

Line Simplification in the Presence of Non-Planar Topological Relationships

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Abstract

A main objective of many line simplification methods is to progressively reduce the scale of shape properties and, in turn, provide a more explicit representation of global shape properties. However, current simplification methods which attempt to achieve this objective, while also maintaining non-planar topological relationships, are restricted and cannot always achieve an optimal result. In this paper, we present a line simplification method which removes these restrictions. This is achieved through the use of a computable set of topological invariants, which is complete and allows the topological consistency of an arbitrary simplification to be determined.

Keywords: line simplification, map generalisation, topology

1 Introduction

Given a detailed map representation it is common to reduce the scale of this representation through the application of a cartographic process known as map generalisation. The primary purpose of performing such a reduction is to transform the map into a representation more suitable for its purpose (Lonergan and Jones 2001). Wilson et al. (2010) demonstrated that if the purpose of the map is to communicate spatial information to a user per-

forming a specific task a suitable reduction in scale can improve efficiency. Map generalisation is performed by applying a set of generalisations operators of which Jones (1997) identified eight categories. These are elimination, simplification, typification, exaggeration, enhancement, collapse, aggregation and displacement. This paper focuses entirely on the generalisation operator of simplification. Simplification performs generalisation by selecting a subset of vertices that represents geometrical objects and does not move the vertices in this subset. (Corcoran et al. 2011). The purpose of any generalisation process is to reduce the scale of the map in question while simultaneously satisfying a set of objectives. Weibel (1996) identified four types of such objectives. These are shape (Gestalt), semantic, metric and topological objectives. The purpose of a shape objective is to reduce the scale of object shape properties to give a more explicit representation of global shape properties. A semantic objective integrates information regarding object semantics when determining the actual type and scale of reduction which should be applied to individual objects. Metric objectives achieve the best possible result in terms of some error criterion. Finally, a topological objective ensures that all generalised maps are topologically equivalent to the original detailed map. Two maps topologically equivalent if a topological or homeomorphism transformation exists between the two maps in question (de Berg et al. 1998, Mortenson 2007), where a topological transformation corresponds to an arbitrary stretching, bending or twisting without tearing of the map. If a homeomorphism exists between a map and its generalised form the generalised map is said to be topologically consistent; otherwise it is said to be topologically inconsistent. The set of all maps which are topologically equivalent form a topological equivalence class.

Many authors have proposed generalisation techniques, which attempt to satisfy a single objective. For example, Douglas and Peucker (1973) and Saalfeld (1999) proposed techniques which attempt to satisfy metric and topological objectives respectively. Kulik et al. (2005) proposed a line simplification technique which satisfies a semantic objective. However in many situations it is necessary to perform generalisation in a manner which satisfies multiple objectives. For example, the generation of destination or metro maps is an application where such a method is necessary (Kopf et al. 2010, Stott et al. 2011, Agrawala and Stolte 2001, Nollenburg and Wolff 2011). For such maps, it is generally accepted that topological equivalence to the original map should be preserved, while only those shape features of the most abstract nature should be preserved. That is, such an application requires a generalisation method, where both shape and topological objectives are satisfied.

In this paper, we focus exclusively on simplification methods which attempt to satisfy both shape and topological objectives. Such simplification methods generally follow a common iterative optimization strategy which begin with an initial solution and then iteratively improve until convergence (Corcoran et al. 2011, Kulik et al. 2005). A single iteration functions as follows. The vertex which contributes least to the overall shape properties, such that its removal does not introduce a topological inconsistency, is determined. This vertex is then removed. The simplification process terminates when the scale of the corresponding shape properties has been reduced sufficiently, or no further vertices can be removed without the introduction of a topological inconsistency.

When performing simplification in a manner which satisfies both shape and topological objectives, it is necessary that the following two tasks can be performed effectively. Firstly, a method for determining the significance of an individual vertex is necessary. This is typically a function of local properties such as the length of both sides adjacent to the vertex in question. Secondly, a method for determining if a given simplification is topologically consistent is necessary. In this paper, we focus on the development of an optimal methodology to perform the second of these tasks in the context of simplifying line features. In a geographical context, line features may correspond to roads, rivers, etc. Although such methods have previously been proposed, as will be discussed later, they are not optimal and can return unsatisfying results. In this paper, we propose a new line simplification method which, under certain assumptions, overcomes this limitation and is, in fact, optimal.

The layout of this paper is as follows. In section 2, we introduce some background material necessary for discussing topological relationships. In section 3, we critique existing line simplification techniques which attempt to satisfy both shape and topological objectives. Section 4 proposes a methodology for determining the topological consistency of two arbitrary scenes. This method is based on the computation of a set of topological invariants. Section 5 states a property associated with the problem of determining if a simplification is topologically consistent. This property reduces the computational complexity of determining the topological consistency of a simplification. Finally in sections 6 and 7 we present results and draw conclusions respectively.

2 Topological relations between lines

In this section, we introduce some concepts which help describe the topological relationships that may exist between a set of lines. Corcoran et al. (2011) proposed that all possible topological relations between objects may be classified as planar or non-planar. A planar topological relationship exists between a set of objects if at all intersection points between the objects a vertex exists and belongs to all objects which intersect at that point. For example, a planar topological relationship exists between the two lines $p=(p_1, p_2, x, p_3, y, p_4, p_5)$ and $r=(r_1, x, r_2, y, r_3, r_4, r_5)$ displayed in Figure 1(a). A non-planar topological relationship exists between a set of objects if the objects intersect without a vertex existing at all intersection points and belonging to all objects which intersect at that point. An example of such a relationship is displayed in Figure 1(b).

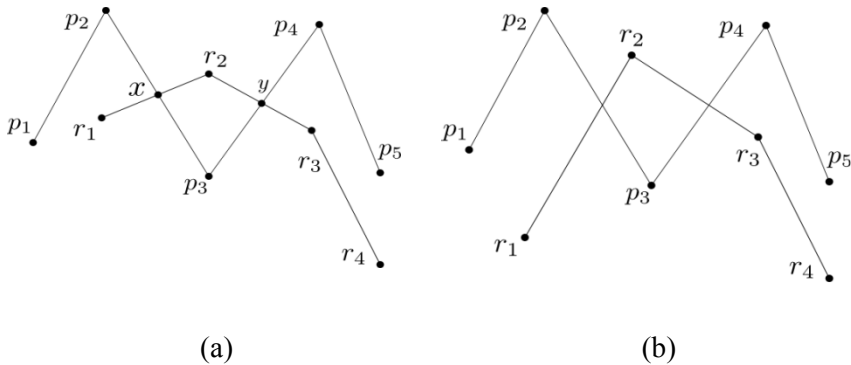


Fig. 1. Planar and non-planar topological relationships exist between the pair of lines in (a) and (b) respectively

When discussing non-planar topological relationships it is important that we define the properties of dimension and multiplicity which describe a particular intersection between lines (Clementini and Di Felice 1998). If the intersection in question takes place in a point, its dimension is zero; this is the case for all intersections in Figure 1(b). If the intersection in question takes place in a line, its dimension is one. The multiplicity of an intersection refers to the number of lines which pass through a given intersection.

Due to the difficulty involved in determining the topological consistency of a simplification, we make the following assumptions regarding all scenes to which the methodology proposed in this paper is applied. We assume that all intersections are of degree zero and that the multiplicity of each intersection is two. We assume that lines do not self-intersect; this

property is common in many spatial datasets such as the road network. Finally, we assume that removing the endpoint of a line feature introduces a topological inconsistency; this is a common assumption made when simplifying lines (Saalfeld 1999). We return to this discussion regarding assumptions made in the conclusions section of this paper.

3 Existing methods for determining topological consistency

In this section, we review existing methods for determining the topological consistency of a given simplification. Before that, we review a framework proposed by Corcoran et al. (2011) for structuring the constraints imposed by such methods. Corcoran et al. (2011) state that any method for determining topological consistency of a given simplification can be summarised in terms of the following three constraints:

- 1) Constraints on the types of topology for which the technique can determine consistency without returning a false-positive; that is, incorrectly classifying a simplification as topologically consistent.
- 2) Constraints on the types of topology for which the technique can determine consistency without returning a false-negative; that is incorrectly classifying a simplification as topologically inconsistent.
- 3) Constraints on the types of simplification to which the technique can be applied.

If a particular method exhibits none of the above constraints, it may be considered optimal. Corcoran et al. (2011) presented a mathematical analysis of existing techniques for determining the topological consistency of an arbitrary simplification. With respect to planar topological relationships, the authors demonstrated that using existing techniques it is possible to determine the topological consistency of an arbitrary simplification in an optimal manner. However, determining the topological consistency of an arbitrary simplification with respect to non-planar topological relationships is still an open research question (Corcoran et al. 2011). Two methods, by Agrawala and Stolte (2001) and Kulik et al. (2005), currently exist for performing this task. In the following two subsections, we present a review of these and demonstrate neither to be optimal with respect to the third constraint of Corcoran et al. (2011) above.

3.1 Agrawala and Stolte (2001) method

The first of these methods was originally proposed by Agrawala and Stolte (2001). This method is optimal with respect to the first two constraints of Corcoran et al. (2011) presented above. However, it is not optimal with respect to the third constraint for the following reason. Before simplification is performed an operation known both as map overlay and planar enforcement (Wise 2002) is applied to the map in question which adds vertices to all objects which intersect at the intersection points in question if such vertices do not already exist. For example, consider the scene in Figure 2(a) which contains the three lines $a=(a_1, a_2, a_3, a_4)$, $b=(b_1, b_2, b_3)$ and $c=(c_1, c_2, c_3, c_4)$. Applying planar enforcement returns the scene displayed in Figure 2(b) containing the three lines $a=(a_1, x, a_2, y, a_3, a_4)$, $b=(b_1, w, x, y, b_2, b_3)$ and $c=(c_1, w, c_2, z, c_3, c_4)$ which are represented using the four additional vertices w, x, y and z . The method of Agrawala and Stolte (2001) is restricted by the fact that it cannot determine the topological consistency of any simplification which does not contain all vertices introduced through planar enforcement.

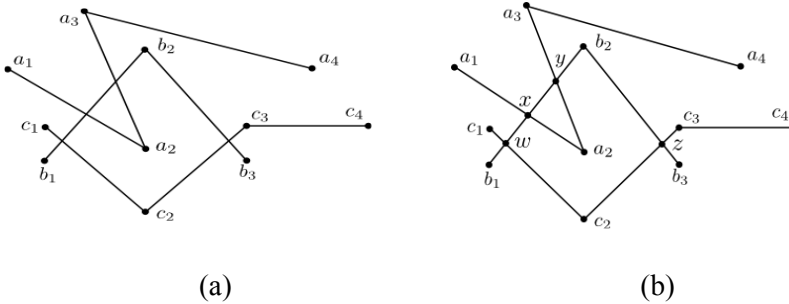


Fig. 2. Planar enforcement is applied to the scene in (a) with the result shown in (b).

To illustrate the lack of optimality exhibited by this method, consider again the scene in Figure 2(a). Applying the simplification step where the line a is simplified by removing the vertices (a_3, a_2) returns the topologically consistent result displayed in Figure 3. It must be noted that the aim of this work is to maintain topological consistency with respect to lines and not the vertices which represent these lines. The method of Agrawala and Stolte (2001) cannot determine if this simplification is topologically consistent because it does not contain the vertices introduced by planar enforcement.

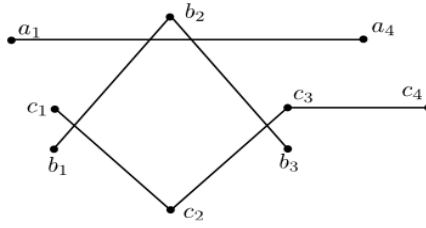


Fig. 3. A topologically consistent simplification of the scene in Figure 2(a) is displayed.

3.2 Kulik et al. (2005) method

The second method, which currently exists for determining topological consistency of a simplification with respect to non-planar topological relationships, was proposed by Kulik et al. (2005) and later used by Weihua (2008) and Corcoran et al. (2011). Again this method is optimal with respect to the first two constraints of Corcoran et al. (2011) presented above. However, it is not optimal with respect to the third constraint for the following reason. Before simplification is performed, those vertices belonging to line segments which intersect without a vertex existing at the intersection point and belonging to all line segments in question are identified. For example, in the context of the scene displayed in Figure 2(a), the vertices $(a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3)$ would be identified as having this property. The method of Kulik et al. (2005) is restricted by the fact that it cannot determine the topological consistency of any simplification which does not contain all vertices identified as having the above property. For example, this method cannot determine if the simplification of Figure 2(a) displayed in Figure 3 is topologically consistent, because it does not contain the vertices belonging to the original intersecting line segments. Since both the methods of Agrawala and Stolte (2001) and Kulik et al. (2005) are constrained they may in turn constrain the corresponding simplification process by forbidding the removal of particular vertices.

4 Topological invariants

Comparing two scenes directly in order to determine if they are topologically equivalent represents an extremely difficult task. To overcome this difficulty, many authors propose the use of topological invariants. A topological invariant is a property of a map which is invariant under a topological transformation. That is, two maps which are topologically equivalent

will both exhibit the same topological invariants (Clementini and Di Felice 1998). The use of invariants, therefore, allows the topology of two scenes to be compared in potentially an effective manner. A set of topological invariants are incomplete if they are necessary but not sufficient for determining topological equivalence. Two scenes, which are not topologically equivalent, may have an equal set of incomplete invariants. The most widely used incomplete sets of invariants are the 6- and 9-intersection matrices of Egenhofer (1991).

A set of topological invariants are complete if they are necessary and sufficient for determining topological equivalence. Therefore in order to insure that two scenes are topologically equivalent, in a manner which is optimal with respect to the first two constraints of Corcoran et al. (2011) presented above, a complete set of invariants must be used. In this section, we define a complete set of topological invariants which contains three elements. This set may be computed for an arbitrary scene and corresponding simplification and, therefore, it is also optimal with respect to the third constraint of Corcoran et al. (2011).

The remainder of this section is structured as follows. In section 4.1, the proposed set of topological invariants is presented and we state the computational complexity of their computation. Section 4.2 proves the necessity of each invariant; that is, no one invariant is implicitly contained in the others. In section 4.3, we prove that the above three invariants form a complete set.

4.1 Invariants

In this subsection, we define three topological invariants in the form of three corresponding definitions. These invariants are entitled intersection sequence (IS), direction sequence (DS) and orientation sequence (OS). These invariants are closely related to those proposed by Clementini and Di Felice (1998) but are specified in the context of line simplification and contain additional computational details.

4.1.1 Intersection Sequence (IS) invariant

Definition 4.1: Let a and b be two lines which intersect in m points. Following the order given by the line a assign the numeric labels $1, \dots, m$ to each intersection. The intersection sequence (IS) invariant is a permutation of the m -tuple $(1, \dots, m)$ which is obtained by traversing the line b in order and recording the labels previously assigned to each intersection.

To illustrate the IS invariant consider the topological relationship which exists between the lines a and b in Figure 4. The intersection sequence in this case is the 5-tuple $(1, 4, 3, 2, 5)$.

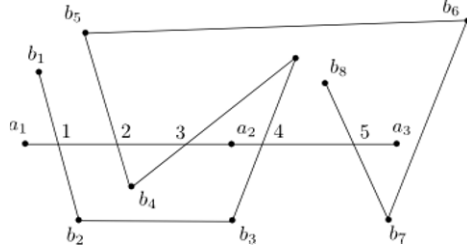


Fig. 4. Each intersection point is labeled in an order obtained by traversing the line a .

Theorem 4.2: Given two simple lines a and b represented by n vertices in total and which intersect k times; the IS invariant can be computed in $O(n^2 + k \log(k))$ time.

Proof: Determining all intersection points requires that all pairs of line segments in a and b be evaluated to determine if an intersection occurs; this requires $O(n^2)$ time. The resulting k intersections are then sorted in terms of distance along a and this operation requires at most $O(k \log(k))$ time. The computational complexity of computing the IS invariant is, therefore, $O(n^2 + k \log(k))$ time.

4.1.2 Direction Sequence (DS) invariant

Definition 4.3: Let a and b be two lines which intersect in m points. Following the order given by the line b assign the numeric labels $1, \dots, m$ to each intersection. The direction sequence (DS) invariant is the m -tuple (c_1, \dots, c_m) where c_i takes the value r if the line b crosses the line a from the right to the left of a as a is traversed at the intersection with numeric label i . Otherwise c_i takes the value l if the line b crosses the line a from the left to the right of a at the intersection with numeric label i .

To illustrate the DS invariant, consider again the topological relationship which exists between the lines a and b in Figure 4. The DS invariant in this case is the 5-tuple (l, r, l, r, r) . The DS invariant can be computed in $O(n^2 + k \log(k))$ time. The proof of this fact is not presented, due to page space limitations.

4.1.3 Orientation Sequence (OS) invariant

Given a sequence of intersections between two lines a and b , each section of the line b between the pair of consecutive intersections (h, k) is called a link and denoted $b(h, k)$ (Clementini and Di Felice 1998). Given two lines a and b and a link $b(h, k)$, consider the cycle obtained by traversing $b(h, k)$ and return to h by traversing a . If such a cycle is counter-clockwise the link orientation $LO_b(h, k)$ takes the value CCW ; on the other hand if such a cycle is clockwise the link orientation $LO_b(h, k)$ takes the value CW . For example, consider the scene in Figure 4; $LO_b(1, 4)$ takes the value CCW while $LO_b(3, 2)$ takes the value CW .

Definition 4.4: Let a and b be two lines which intersect in m points. The orientation sequence (OS) invariant is an $m-1$ tuple containing the sequence of $LO_b(h, k)$ values between each consecutive pair of intersection h and k obtained by traversing the line b .

The OS invariant for the scene displayed in Figure 4 is (CCW, CCW, CW, CW) . The OS invariant can be computed in $O(n^2 + k \log(k))$ time. The proof of this fact is not presented, due to page space limitations.

4.2 Necessity of invariants

In this section, we prove that the IS invariant is necessary. That is, this invariant cannot be expressed unambiguously in terms of the remaining invariants. This is achieved by construct a pair of non-topologically equivalent scenes which have equal topological invariants apart from the IS invariant. Consider the two non-topologically equivalent scenes in Figure 5(a) and Figure 5(b). Both scenes have equal DS and OS invariants of (r, l, r) and (CW, CCW) respectively. However, the scene in Figure 5(a) has an IS invariant of $(1, 2, 3)$ while the scene in Figure 5(b) has a IS invariant of $(2, 3, 1)$. The necessity of the DS and OS invariants can also be proved in a similar manner. The proof of this fact is not presented, due to page space limitations.

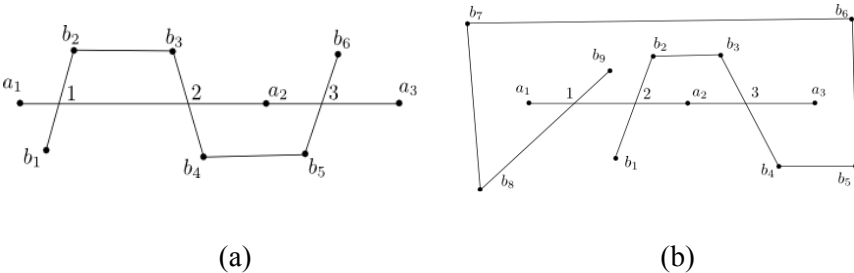


Fig. 5. The scenes (a) and (b) are not topologically equivalent.

4.3 Completeness of invariants

We now prove that the IS, DS and OS invariants form a set of invariants which is complete in the context of the topological relationships which may exist between two lines. This is achieved by proving that each unique set of invariants identifies a class of topologically equivalent scenes. This is in turn proved by giving a procedure to construct a representative scene of the class from such a set of invariants. The uniqueness of such a scene is guaranteed by the fact that each step of the construction process does not exhibit any topological indeterminacy. This form of proof by geometrical construction was originally proposed by Clementini and Di Felice (1998).

The construction process we propose is incremental in the sense that it gradually constructs the topological relationship between two lines, a and b , by adding one intersection at each construction step. The order of construction is specified by the order which the intersections occur along a traversal of the line b . The following theorem defines this construction process.

Theorem 4.5: The set of IS, DS and OS invariants define a class of topologically equivalent scenes.

Proof: Given the set of IS, DS and OS invariants we describe a method to construct a corresponding scene containing two lines. This process either returns a unique scene representing a class of topologically equivalent scenes or an impossible scene. The process contains the following steps:

- 1) Draw a simple line a (all simple lines are topologically equivalent).
- 2) If IS, DS and OS all contain zero elements, draw a line b which does not intersect a . Terminate the construction process.
- 3) Draw the first intersection between a and b so that it is consistent with the first element of DS.
- 4) Mark the next intersection point so that its location is consistent with respect to the corresponding element in IS and the set of intersection labels already added.
- 5) Draw the link between this intersection point and the one previously added such that it is consistent with the corresponding elements of DS and OS.
- 6) Repeat steps 4 and 5 until all intersections have been added.

To demonstrate this construction process, we will construct the scene in Figure 4 which has topological invariants $IS = (1, 4, 3, 2, 5)$, $DS = (l, r, l, r, r)$ and $OS = (CCW, CCW, CW, CW)$. Firstly, we draw the line a as illustrated in Figure 6(a). Next, we draw the first intersection between a and b such that the intersection is consistent with the first element of DS ; that is, the intersection crosses a from the left. This is illustrated in Figure 6(b). Next we mark the location of the next intersection. This has the label 4 in IS and therefore it occurs to the right of the previous intersection along the line a . This step is illustrated in Figure 6(c). Next, we join this intersection with the previous one, created such that the link is consistent with the corresponding elements in DS and OS . That is, it is a counter-clockwise link which intersects a from the right. This step is illustrated in Figure 6(d). Next, we mark the location of the next intersection. This has the label 3 in IS and, therefore, it occurs between the intersections previously marked, which have labels 1 and 4. This is illustrated in Figure 6(e). Next, we join this intersection with the previous one, created such that the link is consistent with the corresponding element in DS and OS . That is, it is a counter-clockwise link which intersects a from the left. This is illustrated in Figure 6(f). This process continues until each of the remaining intersections has been processed; the result of this process is illustrated in Figure 6(g). It is evident that the scene constructed in Figure 6(g) is topologically equivalent to Figure 4.

Therefore, using the proposed set of invariants the topological equivalence of a map and corresponding simplification can be determined indirectly, in a manner which is optimal with respect to the three constraints of Corcoran et al. (2011), by comparison of these invariants. In the following section, we state a property associated with the problem of determining if a scene and corresponding simplification are topologically equivalent. This property allows the associated computational complexity to be reduced.

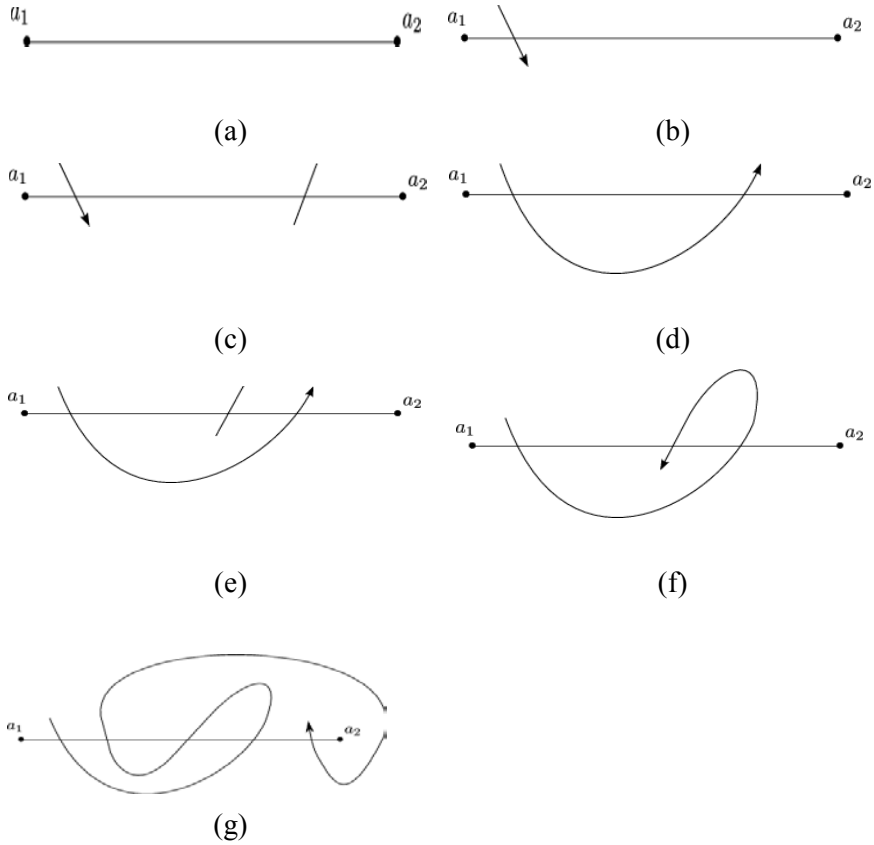


Fig. 6. Steps in the construction of a scene topologically equivalent to Figure 4 are illustrated.

5 Determining topological consistency – associated properties

The problem of determining if two unrelated scenes are topologically equivalent has been studied significantly in the domain of Geographical Information Science (GIS). The problem of determining if a scene and corresponding simplification are topologically equivalent, exhibits a property which makes it distinct from the problem of determining if two unrelated scenes are topological equivalent. This property offers the potential to reduce the difficulty of the problem substantially.

In the context of determining the topological equivalence of two unrelated scenes, in general, the correspondence between objects in each scene

is not known (Clementini and Di Felice 1998). Consider the problem of determining if the unrelated scenes in Figure 7(a) and Figure 7(b) are topologically equivalent where each scene contains three objects. As a first step towards determining topological equivalence, the correspondence between objects in each scene must be determined. If it is determined that the objects a , b and c in Figure 7(a) correspond to objects y , z and x in Figure 7(b) respectively, topological equivalence can subsequently be determined. If any other correspondence was considered, topological equivalence in that case could not be determined. Clementini and Di Felice (1998) propose to overcome this challenge using a depth first search procedure which evaluates many possible correspondences until a suitable one is found or the process terminates.

On the other hand, in the context of determining the topological equivalence of a scene and a corresponding simplification, the correspondence between objects in each scene is known. This, therefore, removes the requirement to perform any searching procedure to determine a suitable correspondence.

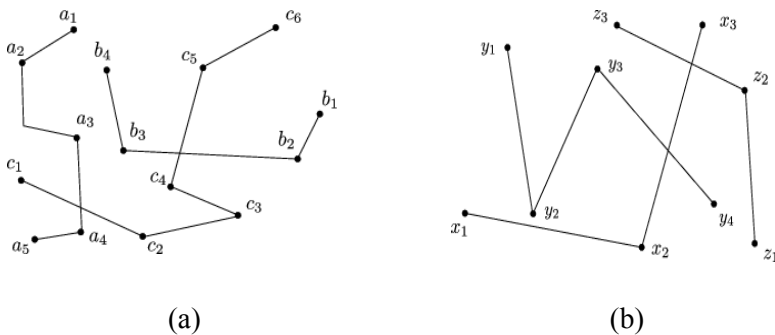


Fig. 7. The scenes in (a) and (b) are unrelated.

6 Results

The line simplification method proposed in this paper functions as follows. At each step, the vertex which contributes least to the overall shape properties such that its removal does not introduce a topological inconsistency is removed. The function by Latecki and Lakmper (1999) is used to determine the significance of a given vertex. Determine the topological equivalence of a map and corresponding simplification involves the following three steps. First the correspondence between the objects in both maps is determined using the property described in section 5. Next the complete set of topological invariants for each topological relationship in the original and simplified scenes are computed using the methods of section 4. Fi-

nally, all corresponding invariants in each scene are compared. If all invariants are equal, it is determined that the simplification is topologically consistent; otherwise, it is determined that the simplification is topologically inconsistent. The simplification process terminates when no further vertices can be removed without the introduction of a topological inconsistency.

In order to demonstrate the effectiveness of the proposed simplification method in the presence of non-planar topological relationships, we used the simplification method of Corcoran et al. (2011) as a benchmark. The method of Corcoran et al. (2011) in turn uses the method of Kulik et al. (2005) (see section 3.2) to ensure all non-planar topological relationships are preserved. That is, those vertices belonging to line segments which intersect without a vertex existing at the intersection point and belonging to all line segments in question cannot be removed through simplification. Three example scenes with corresponding simplification results are displayed in Figures 8, 9 and 10. In each figure the original scene, the result of simplification using the method of Corcoran et al. (2011) and the result of simplification using the proposed method are displayed in sub-figures (a), (b) and (c) respectively.

It is evident from these results that the proposed simplification method returns a more abstract representation of shape properties compared to the simplification method of Corcoran et al. (2011). This is due to the fact that the proposed method is not constrained in terms of the types of simplification to which it can be applied; it is, in fact, optimal with respect to the three constraints of Corcoran et al. (2011) discussed in section 3. The abstract nature of the shapes returned by the proposed method makes it very suitable for applications such as a metro-map generation, where a schematic representation is necessary.

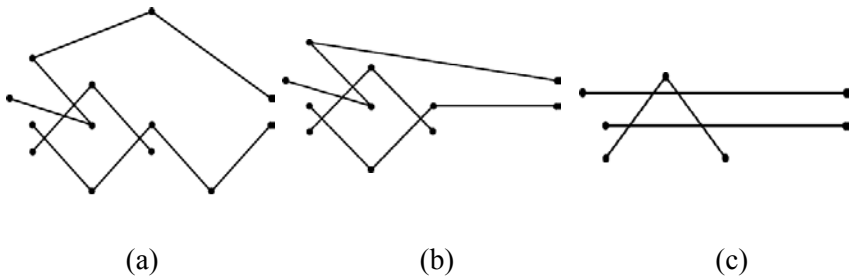


Fig. 8. The scene in (a) is simplified in (b) and (c).

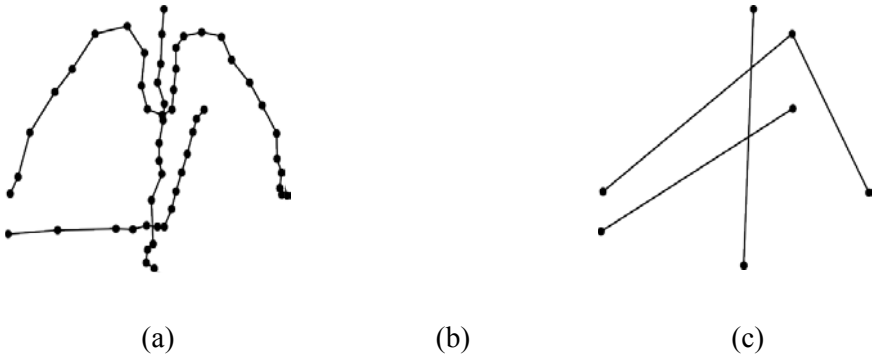


Fig. 9. The scene in (a) is simplified in (b) and (c).

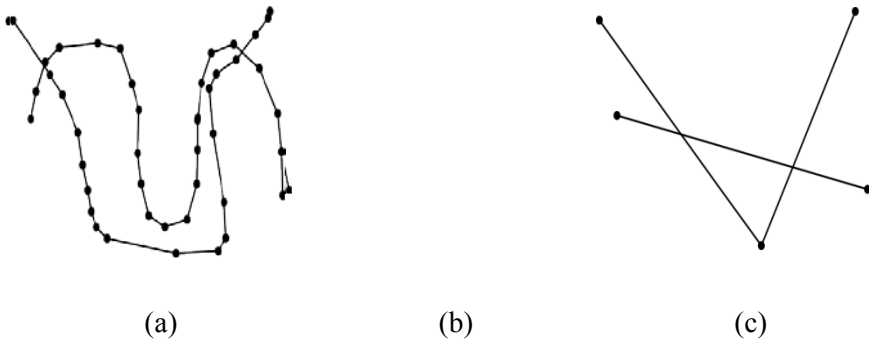


Fig. 10. The scene in (a) is simplified in (b) and (c).

7 Conclusions

This paper proposes a new line simplification method which can simplify lines in the presence of non-planar topological relationships in an optimal manner. The results returned by the proposed method show an improvement when compared to an existing state-of-the-art technique. Despite this, many opportunities to extend and improve the proposed method exist. Currently the proposed method can only simplify lines. In order to be applicable in a more general context, it would be desirable to extend this method so that polygons may also be simplified. Also, currently the proposed method can only simplify scenes where all intersections are of dimension zero and multiplicity two. Again it would be desirable to extend the proposed method so that scenes not satisfying this constraint could be simplified. As a final note, the authors hope that the work presented here will stimulate new research on the topic of line simplification. Despite its long history, as

demonstrated in this paper, this topic is not yet a completely solved problem.

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