)+\Delta C_{ii}(t))\\ \frac{1}{2}(\Delta C_{ij}(t)+\Delta C_{ji}(t))\end{bmatrix}\right)^T\\
&\quad\times\left(\begin{bmatrix}\frac{1}{\sqrt{4(r-1)}}(C_{ii}+C_{ii})\\ \frac{1}{2}(C_{ij}+C_{ji})\end{bmatrix}+\begin{bmatrix}\frac{1}{\sqrt{4(r-1)}}(\Delta C_{ii}(t)+\Delta C_{ii}(t))\\ \frac{1}{2}(\Delta C_{ij}(t)+\Delta C_{ji}(t))\end{bmatrix}\right),
\end{aligned}
\tag{2.28}
$$

where

$$
\begin{aligned}
&\begin{bmatrix}\frac{1}{\sqrt{4(r-1)}}(\Delta C_{ii}(t)+\Delta C_{ii}(t))\\ \frac{1}{2}(\Delta C_{ij}(t)+\Delta C_{ji}(t))\end{bmatrix}\\
&=\begin{bmatrix}\frac{1}{\sqrt{4(r-1)}}X_{Cii}\tilde{\Delta}_C(t)Y_{Cii}\\ \frac{1}{2}X_{Cij}\tilde{\Delta}_C(t)Y_{Cij}\end{bmatrix}\\
&=\begin{bmatrix}\frac{1}{\sqrt{4(r-1)}}X_{Cii} & 0\\ 0 & \frac{1}{2}X_{Cij}\end{bmatrix}\begin{bmatrix}\tilde{\Delta}_C(t) & 0\\ 0 & \tilde{\Delta}_C(t)\end{bmatrix}\begin{bmatrix}Y_{Cii}\\ Y_{Cij}\end{bmatrix}.
\end{aligned}
\tag{2.29}
$$

Then, (2.26) can be written as follows:

$$
\begin{aligned}
&\frac{r}{r-1}\begin{bmatrix}0 & 0\\ 0 & -\gamma^2 I\end{bmatrix}+\left(\frac{1}{2(r-1)}(A_{ii}+A_{ii})+\frac{1}{2}(A_{ij}+A_{ji})\right)^T\begin{bmatrix}P\\0\end{bmatrix}^T\\
&+\begin{bmatrix}P\\0\end{bmatrix}\left(\frac{1}{2(r-1)}(A_{ii}+A_{ii})+\frac{1}{2}(A_{ij}+A_{ji})\right)\\
&+\begin{bmatrix}Y_{Aii}\\Y_{Aij}\end{bmatrix}^T\begin{bmatrix}\tilde{\Delta}_A(t) & 0\\ 0 & \tilde{\Delta}_A(t)\end{bmatrix}^T\begin{bmatrix}\frac{1}{2(r-1)}X_{Aii} & \frac{1}{2}X_{Aij}\end{bmatrix}^T\begin{bmatrix}P\\0\end{bmatrix}^T\\
&+\begin{bmatrix}P\\0\end{bmatrix}\begin{bmatrix}\frac{1}{2(r-1)}X_{Aii} & \frac{1}{2}X_{Aij}\end{bmatrix}\begin{bmatrix}\tilde{\Delta}_A(t) & 0\\ 0 & \tilde{\Delta}_A(t)\end{bmatrix}\begin{bmatrix}Y_{Aii}\\Y_{Aij}\end{bmatrix}\\
&+\left(\begin{bmatrix}\frac{1}{\sqrt{4(r-1)}}(C_{ii}+C_{ii})\\ \frac{1}{2}(C_{ij}+C_{ji})\end{bmatrix}+\begin{bmatrix}\frac{1}{\sqrt{4(r-1)}}X_{Cii} & 0\\ 0 & \frac{1}{2}X_{Cij}\end{bmatrix}\right.\\
&\left.\times\begin{bmatrix}\tilde{\Delta}_C(t) & 0\\ 0 & \tilde{\Delta}_C(t)\end{bmatrix}\begin{bmatrix}Y_{Cii}\\Y_{Cij}\end{bmatrix}\right)^T\\
&\times\left(\begin{bmatrix}\frac{1}{\sqrt{4(r-1)}}(C_{ii}+C_{ii})\\ \frac{1}{2}(C_{ij}+C_{ji})\end{bmatrix}+\begin{bmatrix}\frac{1}{\sqrt{4(r-1)}}X_{Cii} & 0\\ 0 & \frac{1}{2}X_{Cij}\end{bmatrix}\right.\\
&\left.\times\begin{bmatrix}\tilde{\Delta}_C(t) & 0\\ 0 & \tilde{\Delta}_C(t)\end{bmatrix}\begin{bmatrix}Y_{Cii}\\Y_{Cij}\end{bmatrix}\right),\quad i,j=1,2,\ldots,r,\quad i\neq j.
\end{aligned}\tag{2.30}
$$

Using Lemmas 1.2 and 1.4, it is to see that (2.25) is guaranteed if the following inequality is satisfied:

$$
\begin{aligned}
&\frac{r}{r-1}\begin{bmatrix}0 & 0\\ 0 & -\gamma^2 I\end{bmatrix}+\left(\frac{1}{2(r-1)}(A_{ii}+A_{ii})+\frac{1}{2}(A_{ij}+A_{ji})\right)^T\begin{bmatrix}P\\0\end{bmatrix}^T\\
&+\begin{bmatrix}P\\0\end{bmatrix}\left(\frac{1}{2(r-1)}(A_{ii}+A_{ii})+\frac{1}{2}(A_{ij}+A_{ji})\right)\\
&+\varepsilon_{ij}^{-1}\begin{bmatrix}P\\0\end{bmatrix}\begin{bmatrix}\frac{1}{2(r-1)}X_{Aii} & \frac{1}{2}X_{Aij}\end{bmatrix}\begin{bmatrix}\frac{1}{2(r-1)}X_{Aii} & \frac{1}{2}X_{Aij}\end{bmatrix}^T\begin{bmatrix}P\\0\end{bmatrix}^T\\
&+\varepsilon_{ij}\begin{bmatrix}Y_{Aii}\\Y_{Aij}\end{bmatrix}^T\begin{bmatrix}Y_{Aii}\\Y_{Aij}\end{bmatrix}+\begin{bmatrix}\frac{1}{\sqrt{4(r-1)}}(C_{ii}+C_{ii})\\ \frac{1}{2}(C_{ij}+C_{ji})\end{bmatrix}^T\\
&\times\left(I-\sigma_{ij}^{-1}\begin{bmatrix}\frac{1}{\sqrt{4(r-1)}}X_{Cii} & 0\\ 0 & \frac{1}{2}X_{Cij}\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{4(r-1)}}X_{Cii} & 0\\ 0 & \frac{1}{2}X_{Cij}\end{bmatrix}^T\right)^{-1}\\
&\times\begin{bmatrix}\frac{1}{\sqrt{4(r-1)}}(C_{ii}+C_{ii})\\ \frac{1}{2}(C_{ij}+C_{ji})\end{bmatrix}+\sigma_{ij}\begin{bmatrix}Y_{Cii}\\Y_{Cij}\end{bmatrix}^T\begin{bmatrix}Y_{Cii}\\Y_{Cij}\end{bmatrix}<0,\\
&\qquad\qquad\qquad\qquad i,j=1,2,\ldots,r,\quad i\neq j.
\end{aligned}\tag{2.31}
$$

Using Lemma 1.5 and the Schur complement, (2.31) can be verified by

$$\begin{bmatrix} \lambda_{1ij} & * & * & * & * & * & * & * \\ \lambda_{2ij} & -G_{ij}-G_{ij}^T & * & * & * & * & * & * \\ \frac{1}{2(r-1)}X_{Aii}^T\begin{bmatrix} P \\ 0 \end{bmatrix}^T & 0 & -\varepsilon_{ij}I & * & * & * & * & * \\ \frac{1}{2}X_{Aij}^T\begin{bmatrix} P \\ 0 \end{bmatrix}^T & 0 & 0 & -\varepsilon_{ij}I & * & * & * & * \\ \frac{1}{\sqrt{4(r-1)}}(C_{ii}+C_{ii}) & 0 & 0 & 0 & -I & * & * & * \\ \frac{1}{2}(C_{ij}+C_{ji}) & 0 & 0 & 0 & 0 & -I & * & * \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{4(r-1)}}X_{Cii}^T & 0 & -\sigma_{ij}I & * \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}X_{Cij}^T & 0 & -\sigma_{ij}I \end{bmatrix}$$
$$< 0, \quad i,j=1,2,\ldots,r, \quad i\neq j, \tag{2.32}$$

where

$$\begin{aligned} \lambda_{1ij} &= \frac{r}{r-1}\begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + \left(\frac{1}{2(r-1)}(A_{ii}+A_{ii}) + \frac{1}{2}(A_{ij}+A_{ji})\right)^T M_{ij}^T \\ &\quad + M_{ij}\left(\frac{1}{2(r-1)}(A_{ii}+A_{ii}) + \frac{1}{2}(A_{ij}+A_{ji})\right) \\ &\quad + \varepsilon_{ij}\begin{bmatrix} Y_{Aii} \\ Y_{Aij} \end{bmatrix}^T \begin{bmatrix} Y_{Aii} \\ Y_{Aij} \end{bmatrix} + \sigma_{ij}\begin{bmatrix} Y_{Cii} \\ Y_{Cij} \end{bmatrix}^T \begin{bmatrix} Y_{Cii} \\ Y_{Cij} \end{bmatrix}, \\ \lambda_{2ij} &= \begin{bmatrix} P \\ 0 \end{bmatrix}^T - M_{ij}^T + G_{ij}\left(\frac{1}{2(r-1)}(A_{ii}+A_{ii}) + \frac{1}{2}(A_{ij}+A_{ji})\right). \end{aligned}$$

Then, by choosing the matrices defined in (2.22), we obtain the inequality (2.23). Considering Lemma 1.10 with (2.24) and (2.25), we obtain

$$\begin{aligned} &\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t)) \\ &\times\left(A_{ij}^T\begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix}A_{ij} + \Delta A_{ij}^T(t)\begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix}\Delta A_{ij}(t) + \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix}\right. \\ &\left. + (C_{ij}+C_{ji}+\Delta C_{ij}(t)+\Delta C_{ji}(t))^T\frac{1}{4}(C_{ij}+C_{ji}+\Delta C_{ij}(t)+\Delta C_{ji}(t))\right) < 0. \end{aligned} \tag{2.33}$$

From the proof of Theorem 2.1, with the support of (2.33), it can be verified that $\dot{V}(\psi(t)) + e^T(t)e(t) - \gamma^2 w^T(t)w(t) < 0$ for any $\begin{bmatrix} \psi(t) \\ w(t) \end{bmatrix} \neq 0$.

The proof is completed. □

Theorem 2.3 *Consider the filtering error system* (2.3). *For a given scalar* $\gamma > 0$, *if there exist matrices* P_1, P_2, P_3, G, M_{1ij}, M_{2ij}, M_{3ij}, G_{1ij}, G_{2ij}, Υ_{ji}, $\mathscr{A}_{Fj}$, $\mathscr{B}_{Fj}$, $\mathscr{C}_{Fj}$, *and* $\mathscr{D}_{Fj}$, *scalars* ε_{ij} *and* σ_{ij}, *for* $i, j = 1, 2, \ldots, r$ *such that* (2.4) *and the following inequalities hold:*

$$\begin{bmatrix}
\theta_{11ii} - \Upsilon_{ii} - \Upsilon_{ii} & * & * & * & * & * & * & * & * & * & * & * & * & * \\
\theta_{21ii} & \theta_{22ii} & * & * & * & * & * & * & * & * & * & * & * & * \\
\theta_{31ii} & \theta_{32ii} & \theta_{33ii} & * & * & * & * & * & * & * & * & * & * & * \\
\theta_{41ii} & \theta_{42ii} & \theta_{43ii} & \theta_{44ii} & * & * & * & * & * & * & * & * & * & * \\
\theta_{51ii} & \theta_{52ii} & \theta_{53ii} & \theta_{54ii} & \theta_{55} & * & * & * & * & * & * & * & * & * \\
H_{Bi}^T P_2 & H_{Bi}^T P_3 & 0 & 0 & 0 & -\varepsilon_{ii} I & * & * & * & * & * & * & * & * \\
H_{Ai}^T P_2 & H_{Ai}^T P_3 & 0 & 0 & 0 & 0 & -\varepsilon_{ii} I & * & * & * & * & * & * & * \\
H_{Bi}^T P_2 & H_{Bi}^T P_3 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{ii} I & * & * & * & * & * & * \\
H_{Ai}^T P_2 & H_{Ai}^T P_3 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{ii} I & * & * & * & * & * \\
\theta_{101ii} & \theta_{102ii} & \theta_{103ii} & 0 & 0 & 0 & 0 & 0 & 0 & -2I & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -H_{Di}^T & -\sigma_{ii} I & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -H_{Ci}^T & 0 & -\sigma_{ii} I & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -H_{Di}^T & 0 & 0 & -\sigma_{ii} I & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -H_{Ci}^T & 0 & 0 & 0 & -\sigma_{ii} I
\end{bmatrix} < 0,$$

$$i = 1, 2, \ldots, r, \tag{2.34}$$

$$
\begin{bmatrix}
\theta_{11ij}-\varUpsilon_{ji}-\varUpsilon_{ji}^T & * & * & * & * & * & * & * & * & * & * & * & * & * \\
\theta_{21ij} & \theta_{22ij} & * & * & * & * & * & * & * & * & * & * & * & * \\
\theta_{31ij} & \theta_{32ij} & \theta_{33ij} & * & * & * & * & * & * & * & * & * & * & * \\
\theta_{41ij} & \theta_{42ij} & \theta_{43ij} & \theta_{44ij} & * & * & * & * & * & * & * & * & * & * \\
\theta_{51ij} & \theta_{52ij} & \theta_{53ij} & \theta_{54ij} & \theta_{55} & * & * & * & * & * & * & * & * & * \\
H_{Bj}^T P_2 & H_{Bj}^T P_3 & 0 & 0 & 0 & -\varepsilon_{ij} I & * & * & * & * & * & * & * & * \\
H_{Aj}^T P_2 & H_{Aj}^T P_3 & 0 & 0 & 0 & 0 & -\varepsilon_{ij} I & * & * & * & * & * & * & * \\
H_{Bi}^T P_2 & H_{Bi}^T P_3 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{ij} I & * & * & * & * & * & * \\
H_{Ai}^T P_2 & H_{Ai}^T P_3 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{ij} I & * & * & * & * & * \\
\theta_{101ij} & \theta_{102ij} & \theta_{103ij} & 0 & 0 & 0 & 0 & 0 & 0 & -2I & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -H_{Dj}^T & -\sigma_{ij} I & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -H_{Cj}^T & 0 & -\sigma_{ij} I & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -H_{Di}^T & 0 & 0 & -\sigma_{ij} I & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -H_{Ci}^T & 0 & 0 & 0 & -\sigma_{ij} I
\end{bmatrix} < 0,
$$

$$
i, j = 1, 2, \ldots, r, \quad i < j, \tag{2.35}
$$

$$
\begin{bmatrix}
\varUpsilon_{11} & * & \ldots & * \\
\varUpsilon_{21} & \varUpsilon_{22} & \ldots & * \\
\vdots & \vdots & \ddots & \vdots \\
\varUpsilon_{r1} & \varUpsilon_{r2} & \ldots & \varUpsilon_{rr}
\end{bmatrix} < 0, \tag{2.36}
$$

then the prescribed H_∞ performance $\gamma > 0$ is guaranteed. The matrices for an H_∞ filter in the form of (2.2) *are given by* (2.7).

Proof From (2.10), we can know that $\dot{V}(\psi(t)) + e^T(t)e(t) - \gamma^2 w^T(t)w(t) < 0$ for any $\begin{bmatrix} \psi(t) \\ w(t) \end{bmatrix} \neq 0$ if (2.37) holds.

$$
\begin{aligned}
&\sum_{i=1}^{r} h_i^2(\xi(t)) \\
&\quad \times \left(A_{ii}^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} A_{ii} + \Delta A_{ii}^T(t) \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} \Delta A_{ii}(t) + \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \right. \\
&\quad \left. + (C_{ii} + C_{ii} + \Delta C_{ii}(t) + \Delta C_{ii}(t))^T \frac{1}{4} (C_{ii} + C_{ii} + \Delta C_{ii}(t) + \Delta C_{ii}(t)) \right) \\
&\quad + \sum_{i=1}^{r} \sum_{i<j}^{r} h_i(\xi(t)) h_j(\xi(t)) \\
&\quad \times \left(A_{ij}^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} A_{ij} + \Delta A_{ij}^T(t) \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} \Delta A_{ij}(t) + \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \right. \\
&\quad + (C_{ij} + C_{ji} + \Delta C_{ij}(t) + \Delta C_{ji}(t))^T \frac{1}{4} \\
&\quad \left. \times (C_{ij} + C_{ji} + \Delta C_{ij}(t) + \Delta C_{ji}(t)) \right) \\
&\quad + \left(A_{ji}^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} A_{ji} + \Delta A_{ji}^T(t) \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} \Delta A_{ji}(t) + \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \right. \\
&\quad + (C_{ji} + C_{ij} + \Delta C_{ji}(t) + \Delta C_{ij}(t))^T \frac{1}{4} \\
&\quad \left. \times (C_{ji} + C_{ij} + \Delta C_{ji}(t) + \Delta C_{ij}(t)) \right) < 0.
\end{aligned}
\tag{2.37}
$$

Define

$$
\begin{aligned}
\mathcal{N}_{ij} = {} & A_{ij}^T \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} A_{ij} + \Delta A_{ij}^T(t) \begin{bmatrix} P \\ 0 \end{bmatrix}^T + \begin{bmatrix} P \\ 0 \end{bmatrix} \Delta A_{ij}(t) + \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \\
& + (C_{ij} + C_{ji} + \Delta C_{ij}(t) + \Delta C_{ji}(t))^T \frac{1}{4} \\
& \times (C_{ij} + C_{ji} + \Delta C_{ij}(t) + \Delta C_{ji}(t)).
\end{aligned}
$$

By Lemma 1.11, (2.37) holds if the following conditions are fulfilled:

$$
\mathcal{N}_{ii} < \Upsilon_{ii}, \quad i = 1, 2, \ldots, r, \tag{2.38}
$$

$$
\mathcal{N}_{ij} + \mathcal{N}_{ji} \leq \Upsilon_{ji} + \Upsilon_{ji}^T, \quad i, j = 1, 2, \ldots, r, \quad i < j, \tag{2.39}
$$

$$\begin{bmatrix} \Upsilon_{11} & * & \dots & * \\ \Upsilon_{21} & \Upsilon_{22} & \dots & * \\ \vdots & \vdots & \ddots & \vdots \\ \Upsilon_{r1} & \Upsilon_{r2} & \dots & \Upsilon_{rr} \end{bmatrix} < 0. \tag{2.40}$$

Obviously, those LMI properties in the proof of Theorem 2.1 can also be used for inequalities (2.38) and (2.39). Thus, conditions (2.34)–(2.36) are obtained. □

Remark 2.2 Because there are uncertainties in the designed filter, the final LMI conditions are not in the form (2.33) or (2.37). Chang and Yang [1] had verified that the relaxed properties of Lemmas 1.10 and 1.11 cannot be efficiently embodied in Theorems 2.2 and 2.3.

2.2.2 Discrete-Time Case

For the discrete-time case, the filtering error system (2.3) becomes as

$$\begin{aligned} \psi(k+1) &= \tilde{A}(h)\psi(k) + \tilde{B}(h)w(k), \\ e(k) &= \tilde{C}(h)\psi(k) + \tilde{D}(h)w(k). \end{aligned} \tag{2.41}$$

Theorem 2.4 *Consider the filtering error system* (2.41)*. For a given scalar $\gamma > 0$, if there exist matrices P_1, P_2, P_3, G_{1ij}, G, G_{3ij}, $\mathscr{A}_{Fj}$, $\mathscr{B}_{Fj}$, $\mathscr{C}_{Fj}$, and $\mathscr{D}_{Fj}$, scalars σ_{Aij}, and σ_{Cij}, for $i, j = 1, 2, \dots, r$ such that* (2.4) *and the following inequalities hold:*

$$\left[\begin{array}{cccccccc}
\delta_{11ii} & * & * & * & * & * & * & * \\
-2P_2 & \delta_{22ii} & * & * & * & * & * & * \\
\delta_{31ii} & 0 & \delta_{33ii} & * & * & * & * & * \\
\delta_{41ii} & \delta_{42ii} & \delta_{43ii} & \delta_{44ii} & * & * & * & * \\
\delta_{51ii} & \delta_{52ii} & \delta_{53ii} & \delta_{54ii} & \delta_{55} & * & * & * \\
\delta_{61ii} & \delta_{62ii} & \delta_{63ii} & 0 & 0 & -2I & * & * \\
0 & 0 & 0 & H_{Bi}^T G^T & H_{Bi}^T G^T & 0 & -\sigma_{Aii} I & * \\
0 & 0 & 0 & H_{Ai}^T G^T & H_{Ai}^T G^T & 0 & 0 & -\sigma_{Aii} I \\
0 & 0 & 0 & H_{Bi}^T G^T & H_{Bi}^T G^T & 0 & 0 & 0 \\
0 & 0 & 0 & H_{Ai}^T G^T & H_{Ai}^T G^T & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -H_{Di}^T & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -H_{Ci}^T & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -H_{Di}^T & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -H_{Ci}^T & 0 & 0
\end{array}\right.$$

$$
\left.\begin{matrix}
* & * & * & * & * & * \\
* & * & * & * & * & * \\
* & * & * & * & * & * \\
* & * & * & * & * & * \\
* & * & * & * & * & * \\
* & * & * & * & * & * \\
* & * & * & * & * & * \\
* & * & * & * & * & * \\
-\sigma_{Aii} I & * & * & * & * & * \\
0 & -\sigma_{Aii} I & * & * & * & * \\
0 & 0 & -\sigma_{Cii} I & * & * & * \\
0 & 0 & 0 & -\sigma_{Cii} I & * & * \\
0 & 0 & 0 & 0 & -\sigma_{Cii} I & * \\
0 & 0 & 0 & 0 & 0 & -\sigma_{Cii} I
\end{matrix}\right] < 0, \quad i = 1, 2, \ldots, r, \qquad (2.42)
$$

$$
\left[\begin{matrix}
\delta_{11ij} & * & * & * & * & * & * & * \\
-2P_2 & \delta_{22ij} & * & * & * & * & * & * \\
\delta_{31ij} & 0 & \delta_{33ij} & * & * & * & * & * \\
\delta_{41ij} & \delta_{42ij} & \delta_{43ij} & \delta_{44ij} & * & * & * & * \\
\delta_{51ij} & \delta_{52ij} & \delta_{53ij} & \delta_{54ij} & \delta_{55} & * & * & * \\
\delta_{61ij} & \delta_{62ij} & \delta_{63ij} & 0 & 0 & -2I & * & * \\
0 & 0 & 0 & H_{Bj}^T G^T & H_{Bj}^T G^T & 0 & -\sigma_{Aij} I & * \\
0 & 0 & 0 & H_{Aj}^T G^T & H_{Aj}^T G^T & 0 & 0 & -\sigma_{Aij} I \\
0 & 0 & 0 & H_{Bi}^T G^T & H_{Bi}^T G^T & 0 & 0 & 0 \\
0 & 0 & 0 & H_{Ai}^T G^T & H_{Ai}^T G^T & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -H_{Dj}^T & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -H_{Cj}^T & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -H_{Di}^T & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -H_{Ci}^T & 0 & 0
\end{matrix}\right.
$$

$$
\left.\begin{matrix}
* & * & * & * & * & * \\
* & * & * & * & * & * \\
* & * & * & * & * & * \\
* & * & * & * & * & * \\
* & * & * & * & * & * \\
* & * & * & * & * & * \\
* & * & * & * & * & * \\
* & * & * & * & * & * \\
-\sigma_{Aij} I & * & * & * & * & * \\
0 & -\sigma_{Aij} I & * & * & * & * \\
0 & 0 & -\sigma_{Cij} I & * & * & * \\
0 & 0 & 0 & -\sigma_{Cij} I & * & * \\
0 & 0 & 0 & 0 & -\sigma_{Cij} I & * \\
0 & 0 & 0 & 0 & 0 & -\sigma_{Cij} I
\end{matrix}\right] < 0, \qquad (2.43)
$$

$$
i, j = 1, 2, \ldots, r, \quad i < j,
$$

where

$$
\begin{aligned}
\delta_{11ij} =& -2P_1 + \sigma_{Aij} C_i^T E_{Bj}^T E_{Bj} C_i + \sigma_{Aij} C_j^T E_{Bi}^T E_{Bi} C_j \\
& + \sigma_{Cij} C_i^T E_{Dj}^T E_{Dj} C_i + \sigma_{Cij} C_j^T E_{Di}^T E_{Di} C_j, \\
\delta_{22ij} =& -2P_3 + \sigma_{Aij} E_{Aj}^T E_{Aj} + \sigma_{Aij} E_{Ai}^T E_{Ai} + \sigma_{Cij} E_{Cj}^T E_{Cj} + \sigma_{Cij} E_{Ci}^T E_{Ci}, \\
\delta_{31ij} =& \sigma_{Aij} D_i^T E_{Bj}^T E_{Bj} C_i + \sigma_{Aij} D_j^T E_{Bi}^T E_{Bi} C_j \\
& + \sigma_{Cij} D_i^T E_{Dj}^T E_{Dj} C_i + \sigma_{Cij} D_j^T E_{Di}^T E_{Di} C_j, \\
\delta_{33ij} =& \sigma_{Aij} D_i^T E_{Bj}^T E_{Bj} D_i + \sigma_{Aij} D_j^T E_{Bi}^T E_{Bi} D_j \\
& + \sigma_{Cij} D_i^T E_{Dj}^T E_{Dj} D_i + \sigma_{Cij} D_j^T E_{Di}^T E_{Di} D_j - 2\gamma^2 I, \\
\delta_{41ij} =& G_{1ij} A_i + \mathscr{B}_{Fj} C_i + G_{1ij} A_j + \mathscr{B}_{Fi} C_j, \\
\delta_{42ij} =& \mathscr{A}_{Fj} + \mathscr{A}_{Fi}, \\
\delta_{43ij} =& G_{1ij} B_i + \mathscr{B}_{Fj} D_i + G_{1ij} B_j + \mathscr{B}_{Fi} D_j, \\
\delta_{44ij} =& 2\left(-G_{1ij} - G_{1ij}^T + P_1\right), \\
\delta_{51ij} =& G_{3ij} A_i + B_{Fj} C_i + G_{3ij} A_j + B_{Fi} C_j, \\
\delta_{52ij} =& \mathscr{A}_{Fj} + \mathscr{A}_{Fi}, \\
\delta_{53ij} =& G_{3ij} B_i + \mathscr{B}_{Fj} D_i + G_{3ij} B_j + \mathscr{B}_{Fi} D_j, \\
\delta_{54ij} =& 2(-G_{3ij} - G^T + P_2), \\
\delta_{55} =& 2(-G - G^T + P_3), \\
\delta_{61ij} =& L_i - \mathscr{D}_{Fj} C_i + L_j - \mathscr{D}_{Fi} C_j, \\
\delta_{62ij} =& -\mathscr{C}_{Fj} - \mathscr{C}_{Fi}, \\
\delta_{63ij} =& -\mathscr{D}_{Fj} D_i - \mathscr{D}_{Fi} D_j,
\end{aligned}
$$

for $i, j = 1, 2, \ldots, r$, then the prescribed H_∞ performance $\gamma > 0$ is guaranteed. The matrices for an H_∞ filter in the form of (2.2) *are given by* (2.7).

Proof Consider the following Lyapunov function

$$V(\psi(k)) = \psi^T(k) P \psi(k), \quad P > 0.$$

From (2.41), we have

$$
\begin{aligned}
& V(\psi(k+1)) - V(\psi(k)) + e^T(k)e(k) - \gamma^2 w^T(k)w(k) \\
&= \begin{bmatrix} \psi(k) \\ w(k) \end{bmatrix}^T \left(\begin{bmatrix} \tilde{A}(h) & \tilde{B}(h) \end{bmatrix}^T P \begin{bmatrix} \tilde{A}(h) & \tilde{B}(h) \end{bmatrix} + [\tilde{C}(h) \ \tilde{D}(h)]^T [\tilde{C}(h) \ \tilde{D}(h)] \right. \\
&\quad \left. + \begin{bmatrix} -P & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \right) \begin{bmatrix} \psi(k) \\ w(k) \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&= \begin{bmatrix} \psi(k) \\ w(k) \end{bmatrix}^T \Bigg(\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(k)) h_j(\xi(k)) (A_{ij} + \Delta A_{ij})^T P \\
&\quad \times \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(k)) h_j(\xi(k)) (A_{ij} + \Delta A_{ij}) \\
&\quad + \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(k)) h_j(\xi(k)) (C_{ij} + \Delta C_{ij})^T \\
&\quad \times \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(k)) h_j(\xi(k)) (C_{ij} + \Delta C_{ij}) - \begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix} \Bigg) \begin{bmatrix} \psi(k) \\ w(k) \end{bmatrix} \\
&= \begin{bmatrix} \psi(k) \\ w(k) \end{bmatrix}^T \Bigg(\frac{1}{8} \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{d=1}^{r} \sum_{s=1}^{r} h_i(\xi(k)) h_j(\xi(k)) h_d(\xi(k)) h_s(\xi(k)) \\
&\quad \times ((A_{ij} + \Delta A_{ij}(k) + A_{ji} + \Delta A_{ji}(k))^T \\
&\quad \times P^{\frac{1}{2}} P^{\frac{1}{2}} (A_{ds} + \Delta A_{ds}(k) + A_{sd} + \Delta A_{sd}(k)) \\
&\quad + (A_{ds} + \Delta A_{ds}(k) + A_{sd} + \Delta A_{sd}(k))^T \\
&\quad \times P^{\frac{1}{2}} P^{\frac{1}{2}} (A_{ij} + \Delta A_{ij}(k) + A_{ji} + \Delta A_{ji}(k))) \\
&\quad + \frac{1}{8} \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{d=1}^{r} \sum_{s=1}^{r} h_i(\xi(k)) h_j(\xi(k)) h_d(\xi(k)) h_s(\xi(k)) \\
&\quad \times ((C_{ij} + \Delta C_{ij}(k) + C_{ji} + \Delta C_{ji}(k))^T (C_{ds} + \Delta C_{ds}(k) + C_{sd} + \Delta C_{sd}(k)) \\
&\quad + (C_{ds} + \Delta C_{ds}(k) + C_{sd} + \Delta C_{sd}(k))^T (C_{ij} + \Delta C_{ij}(k) + C_{ji} + \Delta C_{ji}(k))) \\
&\quad - \begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix} \Bigg) \begin{bmatrix} \psi(k) \\ w(k) \end{bmatrix},
\end{aligned}
$$

where

$$
\begin{aligned}
A_{ij} &= \begin{bmatrix} A_i & 0 & B_i \\ B_{Fj} C_i & A_{Fj} & B_{Fj} D_i \end{bmatrix}, \\
\Delta A_{ij}(k) &= \begin{bmatrix} 0 & 0 & 0 \\ H_{Bj} \Delta_B(k) E_{Bj} C_i & H_{Aj} \Delta_A(k) E_{Aj} & H_{Bj} \Delta_B(k) E_{Bj} D_i, \end{bmatrix}, \\
C_{ij} &= [L_i - D_{Fj} C_i \;\; -C_{Fj} \;\; -D_{Fj} D_i], \\
\Delta C_{ij}(k) &= [-H_{Dj} \Delta_D(k) E_{Dj} C_i \;\; -H_{Cj} \Delta_C(k) E_{Cj} \;\; -H_{Dj} \Delta_D(k) E_{Dj} D_i].
\end{aligned}
$$

Using Lemma 1.2 with $\varepsilon = 1$, we have

$$
\begin{aligned}
&V(\psi(k+1)) - V(\psi(k)) + e^T(k)e(k) - \gamma^2 w^T(k)w(k) \\
&\le \begin{bmatrix} \psi(k) \\ w(k) \end{bmatrix}^T \left(\frac{1}{8} \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{d=1}^{r} \sum_{s=1}^{r} h_i(\xi(k)) h_j(\xi(k)) h_d(\xi(k)) h_s(\xi(k)) \right. \\
&\quad \times \Bigg((A_{ij} + \Delta A_{ij}(k) + A_{ji} + \Delta A_{ji}(k))^T P (A_{ij} + \Delta A_{ij}(k) + A_{ji} + \Delta A_{ji}(k)) \\
&\quad + (A_{ds} + \Delta A_{ds}(k) + A_{sd} + \Delta A_{sd}(k))^T P (A_{ds} + \Delta A_{ds}(k) + A_{sd} + \Delta A_{sd}(k)) \\
&\quad + (C_{ij} + \Delta C_{ij}(k) + C_{ji} + \Delta C_{ji}(k))^T (C_{ij} + \Delta C_{ij}(k) + C_{ji} + \Delta C_{ji}(k)) \\
&\quad + (C_{ds} + \Delta C_{ds}(k) + C_{sd} + \Delta C_{sd}(k))^T (C_{ds} + \Delta C_{ds}(k) + C_{sd} + \Delta C_{sd}(k)) \\
&\quad \left. - \begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix} \Bigg) \right) \begin{bmatrix} \psi(k) \\ w(k) \end{bmatrix} \\
&= \begin{bmatrix} \psi(k) \\ w(k) \end{bmatrix}^T \left(\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(k)) h_j(\xi(k)) \right. \\
&\quad \times \Big((A_{ij} + \Delta A_{ij}(k) + A_{ji} + \Delta A_{ji}(k))^T \frac{P}{4} (A_{ij} + \Delta A_{ij}(k) + A_{ji} + \Delta A_{ji}(k)) \\
&\quad + (C_{ij} + \Delta C_{ij}(k) + C_{ji} + \Delta C_{ji}(k))^T \frac{1}{4} (C_{ij} + \Delta C_{ij}(k) + C_{ji} + \Delta C_{ji}(k)) \\
&\quad \left. - \begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix} \Big) \right) \begin{bmatrix} \psi(k) \\ w(k) \end{bmatrix} \\
&= \begin{bmatrix} \psi(k) \\ w(k) \end{bmatrix}^T \left(\sum_{i=1}^{r} h_i^2(\xi(k)) \right. \\
&\quad \times \Big((A_{ii} + \Delta A_{ii}(k) + A_{ii} + \Delta A_{ii}(k))^T \frac{P}{4} (A_{ii} + \Delta A_{ii}(k) + A_{ii} + \Delta A_{ii}(k)) \\
&\quad + (C_{ii} + \Delta C_{ii}(k) + C_{ii} + \Delta C_{ii}(k))^T \frac{1}{4} (C_{ii} + \Delta C_{ii}(k) + C_{ii} + \Delta C_{ii}(k)) \\
&\quad - \begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix} \Big) + \sum_{i=1}^{r} \sum_{i<j}^{r} h_i(\xi(k)) h_j(\xi(k)) \\
&\quad \times \Bigg((A_{ij} + \Delta A_{ij}(k) + A_{ji} + \Delta A_{ji}(k))^T \frac{P}{2} (A_{ij} + \Delta A_{ij}(k) + A_{ji} + \Delta A_{ji}(k)) \\
&\quad + (C_{ij} + \Delta C_{ij}(k) + C_{ji} + \Delta C_{ji}(k))^T \frac{1}{2} (C_{ij} + \Delta C_{ij}(k) + C_{ji} + \Delta C_{ji}(k)) \\
&\quad \left. - 2 \begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix} \Bigg) \right) \begin{bmatrix} \psi(k) \\ w(k) \end{bmatrix}.
\end{aligned}
$$

Thus, $V(\psi(k+1)) - V(\psi(k)) + e^T(k)e(k) - \gamma^2 w^T(k)w(k) < 0$ for any $\begin{bmatrix} \psi(k) \\ w(k) \end{bmatrix} \neq 0$ if

$$
\begin{aligned}
&(A_{ii} + A_{ii} + X_{Aii}\bar{\Delta}A(k)Y_{Aii})^T \frac{P}{2}(A_{ii} + A_{ii} + X_{Aii}\bar{\Delta}A(k)Y_{Aii}) \\
&+ (C_{ii} + C_{ii} + X_{Cii}\bar{\Delta}C(k)Y_{Cii})^T \frac{1}{2}(C_{ii} + C_{ii} + X_{Cii}\bar{\Delta}C(k)Y_{Cii}) \\
&- 2\begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix} < 0, \quad i = 1, 2, \ldots, r,
\end{aligned} \tag{2.44}
$$

and

$$
\begin{aligned}
&(A_{ij} + A_{ji} + X_{Aij}\bar{\Delta}A(k)Y_{Aij})^T \frac{P}{2}(A_{ij} + A_{ji} + X_{Aij}\bar{\Delta}A(k)Y_{Aij}) \\
&+ (C_{ij} + C_{ji} + X_{Cij}\bar{\Delta}C(k)Y_{Cij})^T \frac{1}{2}(C_{ij} + C_{ji} + X_{Cij}\bar{\Delta}C(k)Y_{Cij}) \\
&- 2\begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix} < 0, \quad i, j = 1, 2, \ldots, r, \quad i < j,
\end{aligned} \tag{2.45}
$$

where

$$
\begin{aligned}
X_{Aij} &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ H_{Bj} & H_{Aj} & H_{Bi} & H_{Ai} \end{bmatrix}, \quad Y_{Aij} = \begin{bmatrix} E_{Bj}C_i & 0 & E_{Bj}D_i \\ 0 & E_{Aj} & 0 \\ E_{Bi}C_j & 0 & E_{Bi}D_j \\ 0 & E_{Ai} & 0 \end{bmatrix}, \\
\bar{\Delta}A(k) &= \mathrm{diag}\{\Delta_B(k),\ \Delta_A(k),\ \Delta_B(k),\ \Delta_A(k)\}, \\
X_{Cij} &= [-H_{Dj} \ \ -H_{Cj} \ \ -H_{Di} \ \ -H_{Ci}], \\
Y_{Cij} &= \begin{bmatrix} E_{Dj}C_i & 0 & E_{Dj}D_i \\ 0 & E_{Cj} & 0 \\ E_{Di}C_j & 0 & E_{Di}D_j \\ 0 & E_{Ci} & 0 \end{bmatrix}, \\
\bar{\Delta}C(k) &= \mathrm{diag}\{\Delta_D(k),\ \Delta_C(k),\ \Delta_D(k),\ \Delta_C(k)\}.
\end{aligned}
$$

for $i, j = 1, 2, \ldots, r$.

Using Lemma 1.4, we have

$$
\begin{aligned}
&(A_{ii} + A_{ii} + X_{Aii}\bar{\Delta}A(k)Y_{Aii})^T \frac{P}{2}(A_{ii} + A_{ii} + X_{Aii}\bar{\Delta}A(k)Y_{Aii}) \\
&+ (C_{ii} + C_{ii} + X_{Cii}\bar{\Delta}C(k)Y_{Cii})^T \\
&\times \frac{1}{2}(C_{ii} + C_{ii} + X_{Cii}\bar{\Delta}C(k)Y_{Cii}) - 2\begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix}
\end{aligned}
$$

$$\leq (A_{ii} + A_{ii})^T \left(2P^{-1} - \sigma_{Aii}^{-1} X_{Aii} X_{Aii}^T\right)^{-1} (A_{ii} + A_{ii}) + \sigma_{Aii} Y_{Aii}^T Y_{Aii}$$
$$+ (C_{ii} + C_{ii})^T \left(2I - \sigma_{Cii}^{-1} X_{Cii} X_{Cii}^T\right)^{-1} (C_{ii} + C_{ii})$$
$$+ \sigma_{Aii} Y_{Cii}^T Y_{Cii} - 2 \begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix}, \quad i = 1, 2, \ldots, r, \tag{2.46}$$

and

$$(A_{ij} + A_{ji} + X_{Aij} \bar{\Delta} A(k) Y_{Aij})^T \frac{P}{2} (A_{ij} + A_{ji} + X_{Aij} \bar{\Delta} A(k) Y_{Aij})$$
$$+ (C_{ij} + C_{ji} + X_{Cij} \bar{\Delta} C(k) Y_{Cij})^T \frac{1}{2} (C_{ij} + C_{ji} + X_{Cij} \bar{\Delta} C(k) Y_{Cij})$$
$$- 2 \begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix}$$
$$\leq (A_{ij} + A_{ji})^T \left(2P^{-1} - \sigma_{Aij}^{-1} X_{Aij} X_{Aij}^T\right)^{-1} (A_{ij} + A_{ji}) + \sigma_{Aij} Y_{Aij}^T Y_{Aij}$$
$$+ (C_{ij} + C_{ji})^T \left(2I - \sigma_{Cij}^{-1} X_{Cij} X_{Cij}^T\right)^{-1} (C_{ij} + C_{ji}) + \sigma_{Cij} Y_{Cij}^T Y_{Cij}$$
$$- 2 \begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix}, \quad i, j = 1, 2, \ldots, r, \quad i < j. \tag{2.47}$$

The conditions (2.44) and (2.45) hold if the following conditions are satisfied:

$$(A_{ii} + A_{ii})^T \left(2P^{-1} - \sigma_{Aii}^{-1} X_{Aii} X_{Aii}^T\right)^{-1} (A_{ii} + A_{ii}) + \sigma_{Aii} Y_{Aii}^T Y_{Aii}$$
$$+ (C_{ii} + C_{ii})^T \left(2I - \sigma_{Cii}^{-1} X_{Cii} X_{Cii}^T\right)^{-1} (C_{ii} + C_{ii})$$
$$+ \sigma_{Cii} Y_{Cii}^T Y_{Cii} - 2 \begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix} < 0, \quad i = 1, 2, \ldots, r, \tag{2.48}$$

and

$$(A_{ij} + A_{ji})^T \left(2P^{-1} - \sigma_{Aij}^{-1} X_{Aij} X_{Aij}^T\right)^{-1} (A_{ij} + A_{ji}) + \sigma_{Aij} Y_{Aij}^T Y_{Aij}$$
$$+ (C_{ij} + C_{ji})^T \left(2I - \sigma_{Cij}^{-1} X_{Cij} X_{Cij}^T\right)^{-1} (C_{ij} + C_{ji})$$
$$+ \sigma_{Cij} Y_{Cij}^T Y_{Cij} - 2 \begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix} < 0, \quad i, j = 1, 2, \ldots, r, \quad i < j. \tag{2.49}$$

Then, by Lemma 1.1 to (2.48) and (2.49), we have

$$\begin{bmatrix} -2\begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix} + \sigma_{Aii} Y_{Aii}^T Y_{Aii} + \sigma_{Cii} Y_{Cii}^T Y_{Cii} & * & * & * & * \\ A_{ii} + A_{ii} & -2P^{-1} & * & * & * \\ C_{ii} + C_{ii} & 0 & -2I & * & * \\ 0 & X_{Aii}^T & 0 & -\sigma_{Aii} I & * \\ 0 & 0 & X_{Cii}^T & 0 & -\sigma_{Cii} I \end{bmatrix} < 0, \quad i = 1, 2, \ldots, r, \tag{2.50}$$

and

$$\begin{bmatrix} -2\begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix} + \sigma_{Aij} Y_{Aij}^T Y_{Aij} + \sigma_{Cij} Y_{Cij}^T Y_{Cij} & * & * & * & * \\ A_{ij} + A_{ji} & -2P^{-1} & * & * & * \\ C_{ij} + C_{ji} & 0 & -2I & * & * \\ 0 & X_{Aij}^T & 0 & -\sigma_{Aij} I & * \\ 0 & 0 & X_{Cij}^T & 0 & -\sigma_{Cij} I \end{bmatrix} < 0, \quad i, j = 1, 2, \ldots, r, \quad i < j. \tag{2.51}$$

Pre- and post-multiplying (2.50) diag$\{I, G_{ii}, I, I, I\}$ and its transpose, respectively, (2.50) is equivalent to the following matrix inequality:

$$\begin{bmatrix} -2\begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix} + \sigma_{Aii} Y_{Aii}^T Y_{Aii} + \sigma_{Cii} Y_{Cii}^T Y_{Cii} & * & * & * & * \\ G_{ii}(A_{ii} + A_{ii}) & -2G_{ii} P^{-1} G_{ii}^T & * & * & * \\ C_{ii} + C_{ii} & 0 & -2I & * & * \\ 0 & X_{Aii}^T G_{ii}^T & 0 & -\sigma_{Aii} I & * \\ 0 & 0 & X_{Cii}^T & 0 & -\sigma_{Cii} I \end{bmatrix} < 0, \quad i = 1, 2, \ldots, r. \tag{2.52}$$

Pre- and post-multiplying (2.51) diag$\{I, G_{ij}, I, I, I\}$ and its transpose, respectively, (2.51) is equivalent to the following matrix inequality:

$$\begin{bmatrix} -2\begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix} + \sigma_{Aij} Y_{Aij}^T Y_{Aij} + \sigma_{Cij} Y_{Cij}^T Y_{Cij} & * & * & * & * \\ G_{ij}(A_{ij} + A_{ji}) & -2G_{ij} P^{-1} G_{ij}^T & * & * & * \\ C_{ij} + C_{ji} & 0 & -2I & * & * \\ 0 & X_{Aij}^T G_{ij}^T & 0 & -\sigma_{Aij} I & * \\ 0 & 0 & X_{Cij}^T & 0 & -\sigma_{Cij} I \end{bmatrix} < 0, \quad i, j = 1, 2, \ldots, r, \quad i < j. \tag{2.53}$$

The inequality $-(M-N)^T M^{-1}(M-N) \leq 0$ implies that $-N^T M^{-1} N \leq -N - N^T + M$, then (2.52) and (2.53) hold if

$$\begin{bmatrix} -2\begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix} + \sigma_{Aii} Y_{Aii}^T Y_{Aii} + \sigma_{Cii} Y_{Cii}^T Y_{Cii} & * & * & * & * \\ G_{ii}(A_{ii} + A_{ii}) & 2(-G_{ii} - G_{ii}^T + P) & * & * & * \\ C_{ii} + C_{ii} & 0 & -2I & * & * \\ 0 & X_{Aii}^T G_{ii}^T & 0 & -\sigma_{Aii} I & * \\ 0 & 0 & X_{Cii}^T & 0 & -\sigma_{Cii} I \end{bmatrix} < 0, \quad i = 1, 2, \ldots, r, \tag{2.54}$$

and

$$\begin{bmatrix} \varphi & * & * & * & * \\ G_{ij}(A_{ij} + A_{ji}) & 2(-G_{ij} - G_{ij}^T + P) & * & * & * \\ C_{ij} + C_{ji} & 0 & -2I & * & * \\ 0 & X_{Aij}^T G_{ij}^T & 0 & -\sigma_{Aij} I & * \\ 0 & 0 & X_{Cij}^T & 0 & -\sigma_{Cij} I \end{bmatrix} < 0, \quad i, j = 1, 2, \ldots, r, \quad i < j, \tag{2.55}$$

hold, where $\varphi = -2\begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix} + \sigma_{Aij} Y_{Aij}^T Y_{Aij} + \sigma_{Cij} Y_{Cij}^T Y_{Cij}$.

Now, we assume that matrices P, G_{ii}, and G_{ij} are of the following form:

$$P = \begin{bmatrix} P_1 & * \\ P_2 & P_3 \end{bmatrix} > 0, \quad G_{ii} = \begin{bmatrix} G_{1ii} & G \\ G_{3ii} & G \end{bmatrix}, \quad G_{ij} = \begin{bmatrix} G_{1ij} & G \\ G_{3ij} & G \end{bmatrix}. \tag{2.56}$$

From (2.54) to (2.56), we obtain (2.42) and (2.43) with $\mathscr{A}_{Fj} = GA_{Fj}$, $\mathscr{B}_{Fj} = GB_{Fj}$, $\mathscr{C}_{Fj} = C_{Fj}$, and $\mathscr{D}_{Fj} = D_{Fj}$.

Then, we have $V(\psi(k+1)) - V(\psi(k)) + e^T(k)e(k) - \gamma^2 w^T(k)w(k) < 0$ for any $\begin{bmatrix} \psi(k) \\ w(k) \end{bmatrix} \neq 0$, which implies that

$$V(\psi(\infty)) - V(\psi(0)) + \sum_{k=0}^{\infty} e^T(k)e(k) - \gamma^2 \sum_{k=0}^{\infty} w^T(k)w(k) < 0.$$

With zero initial condition $\psi(0) = 0$ and $V(\psi(\infty)) > 0$, we obtain $\sum_{k=0}^{\infty} e^T(k)e(k) < \gamma^2 \sum_{k=0}^{\infty} w^T(k)w(k)$ for any nonzero $w(k) \in l_2[0, \infty)$. Thus, the proof is complete. □

Remark 2.3 In contrast to the our existing results [2], Theorem 2.4 contributes in two aspects:

1. From the proof of Theorem 2.4, we can see that product terms

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(k))h_j(\xi(k))(A_{ij}+\Delta A_{ij})^T P \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(k))h_j(\xi(k))(A_{ij}+\Delta A_{ij})$$

and

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(k))h_j(\xi(k))(C_{ij}+\Delta C_{ij})^T \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(k))h_j(\xi(k))(C_{ij}+\Delta C_{ij})$$

are processed separately. This implies that the scalar $\tilde{\varepsilon}_{ij}$ in [2] are not shared, which brings more relaxed design conditions.

2. The dimension of the matrix consisting of X_{Aij} and X_{Cij} in Theorem 2.4 is smaller than $\tilde{\mathscr{H}}_{ij}$ in [2].

2.3 Numerical Example

A discrete-time T–S fuzzy plant model with the following two fuzzy rules is considered [5]:

$$\begin{aligned}
R^1 : &\text{ if } x_1(k) \text{ is } M_{11},\\
&\text{ then } x(k+1) = A_1x(k) + B_1w(k),\\
&\qquad y(k) = C_1x(k) + D_1w(k),\\
&\qquad z(k) = L_1x(k),\\
R^2 : &\text{ if } x_1(k) \text{ is } M_{12},\\
&\text{ then } x(k+1) = A_2x(k) + B_2w(k),\\
&\qquad y(k) = C_2x(k) + D_2w(k),\\
&\qquad z(k) = L_2x(k),
\end{aligned} \tag{2.57}$$

where

$$\begin{aligned}
A_1 &= \begin{bmatrix} 1.0500 & 0.3500 \\ -0.4200 & -0.0700 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.7920 & -0.4320 \\ -0.3600 & 0 \end{bmatrix},\\
B_1 &= \begin{bmatrix} -0.1000 \\ -0.0040 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.0100 \\ -0.1000 \end{bmatrix},\\
C_1 &= [1.7100\ 2.8500], \quad C_2 = [-1.9000\ 2.2800],\\
D_1 &= 0.005, \quad D_2 = 0.005,\\
L_1 &= [0.8100\ 0.2700], \quad L_2 = [0.4000\ 1.2000].
\end{aligned}$$

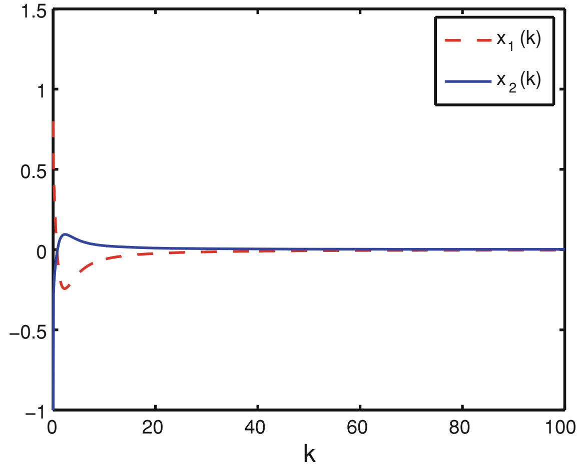

Fig. 2.2 State response of $x(k)$

The membership functions $h_1(k)$ and $h_2(k)$ are given, respectively, by

$$h_1(k) = \begin{cases} \left|\frac{\sin(x_1(k))}{x_1(k)}\right| & \text{for } x_1(k) \neq 0, \\ 1 & \text{for } x_1(k) = 0, \end{cases}$$

and

$$h_2(k) = 1 - h_1(k).$$

We give the known parameters in (2.2) as

$$\begin{aligned} H_{A1} &= \begin{bmatrix} 0.05 \\ 0.162 \end{bmatrix}, \quad H_{A2} = \begin{bmatrix} 0.264 \\ 0.123 \end{bmatrix}, \\ E_{A1} &= [0.15\ 0.2], \quad E_{A2} = [0.25\ 0.1], \\ H_{B1} &= \begin{bmatrix} 0.354 \\ 0.132 \end{bmatrix}, \quad H_{B2} = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}, \\ E_{B1} &= 0.45, \quad E_{B2} = 0.15, \\ H_{C1} &= 0.35, \quad H_{C2} = 0.45, \\ E_{C1} &= [0.23\ 0.35], \quad E_{C2} = [0.2\ 0.4], \\ H_{D1} &= 0.5, \quad H_{D2} = 0.3, \\ E_{D1} &= 0.1, \quad E_{D2} = 0.3. \end{aligned}$$

By using the Matlab LMI Control Toolbox [3] to solve (2.4), (2.42), and (2.43) in Theorem 2.4, we obtain the minimum H_∞ performance $\gamma_{\min}$ is 0.7996 and

$$\mathscr{A}_{F1} = 10^{-3} \times \begin{bmatrix} 0.0908 & 0.1431 \\ 0.2226 & -0.3691 \end{bmatrix}, \quad \mathscr{B}_{F1} = \begin{bmatrix} 0.0021 \\ -0.0024 \end{bmatrix},$$

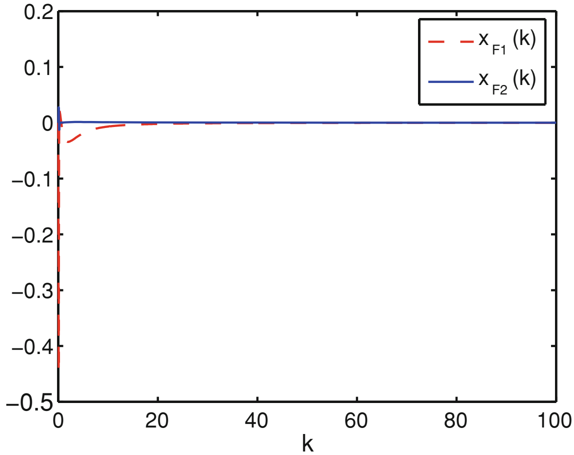

Fig. 2.3 State response of $x_F(k)$

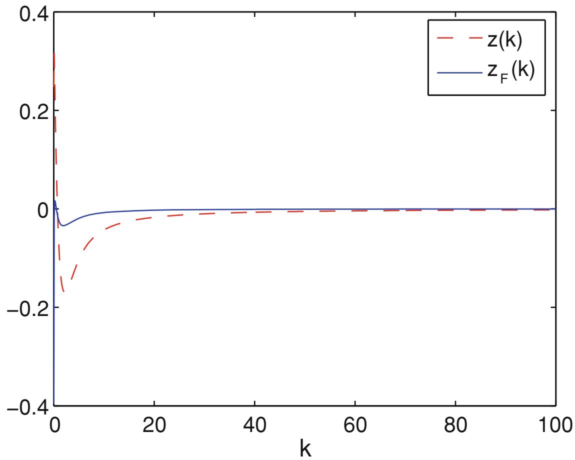

Fig. 2.4 Response of $z(k)$ and $z_F(k)$

$$\mathscr{C}_{F1} = [-0.0312\ 0.0342], \quad \mathscr{D}_{F1} = 0.1905,$$

$$\mathscr{A}_{F2} = \begin{bmatrix} -0.0055 & -0.0108 \\ 0.0041 & 0.0093 \end{bmatrix}, \quad \mathscr{B}_{F2} = \begin{bmatrix} 0.0073 \\ -0.0007 \end{bmatrix},$$

$$\mathscr{C}_{F2} = [-0.0041\ 0.0124], \quad \mathscr{D}_{F2} = 0.0516,$$

$$G = \begin{bmatrix} 0.1190\ 0.1459 \\ 0.2745\ 0.4476 \end{bmatrix}.$$

Substituting $\mathscr{A}_{F1}$, $\mathscr{B}_{F1}$, $\mathscr{C}_{F1}$, $\mathscr{D}_{F1}$, $\mathscr{A}_{F2}$, $\mathscr{B}_{F2}$, $\mathscr{C}_{F2}$, $\mathscr{D}_{F2}$, and G into (2.7), the filter matrices can be given as follows:

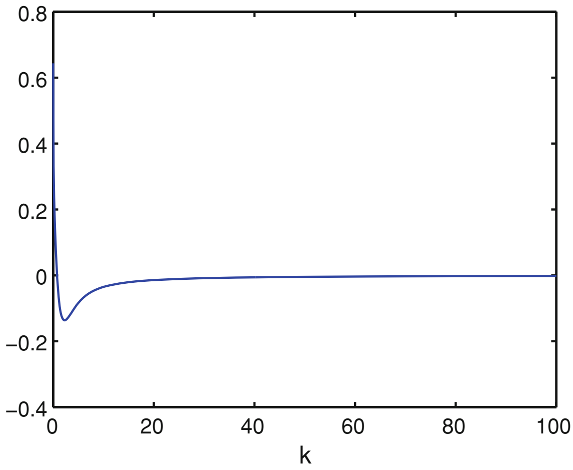

Fig. 2.5 Error response of $e(k)$

$$A_{F1} = \begin{bmatrix} 0.0006 & 0.0089 \\ 0.0001 & -0.0063 \end{bmatrix}, \quad B_{F1} = \begin{bmatrix} 0.0991 \\ -0.0661 \end{bmatrix},$$
$$C_{F1} = [-0.0312 \; 0.0342], \quad D_{F1} = 0.1905,$$
$$A_{F2} = \begin{bmatrix} -0.2306 & -0.4675 \\ 0.1505 & 0.3074 \end{bmatrix}, \quad B_{F2} = \begin{bmatrix} 0.2544 \\ -0.1576 \end{bmatrix},$$
$$C_{F2} = [-0.0041 \; 0.0124], \quad D_{F2} = 0.0516.$$

The external disturbance $w(k)$ is defined as $w(k) = (2 + k^{1.3})^{-1}, k = 1, 2, \ldots$ and the initial conditions are chosen as $x(0) = [0.8 \;\; -1]^T$, $x_F(0) = [0 \; 0]^T$. By considering $\Delta_\alpha = e^{-0.05k}$, $\alpha = A, B, C, D$, the simulation results of the state responses of the plant and filter are shown in Figs. 2.2 and 2.3, respectively. The simulation results of $z(k)$ and $z_F(k)$ are given in Fig. 2.4. Figure 2.5 shows the response of the filtering error $e(k)$.

The ratio of $\gamma(k) = \sqrt{\sum_{k=0}^{\infty} e^T(k)e(k) < \gamma^2 \sum_{k=0}^{\infty} w^T(k)w(k)}$ can show the influence of the disturbance $w(k)$ on the filter error $e(k)$, and the plot of the ratio is shown in Fig. 2.6. It can be seen that the ratio tends to a constant value 0.6145, which is less than the prescribed value, i.e., 0.7996.

From Figs. 2.2, 2.3, 2.4, 2.5, and 2.6, we can easily find that when the fuzzy filter has additive gain variations, the proposed designed filter is performed well for guaranteeing the H_∞ performance of the filtering error system.

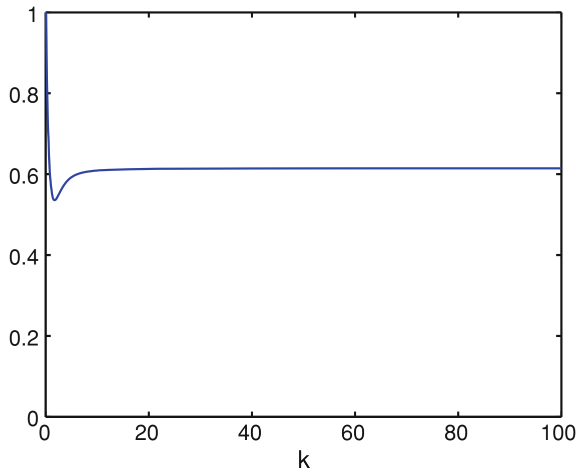

Fig. 2.6 The value of γ

2.4 Conclusion

The non-fragile filtering problem for T–S fuzzy systems has been studied, where the filters are assumed to have additive gain variations. The LMI technique has been used to design the non-fragile filters such that filtering error systems are asymptotically stable with prescribed H_∞ performances. Some slack matrix variables have been introduced to facilitate the design procedure of the non-fragile filters. A numerical example has been given to show the merits of the proposed approaches.

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