
Preface

ἀγεωμέτρητος μηδεὶς εἰσίτω

(literally: “non-geometers don’t enter.” Inscription at the entrance to Plato’s Academy 387 B.C. (perhaps only a legend), and on the frontispiece of Copernicus’ *De revolutionibus* 1543 A.D.)

Geometry, so named at least since Plato’s times, is the oldest branch of mathematics. It contains many beautiful results, elegant ideas and surprising connections, to which many great thinkers have contributed through the centuries. Among these are Thales, Pythagoras, Euclid, Apollonius, Archimedes, Ptolemy, Pappus, the Arabs, Regiomontanus, Copernicus, Viète, Kepler, Descartes, Newton, the Bernoullis, Euler, Monge, Poncelet and Steiner. In this book, we study geometry from the texts of these masters as closely as we judge useful and roughly in historic order, in accordance with the famous maxim of Niels Henrik Abel, “*I learned from the masters and not from the pupils*”. This explains “by Its History” in the title, without aiming at a complete history of the subject.¹

Geometry arose in the dawn of science and later became one of the *septem artes liberales* as part of the *quadrivium*. We start from the beginning of geometry, motivated by practical problems of measurement (μετρέω), then follow its development into a rigorous abstract science by the Greek philosophers, until the rich period with more and more sophisticated problems and methods in the later Greek and Arab period. The second part of the book then describes the victory of the methods of algebra and linear algebra, the growing audacity in dealing with infinite processes and we see how all other branches of science grew out of geometry: algebra, calculus, mechanics (in particular celestial mechanics). So we hope that the book not only constitutes a good introduction to the study of higher geometry, but also to the study of other branches of science, especially for those students who intend to become teachers.

However, due to the rapid success of algebraic methods, synthetic geometry slowly lost its place in university education, a development already deplored by Newton, more than three centuries ago (first sentence of his *Treatise of the Methods of Series and Fluxions*, 1671, p. 33) ...

“Observing that the majority of geometers, with an almost complete neglect of the ancients’ synthetical method, now for the most part apply

¹Michel Chasles’ *Aperçu historique* has 800 pages and weighs five pounds.

themselves to the cultivation of analysis [...] I found it not amiss, for the satisfaction of learners, to draw up the following short tract ...

... a development which accelerated during the twentieth century (Coxeter, *Introduction to Geometry*, 1961, p. iv):

“For the last thirty or forty years, most Americans have somehow lost interest in geometry. The present book constitutes an attempt to revitalize this sadly neglected subject.”

What is not honoured at the university also disappears, a generation later, from the high-school. We quote from A. Connes (*Newsletter of the EMS*, March 2008, p. 32):

“We must absolutely train very young people to do mathematical exercises, in particular geometry exercises — this is very good training. I find it awful when I see that, in school, kids are taught recipes, just recipes, and aren’t encouraged to think. When I was at school, I remember that we were given problems of [...] geometry. We went to a lot of trouble to solve them. It wasn’t baby geometry. [...] It’s a shame we don’t do it anymore.”

We have made all efforts to produce an interesting and enjoyable book, intended mainly for students of science (at the beginning) and teachers (throughout their career), by including many illustrations, figures, exercises and references to the literature, so that we suggest with Copernicus’ *De revolutionibus*

Igitur eme, lege, frueri [Therefore buy, read and enjoy].

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Our special thanks, however, go to J. Steinig, who read the entire book *four times (!)* and suggested thousands of corrections and grammatical improvements. He pointed out errors and sloppy arguments, and supplied us with many references and better proofs. And if ever “eme, lege, frueri” is really justified, it is also his merit.



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