

# Preface

After brilliant studies in the most renowned turkish institutions, Ali Suleyman Üstünel was longing to become a physicist when Hayri Körezlioglu convinced him to switch to mathematics. This was the beginning of a long and deep collaboration and friendship.

A.S. Üstünel finally defended his Ph.D. in probability in Paris in 1981 with Laurent Schwarz as an examiner. He first began to work at Centre National d'Études en Télécommunications (now Orange Labs) and then at Ecole Nationale Supérieure des Télécommunications (now Télécom Paristech). His first works were related to nuclear-valued processes. The strong topological properties of nuclear spaces induce that many properties only have to be verified “cylindrically” to hold in full generality: For instance, a process  $(X(t), t \geq 0)$  with values in the set of tempered distributions is continuous if and only if for any  $\varphi$  rapidly decreasing, the real-valued process  $(\langle X(t), \varphi \rangle, t \geq 0)$  is continuous. The work of A.S. Üstünel culminated in the “three operators lemma” which states that when three Hilbert–Schmidt operators are applied in a row to a cylindrical semi-martingale, it becomes a true semi-martingale.

In the mid-1980s, he was one of the pioneering researchers to investigate thoroughly the newly born Malliavin Calculus, a field where he quickly became (and still is!) a world renowned expert. From 1986, H. Korezlioglu and A.S. Üstünel organized the “Stochastic analysis and related topics” workshop whose first occurrences took place in Silivri (Turkey) every 2 years. The “Silivri band” (mainly M. Chaleyat-Maurel, A. Grorud, A. Millet, D. Nualart, E. Pardoux, M. Pontier, M. Sanz) played a major role in the development of Malliavin calculus and its applications. At the same time, A.S. Üstünel and Moshe Zakai started a collaboration which was to last for the next 20 years. Their main subject of investigation has been the absolute continuity of shift transformations in the Wiener space. It is well known that the law of Brownian motion with an adapted, square integrable drift is absolutely continuous with respect to the law of the Brownian motion. They devoted their whole energy to extend the family of admissible drifts, that is to say drifts such that the absolute continuity property still holds. The main question is to get rid of the adaptability. They showed that this can be replaced by, for instance, either monotony or some

regularity on the Malliavin derivative of the drift. Most of their results are contained in the beautiful book they coauthored.

The reciprocal problem can be informally stated as: Given a measure equivalent to the Wiener measure, does there exist a shift transformation which realizes this measure? It turned out that the optimal transportation theory which was regaining interest after the work of Brenier in the end of the previous millenium yielded an answer to this problem. Using his previously defined notion of  $H$ -convexity, A.S. Üstünel, in collaboration with Denis Feyel, solved the so-called Monge–Kantorovitch problem in the Wiener space for the Cameron–Martin cost. Once again, Malliavin calculus provided the convenient concepts to generalize almost word for word, the results known in finite dimension. Surprisingly, the proofs of some results such as Talagrand or Poincaré inequalities appeared to be even simpler in infinite dimension due to the availability of the Itô calculus. In several papers, they showed different properties of the solution of the Monge–Kantorovitch problem, which yielded in turn several functional inequalities.

Combining all his earlier results, A.S. Üstünel found a criterion which ensures the invertibility of a shift transformation on the Wiener space: If the kinetic energy of the drift  $u$  is equal to the entropy of the measure induced by the corresponding shift transformation, then the map  $\omega \mapsto \omega + u(\omega)$  is invertible. Such a result can be interpreted as a construction of a strong solution of the stochastic differential equation  $dX(t) = -\dot{u}(X, t) dt + dB(t)$  for very general  $u$ .

This quick glance at A.S. Üstünel’s work does not give justice to his other numerous contributions to control, filtering, functional inequalities, fractional Brownian motion, etc. but it shows a strong line of thought and a constant will to focus on deep problems. To borrow one of his favorite metaphor: Instead of looking to the hole which corresponds to the key, he rather prefers to seek for the key which fits into the hole.

Besides his own research activities, A.S. Üstünel has been the professor of several generations of students at Telecom ParisTech and the Ph.D. advisor of many students. We have all been impressed by not only his passion to mathematics, his wide knowledge, but also his generosity, his kindness and the relevance of his advice.

On a more artistic side, Süleyman and his wife, Jacqueline, became world-renowned specialists of the Turkish painter Fikret Moualla, many paintings of whom can be seen at their gallery in Paris. But that is another story. We take this opportunity to deeply thank Jacqueline, whose help was crucial to organize this workshop, or more precisely, to convince Süleyman to participate in this workshop organized on the occasion of his 60th birthday.

Happy birthday Süleyman!

Paris, France

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