Wrinkling Analysis of Rectangular Soft-Core Composite Sandwich Plates

Mohammad Mahdi Kheirikhah and Mohammad Reza Khalili

Abstract In the present chapter, a new improved higher-order theory is presented for wrinkling analysis of sandwich plates with soft orthotropic core. Third-order plate theory is used for face sheets and quadratic and cubic functions are assumed for transverse and in-plane displacements of the core, respectively. Continuity conditions for transverse shear stresses at the interfaces as well as the conditions of zero transverse shear stresses on the upper and lower surfaces of plate are satisfied. The nonlinear von Kármán type relations are used to obtain strains. Also, transverse flexibility and transverse normal strain and stress of the orthotropic core are considered. An analytical solution for static analysis of simply supported sandwich plates under uniaxial in-plane compressive load is presented using Navier's solution. The effect of geometrical parameters and material properties of face sheets and core are studied on the face wrinkling of sandwich plates. Comparison of the present results with those of plate theories confirms the accuracy of the proposed theory.

Keywords Overall Buckling · Wrinkling · Sandwich plate · Analytical solution · Soft core

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1 Introduction

Sandwich plates are widely used in many engineering applications such as aerospace, automobile, and shipbuilding because of their high strength and stiffness, low weight and durability. These plates consist generally of two stiff face sheets and a soft core, which are bonded together. In most cases, the core consist of a thick foam polymer or honeycomb material, while thin composite laminates are commonly used as the face sheets. Sandwich plates experience some failure modes not occurring in metallic sheets or laminated plates. Face wrinkling is one of the important behaviors of these plates subjected to in-plane compressive loads. In this phenomenon, the faces buckle in shorter wavelength than those associated with overall buckling of the plate [1].

There are three different modes of wrinkling. The mode I which is named 'rigid base' may occur when only one of the face sheets buckle. The mode II or 'anti-symmetric Wrinkling' may happen when the mode-shape of the face sheets are the same. In this mode the mid-plane of the core deforms. In the mode III which is called 'symmetric wrinkling', the mode-shape of the face sheets is symmetric about mid-plane of sandwich plate. Such symmetrical modes can only occur in sandwich plates with a soft core material [2].

The first studies on wrinkling analysis of soft-core sandwich panels began in 1930s decade. Gough et al. [3] used the Winkler elastic foundation model to study sandwich panels with a compliant core material. They neglected the compressive stresses of the core in the direction of the applied load. The symmetric and anti-symmetric wrinkling for sandwich struts with isotropic facings and solid cores were investigated by Hoff and Mautner [4] using a new model. In this model, the through thickness deformation decays linearly from the face sheet into the core. Plantema [5] proposed an exponential decay for the through thickness deformation in his book. Allen [6] studied the 2D wrinkling problem of sandwich beams or plates in cylindrical bending. He solved the governing differential equation and assumed that the core stress field has to satisfy the Airy's stress function under 2D conditions. Also, Zenkert [7] and Vinson [8] summarized sandwich wrinkling statements in their textbooks.

Benson and Mayers [9] presented a unified theory for the overall buckling and face wrinkling of sandwich panels with isotropic facings. This theory was expanded by Hadi and Matthews [10] for the wrinkling of anisotropic sandwich panels. Their approach was able to simultaneously calculate the anti-symmetric and symmetric wrinkling loads. Niu and Talreja [11] studied the wrinkling of composite sandwich plates. They presented a unified wrinkling model which combined three modes of wrinkling and also showed that the critical mode of wrinkling is the anti-symmetrical mode.

Frostig [12] developed a theory using the classical laminated plate theory (CLPT) for the face sheets and postulated a stress distribution in the core for overall and local buckling analysis of soft core sandwich plates. Analytical solutions were presented for simply supported soft-core sandwich plates, but the transverse stress continuity conditions were neglected. In two papers, Dawe and Yuan [13, 14] provided a model which uses a quadratic and linear expansion of the in-plane and transverse displacements of the core and represented the face sheets as either FSDT or CLPT. A B-spline finite stripe method (FSM) was used for buckling and wrinkling of rectangular sandwich plates subjected to in-plane compressive and shears loads applied to the face sheets. Vonach and Rammerstorfer [15] studied the problem of the wrinkling of orthotropic sandwich panels under general loading. They assumed infinite thickness for the core and a sinusoidal wrinkling wave at the interface of the face sheet and the core. A high-order layer-wise model was proposed by Dafedar et al. [16] for buckling analysis of multi-core sandwich plates. They assumed cubic polynomial functions for all displacement components in any layer. As a large number of unknowns were involved, they proposed a simplified model and calculated critical loads based on the geometric stiffness matrix concept.

Leotoing et al. [17] proposed a single model for local and global buckling of sandwich beams with facings and core made of homogeneous isotropic linear elastic materials. In this model, a linear distribution was assumed for the transverse shear stress through the beam thickness. Biaxial wrinkling of sandwich panels with composite face sheets was investigated by Birman and Bert [1]. They used three different models for the core: a simple Winkler elastic foundation model, the Hoff and Mautner [4] model which assumed a linear decay for the through thickness deformation from the face sheet into the core, and the Plantema [5] model with exponential decay. Birman [18] also analyzed wrinkling of a large aspect ratio simply supported sandwich panel with cross-ply facings subjected to an elevated temperature or heat flux on one of the surfaces. Fagerberg and Zenkert [19] studied imperfection-induced wrinkling material failure in sandwich panels based on Allen's model [6]. Also, effects of anisotropy and biaxial loading on the wrinkling of sandwich panels were considered by Fagerberg and Zenkert [20]. Elastic and elastic–plastic skin wrinkling of graded and layered foam core sandwich panels were studied Grenestedt and Danielsson [21]. Kardomateas [22] presented a 2D elasticity solution for the wrinkling analysis of sandwich beams or wide sandwich panels subjected to axially compressive loading. The sandwich section was assumed symmetric and the facings and the core were considered to be orthotropic. Solutions for global buckling and face wrinkling of sandwich plates under transient loads were presented by Hohe [23] using the Galerkin method. He used the first-order shear deformable plate theory (FSDT) for the face sheets and linear and quadratic functions for transverse and in-plane displacements of the core. Meyer-Piening [24] presented two linear wrinkling formulations for sandwich plates with thin and thick orthotropic facings based on the analytical formulations of Zenkert [7]. He modified the models to account for unequal facings and orthotropic properties in the face layers and compared the obtained results with the finite element method (FEM).

Aiello and Ombres [25] presented an analytical approach for evaluating the buckling load of sandwich panels made of hybrid laminated faces and a transversely flexible core. A priori assumption of the displacement field through the thickness was applied which was a superposition of symmetric and anti-symmetric components besides a pure compressive mode. Lopatin and Morzov [2] presented the solution of the face wrinkling problem for a sandwich panel with composite facings and an orthotropic core based on the energy method. They developed a new model of the elastic core which allowed for the transverse compression and shear of the core material as well as nonlinear through-the-thickness decay of the lateral normal displacements at wrinkling. Shariyat [26] studied nonlinear dynamic thermo-mechanical buckling and wrinkling of the imperfect sandwich plates using the finite element method. He introduced a generalized global–local plate theory (GLPT) that guarantees the continuity conditions of all displacements and transverse stress components and considered the transverse flexibility of sandwich plates.

Some researchers studied the face wrinkling problem of sandwich plates experimentally. Pearce and Webber [27] presented the overall buckling and wrinkling loads for different four-edges simply supported sandwich plates by experiments. Wadee [28] investigated localized cylindrical buckling of sandwich panels experimentally and compared the obtained results with the theoretical solutions. Also, Wadee [29] studied the effect of localized imperfections on the wrinkling of sandwich panels. Gdoutos et al. [30] measured face wrinkling failure loads of sandwich columns under compression, beams in three- and four-point bending and cantilever beams under end loading.

Noor et al. [31] presented three-dimensional elasticity solutions for global buckling of simply supported sandwich panels with composite face sheets. But, they did not present a wrinkling analysis of the sandwich plates. Kardomateas [32] presented a 2D elasticity solution for the wrinkling analysis of sandwich beams or wide sandwich panels subjected to axially compressive loading. The sandwich section was assumed symmetric and the facings and the core were considered to be orthotropic. Ji and Waas [33] studied the elastic stability of a sandwich panel (wide beam) using 2D classical elasticity. They obtained global buckling and wrinkling loads of 2D sandwich panels.

Based on the above discussions, it can be concluded that initial works on wrinkling of sandwich plates [3–6] modeled the supporting action of the core by a simple Winkler elastic foundation. In these models, the effect of the other face sheet is neglected and face sheets are assumed isotropic. Also, in this approach, the sandwich plate wrinkle in a 2D manner such as a sandwich beam or a sandwich plate in cylindrical bending. Based on this approach, some authors [1, 18–20] studied the wrinkling of sandwich plates with anisotropic or orthotropic face sheets. 2D wrinkling analysis of sandwich beams or sandwich plates in cylindrical bending was only presented using an elasticity solution [22] or an energy method [2]. Some investigators [12–14, 16, 23 and 26] assumed the layered sandwich plates consisting of two laminated composite face sheets and a soft flexible core and postulated polynomial functions for in-plane and transverse displacements of each layer. Frostig [12], Dafedar et al. [16] and Hohe [23] presented analytical solutions for wrinkling analysis of sandwich plates, but they used some simplifications which resulted in a lower accuracy. The higher-order global–local theory of Shariyat [26] is sufficient and accurate for the solution of sandwich plates. But, his solution for buckling and wrinkling problem was not presented analytically.

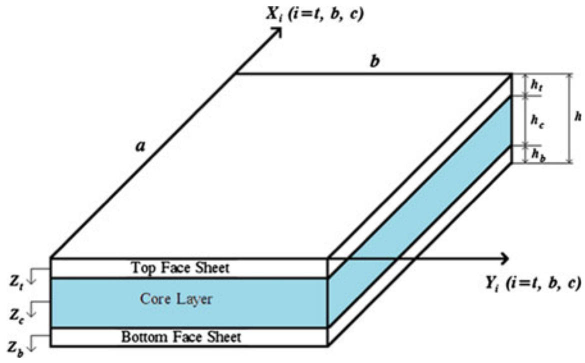
Therefore, it seems there are no published papers on an analytical solution for wrinkling analysis of composite-faced sandwich plates using an accurate 3D theory. The purpose of this chapter is to present a higher-order plate theory for the wrinkling analysis of composite-faced sandwich plates with a soft orthotropic core. There are some important points to be noted for more accuracy in the modeling and analysis of sandwich structures. The continuity conditions of the displacements and the inter-laminar transverse shear stresses should be satisfied to accurately model the mechanical behavior of these plates. But, variation of material properties between the core and the face sheets causes slopes of displacements and transverse shear stresses to change in the face sheet-core interfaces. In addition, the conditions of zero transverse shear stresses on the upper and lower surfaces must be enforced. Polymer foam cores are very flexible relative to the face sheets. As such, this behavior leads to un-identical displacement patterns through the depth of the panel and also the displacements of the upper face sheet differ from those of the lower one [34].

In the present chapter, third-order plate theory is used for the face sheets and quadratic and cubic functions are assumed for the transverse and in-plane displacements of the core. The nonlinear von Kármán type relations are used to obtain strains. Continuity conditions of transverse shear stresses at the interfaces as well as the conditions of zero transverse shear stresses on the upper and lower surfaces of the plate are satisfied. Also, transverse flexibility and transverse normal strain and stress of the core are considered. The equations of motion and boundary conditions are derived via the principle of minimum potential energy. An analytical solution for static analysis of simply supported sandwich plates under in-plane compressive loads is presented using Navier's solution. Wrinkling loads are obtained for various sandwich plates. The effect of geometrical parameters of face sheets and core are studied on the wrinkling behavior of sandwich plates.

2 Mathematical Formulations

A rectangular sandwich plate with plane dimensions $a \times b$ and total thickness of h is considered, as shown in Fig. 1. The sandwich is composed of three layers: the top and the bottom face sheets and the core layer. All layers are assumed with a uniform thickness and the z-coordinate of each layer is measured downward from its mid-plane. The face sheets are generally unequal in thickness i.e. h_t and h_b are the thicknesses of the top and bottom face sheets respectively. The face sheets are assumed to be laminated composites. The core is also assumed as a soft orthotropic material with thickness h_c.

Fig. 1 A typical sandwich plate and its dimensions



2.1 Kinematic Relations

In the present structural model for sandwich plates, the third-order shear deformable theory is adopted for the face sheets. Hence, the displacement components of the top and bottom face sheets $(j = t\,,\, b)$ are represented as [34]:

$$\begin{aligned} u_j(x,y,z_t) &= u_{0j}(x,y) + z_j u_{1j}(x,y) + z_j^2 u_{2j}(x,y) + z_j^3 u_{3j}(x,y) \\ v_j(x,y,z_t) &= v_{0j}(x,y) + z_j v_{1j}(x,y) + z_j^2 v_{2j}(x,y) + z_j^3 v_{3j}(x,y) \\ w_j(x,y,z_t) &= w_{0j}(x,y) \end{aligned} \tag{1}$$

where u_{kj} and v_{kj} $(k = 0, 1, 2, 3)$ are the unknowns of the in-plane displacements of each face sheet and w_{0j} are the unknowns of its vertical displacements, respectively.

The core layer is much thicker and softer than the face sheets. Thus, the displacements fields for the core are assumed as a cubic pattern for the in-plane displacement components and as a quadratic one for the vertical component:

$$\begin{aligned} u_c(x,y,z_c) &= u_{0c}(x,y) + z_c u_{1c}(x,y) + z_c^2 u_{2c}(x,y) + z_c^3 u_{3c}(x,y) \\ v_c(x,y,z_c) &= v_{0c}(x,y) + z_c v_{1c}(x,y) + z_c^2 v_{2c}(x,y) + z_c^3 v_{3c}(x,y) \\ w_c(x,y,z_c) &= w_{0c}(x,y) + z_c w_{1c}(x,y) + z_c^2 w_{2c}(x,y) \end{aligned} \tag{2}$$

where u_{kc} and v_{kc} $(k = 0,\ 1,\ 2,\ 3)$ are the unknowns of the in-plane displacement components of the core and w_{lc} $(l = 0,\ 1,\ 2)$ are the unknowns of its vertical displacements, respectively. Therefore, the face sheets are assumed as in-plane flexible and transversely rigid plates. Also, the core is assumed as in-plane and transversely flexible layer. Finally, in this model there are twenty-nine displacement unknowns: nine unknowns for each face sheet and eleven unknowns for the core.

2.2 Compatibility Conditions

In the present sandwich plate theory, the core is perfectly bonded to the face sheets. Hence, there are three interface displacement continuity requirements in each face sheet-core interface which can be obtained as follows:

$$\begin{aligned} u_t\left(z_t=\frac{h_t}{2}\right) &= u_c\left(z_c=\frac{-h_c}{2}\right), u_b\left(z_b=\frac{-h_b}{2}\right) = u_c\left(z_c=\frac{h_c}{2}\right) \\ v_t\left(z_t=\frac{h_t}{2}\right) &= v_c\left(z_c=\frac{-h_c}{2}\right), v_b\left(z_b=\frac{-h_b}{2}\right) = v_c\left(z_c=\frac{h_c}{2}\right) \\ w_t\left(z_t=\frac{h_t}{2}\right) &= w_c\left(z_c=\frac{-h_c}{2}\right), w_b\left(z_b=\frac{-h_b}{2}\right) = w_c\left(z_c=\frac{h_c}{2}\right) \end{aligned} \tag{3}$$

2.3 Strains

The nonlinear von Kármán strain–displacement relations for the face sheets ($j = t, b$) can be expressed as:

$$\begin{aligned} \varepsilon^j_{xx} &= u_{0j,x} + z_j u_{1j,x} + z_j^2 u_{2j,x} + z_j^3 u_{3j,x} + \frac{1}{2}\left(w_{0j,x}\right)^2 \\ \varepsilon^j_{yy} &= v_{0j,y} + z_t v_{1j,y} + z_j^2 v_{2j,y} + z_j^3 v_{3j,y} + \frac{1}{2}\left(w_{0j,y}\right)^2 \\ \varepsilon^j_{zz} &= 0 \\ \gamma^j_{xy} &= v_{0j,x} + z_j v_{1j,x} + z_j^2 v_{2j,x} + z_j^3 v_{3j,x} + u_{0j,y} + z_j u_{1j,y} + z_j^2 u_{2j,y} + z_j^3 u_{3j,y} + w_{0j,x} w_{0j,y} \\ \gamma^j_{xz} &= u_{1j} + 2z_j u_{2j} + 3z_j^2 u_{3j} + w_{0j,x} \\ \gamma^j_{yz} &= v_{1j} + 2z_j v_{2j} + 3z_j^2 v_{3j} + w_{0j,y} \end{aligned} \tag{4}$$

and the nonlinear von Kármán strain–displacement relations for the core can be defined as:

$$\begin{aligned} \varepsilon^c_{xx} &= u_{0c,x} + z_c u_{1c,x} + z_c^2 u_{2c,x} + z_c^3 u_{3c,x} + \frac{1}{2}\left(w_{0c,x}\right)^2 \\ \varepsilon^c_{yy} &= v_{0c,y} + z_c v_{1c,y} + z_c^2 v_{2c,y} + z_c^3 v_{3c,y} + \frac{1}{2}\left(w_{0c,y}\right)^2 \\ \varepsilon^c_{zz} &= w_{1c} + 2z_c w_{2c} \\ \gamma^c_{xy} &= v_{0c,x} + z_c v_{1c,x} + z_c^2 v_{2c,x} + z_c^3 v_{3c,x} + u_{0c,y} + z_c u_{1c,y} + z_c^2 u_{2c,y} + z_c^3 u_{3c,y} + w_{0c,x} w_{0c,y} \\ \gamma^c_{xz} &= u_{1c} + 2z_c u_{2c} + 3z_c^2 u_{3c} + w_{0c,x} + z_c w_{1c,x} + z_c^2 w_{2c,x} \\ \gamma^c_{yz} &= v_{1c} + 2z_c v_{2c} + 3z_c^2 v_{3c} + w_{0c,y} + z_c w_{1c,y} + z_c^2 w_{2c,y} \end{aligned} \tag{5}$$

2.4 Transverse Shear Stresses

For an orthotropic lamina of laminated composite plates such as the top and bottom face sheets, the reduced stress–strain relationships can be defined as follows [34]:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & Q_{16} \\ Q_{12} & Q_{22} & 0 & 0 & Q_{26} \\ 0 & 0 & Q_{44} & Q_{45} & 0 \\ 0 & 0 & Q_{45} & Q_{55} & 0 \\ Q_{16} & Q_{26} & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \tag{6}$$

where $Q_{mn}(m, n = 1, 2, 6)$ are the reduced stiffness coefficients and $Q_{kl}(k, l = 4, 5)$ are the transverse shear stiffness coefficients. The transverse shear stresses on the upper surface of the sandwich plate are zero [32]. Hence:

$$\begin{cases} Q_{44}^{tu}\gamma_{yz}^{t}\left(z_t = \dfrac{-h_t}{2}\right) + Q_{45}^{tu}\gamma_{xz}^{t}\left(z_t = \dfrac{-h_t}{2}\right) = 0 \\ Q_{45}^{tu}\gamma_{yz}^{tu}\left(z_t = \dfrac{-h_t}{2}\right) + Q_{55}^{tu}\gamma_{xz}^{t}\left(z_t = \dfrac{-h_t}{2}\right) = 0 \end{cases} \tag{7}$$

where $Q_{mn}^{tu}(m, n = 4, 5)$ and $\gamma_{kz}^{t}(k = x, y)$ are the transverse shear stiffness coefficients and transverse shear strains of the upper lamina of the top face sheet, respectively. From the above equations, it can be simply drawn that:

$$\gamma_{yz}^{t}\left(z_t = \frac{-h_t}{2}\right) = \gamma_{xz}^{t}\left(z_t = \frac{-h_t}{2}\right) = 0 \tag{8}$$

Similarly, for the lower lamina of the bottom face sheet:

$$\gamma_{yz}^{b}\left(z_b = \frac{h_b}{2}\right) = \gamma_{xz}^{b}\left(z_b = \frac{h_b}{2}\right) = 0 \tag{9}$$

In addition, the continuity of transverse shear stresses at the top and bottom face sheet-core interfaces must be satisfied. The stress–strain relationships for the orthotropic core can be read as follows [34]:

$$\begin{Bmatrix} \sigma_{xx}^{c} \\ \sigma_{yy}^{c} \\ \sigma_{zz}^{c} \\ \tau_{yz}^{c} \\ \tau_{xz}^{c} \\ \tau_{xy}^{c} \end{Bmatrix} = \begin{bmatrix} C_{11}^{c} & C_{12}^{c} & C_{13}^{c} & 0 & 0 & 0 \\ C_{21}^{c} & C_{22}^{c} & C_{23}^{c} & 0 & 0 & 0 \\ C_{31}^{c} & C_{32}^{c} & C_{33}^{c} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44}^{c} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55}^{c} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66}^{c} \end{bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^{c} \\ \varepsilon_{yy}^{c} \\ \varepsilon_{zz}^{c} \\ \gamma_{yz}^{c} \\ \gamma_{xz}^{c} \\ \gamma_{xy}^{c} \end{Bmatrix} \tag{10}$$

where $C_{mn}^{c}(m, n = 1, \ldots, 6)$ are the stiffness coefficients of the core. Therefore, at the top face sheet-core interface, the transverse shear stress continuity conditions can be read as:

$$\begin{cases} Q_{44}^{tl}\gamma_{yz}^{t}\left(z_t = \frac{h_t}{2}\right) + Q_{45}^{tl}\gamma_{xz}^{t}\left(z_t = \frac{h_t}{2}\right) = C_{44}^{c}\gamma_{yz}^{c}\left(z_c = \frac{-h_c}{2}\right) \\ Q_{45}^{tl}\gamma_{yz}^{t}\left(z_t = \frac{h_t}{2}\right) + Q_{55}^{tl}\gamma_{xz}^{t}\left(z_t = \frac{h_t}{2}\right) = C_{55}^{c}\gamma_{xz}^{c}\left(z_c = \frac{-h_c}{2}\right) \end{cases} \tag{11}$$

In the above equations, $Q_{mn}^{tl}(m, n = 4, 5)$ and $\gamma_{kz}^{t}(k = x, y)$ are the transverse shear stiffness coefficients and the transverse shear strains of the lower lamina of the top face sheet, respectively. $C_{mm}^{c}(m = 4, 5)$ are the transverse shear stiffness coefficients of the core and $\gamma_{kz}^{c}(k = x, y)$ are the transverse shear strains of the core. Similarly, at the bottom face sheet-core interface:

$$\begin{cases} Q_{44}^{bu}\gamma_{yz}^{b}\left(z_b = \frac{-h_b}{2}\right) + Q_{45}^{bu}\gamma_{xz}^{b}\left(z_b = \frac{-h_b}{2}\right) = C_{44}^{c}\gamma_{yz}^{c}\left(z_c = \frac{h_c}{2}\right) \\ Q_{45}^{bu}\gamma_{yz}^{b}\left(z_b = \frac{-h_b}{2}\right) + Q_{55}^{bu}\gamma_{xz}^{b}\left(z_b = \frac{-h_b}{2}\right) = C_{55}^{c}\gamma_{xz}^{c}\left(z_c = \frac{h_c}{2}\right) \end{cases} \tag{12}$$

where $Q_{mn}^{bu}(m, n = 4, 5)$ and $\gamma_{kz}^{b}(k = x, y)$ are the transverse shear stiffness coefficients and transverse shear strains of the upper lamina of the bottom face sheet, respectively. Finally, eight Eqs. (8, 9, 11, 12) are obtained for satisfying the continuity conditions of transverse shear stresses at the interfaces as well as the conditions of zero transverse shear stresses on the upper and lower surfaces of the sandwich plate. In the case of cross-ply laminated face sheets, these equations can be reduced by applying $Q_{45}^{tu} = Q_{45}^{tl} = Q_{45}^{bu} = Q_{45}^{bl} = 0$. In addition, for an orthotropic core $\mathrm{C}_{44}^{\mathrm{c}} = \mathrm{G}_{\mathrm{yz}}^{\mathrm{c}}$ and $\mathrm{C}_{55}^{\mathrm{c}} = \mathrm{G}_{\mathrm{xz}}^{\mathrm{c}}$.

2.5 Governing Equations

The governing equations of motion for the face sheets and the core are derived through the principle of minimum potential energy:

$$\delta\Pi = \delta U + \delta V = 0 \tag{13}$$

where U is the total strain energy, V is the potential of the external loads and δ denotes the variation operator. The variation of the external work equals to:

$$\delta V = -\int_0^a\int_0^b \left[\overline{n_{xt}}\delta u_{0t} + \overline{n_{yt}}\delta v_{0t} + q_t\delta w_{0t} + \overline{n_{xb}}\delta u_{0b} + \overline{n_{yb}}\delta v_{0b} + q_b\delta w_{0b}\right]dxdy \tag{14}$$

where u_{0j}, v_{0j} and $w_{0j}(j = t, b)$ are the displacements of the mid-plane of the face sheets in the longitudinal, transverse and vertical directions, respectively; $\overline{n_{xj}}$ and $\overline{n_{yj}}(j = t, b)$ are the in-plane external loads of the top and bottom face sheets and q_t and q_b are the vertical distributed loads applied on the top and bottom face sheets, respectively.

The first variation of the total strain energy can be expressed in terms of all stresses and strains of the face sheets and the core. In addition, six compatibility conditions at the interfaces, four conditions of zero transverse shear stresses on the upper and lower surfaces of the plate and four continuity conditions of transverse shear stresses at the interfaces are fulfilled by using fourteen Lagrange multipliers. Thus, the variation of the strain energy for the sandwich plate with cross-ply laminated face sheets and an orthotropic core can be modified by using the following equation [12]:

$$
\begin{aligned}
\delta U &= \int_{v_t}\left(\sigma_{xx}^t\delta\varepsilon_{xx}^t + \sigma_{yy}^t\delta\varepsilon_{yy}^t + \tau_{xy}^t\delta\gamma_{xy}^t + \tau_{xz}^t\delta\gamma_{xz}^t + \tau_{yz}^t\delta\gamma_{yz}^t\right)dv \\
&+ \int_{v_b}\left(\sigma_{xx}^b\delta\varepsilon_{xx}^b + \sigma_{yy}^b\delta\varepsilon_{yy}^b + \tau_{xy}^b\delta\gamma_{xy}^b + \tau_{xz}^b\delta\gamma_{xz}^b + \tau_{yz}^b\delta\gamma_{yz}^b\right)dv \\
&+ \int_{v_c}\left(\sigma_{zz}^c\delta\varepsilon_{zz}^c + \tau_{xz}^c\delta\gamma_{xz}^c + \tau_{yz}^c\delta\gamma_{yz}^c + \sigma_{xx}^c\delta\varepsilon_{xx}^c + \sigma_{yy}^c\delta\varepsilon_{yy}^c + \tau_{xy}^c\delta\gamma_{xy}^c\right)dv \\
&+ \delta\left\{\int_0^a\int_0^b\left[\lambda_x^t\left(u_t\left(z_t=\frac{h_t}{2}\right) - u_c\left(z_c=\frac{-h_c}{2}\right)\right) + \lambda_y^t\left(v_t\left(z_t=\frac{h_t}{2}\right) - v_c\left(z_c=\frac{-h_c}{2}\right)\right)\right.\right. \\
&+ \lambda_z^t\left(w_t\left(z_t=\frac{h_t}{2}\right) - w_c\left(z_c=\frac{-h_c}{2}\right)\right) + \lambda_x^b\left(u_b\left(z_b=\frac{-h_b}{2}\right) - u_c\left(z_c=\frac{h_c}{2}\right)\right) \\
&\left.\left.+\lambda_y^b\left(v_b\left(z_b=\frac{-h_b}{2}\right) - v_c\left(z_c=\frac{h_c}{2}\right)\right) + \lambda_z^b\left(w_b\left(z_b=\frac{-h_b}{2}\right) - w_c\left(z_c=\frac{h_c}{2}\right)\right)\right]dxdy\right\} \\
&+ \delta\left\{\int_0^a\int_0^b\left[\lambda_{xz}^t\left(\gamma_{xz}^t\left(z_t=\frac{-h_t}{2}\right)\right) + \lambda_{yz}^t\left(\gamma_{yz}^t\left(z_t=\frac{-h_t}{2}\right)\right) + \lambda_{xz}^b\left(\gamma_{xz}^b\left(z_b=\frac{h_b}{2}\right)\right)\right.\right. \\
&+ \lambda_{yz}^b\left(\gamma_{yz}^b\left(z_b=\frac{h_b}{2}\right)\right) + \lambda_{yz}^{tc}\left(Q_{44}^t\gamma_{yz}^t\left(z_t=\frac{h_t}{2}\right) - G_{yz}^c\gamma_{yz}^c\left(z_c=\frac{-h_c}{2}\right)\right) \\
&+ \lambda_{xz}^{tc}\left(Q_{55}^t\gamma_{xz}^t\left(z_t=\frac{h_t}{2}\right) = G_{xz}^c\gamma_{xz}^c\left(z_c=\frac{-h_c}{2}\right)\right) + \lambda_{yz}^{bc}\left(Q_{44}^b\gamma_{yz}^b\left(z_b=\frac{-h_b}{2}\right) = G_{yz}^c\gamma_{yz}^c\left(z_c=\frac{h_c}{2}\right)\right) \\
&\left.\left.+\lambda_{xz}^{bc}\left(Q_{55}^b\gamma_{xz}^b\left(z_t=\frac{-h_b}{2}\right) = G_{xz}^c\gamma_{xz}^c\left(z_c=\frac{h_c}{2}\right)\right)\right]dxdy\right\}
\end{aligned}
\tag{15}
$$

where $\lambda_i^j(i=x,y,z),(j=t,b)$ are the six Lagrange multipliers for compatibility conditions at the top and bottom interfaces; $\lambda_{iz}^j(i=x,y),(j=t,b)$ are the four Lagrange multipliers for conditions of zero transverse shear stresses on the upper and lower surfaces of the plate and $\lambda_{iz}^{jc}(i=x,y),(j=t,b)$ are the four Lagrange multipliers for continuity conditions of transverse shear stresses at the interfaces. The stress resultants for the two face sheets and the core $(j=t,b,c)$ can be defined as:

$$
\begin{Bmatrix} N_{xx}^j & N_{yy}^j & N_{xy}^j \\ M_{xx}^j & M_{yy}^j & M_{xy}^j \\ P_{xx}^j & P_{yy}^j & P_{xy}^j \\ R_{xx}^j & R_{yy}^j & R_{xy}^j \end{Bmatrix} = \int_{\frac{-h_j}{2}}^{\frac{h_j}{2}} \begin{bmatrix} \sigma_{xx}^j & \sigma_{yy}^j & \tau_{xy}^j \end{bmatrix} \begin{Bmatrix} 1 \\ z_j \\ z_j^2 \\ z_j^3 \end{Bmatrix} dz_j \tag{16}
$$

$$\begin{Bmatrix} Q^j_{xz} & Q^j_{yz} \\ S^j_{xz} & S^j_{yz} \\ T^j_{xz} & T^j_{yz} \end{Bmatrix} = \int_{\frac{-h_j}{2}}^{\frac{h_j}{2}} \begin{bmatrix} \tau^j_{xz} & \tau^j_{yz} \end{bmatrix} \begin{Bmatrix} 1 \\ z_j \\ z_j^2 \end{Bmatrix} dz_j \tag{17}$$

Also, the transverse normal stress resultants for the core can be defined as:

$$\begin{Bmatrix} N^c_{zz} \\ M^c_{zz} \end{Bmatrix} = \int_{\frac{-h_c}{2}}^{\frac{h_c}{2}} \sigma^c_{zz} \begin{Bmatrix} 1 \\ z_c \end{Bmatrix} dz_c \tag{18}$$

Integrating by part and doing some mathematical operations, the equations of motion for the top face sheet can be calculated as:

$$\begin{aligned}
&-N^t_{xx,x} - N^t_{xy,y} + \lambda^t_x - \overline{n_{xt}} = 0 \\
&-M^t_{xx,x} - M^t_{xy,y} + Q^t_{xz} + \frac{h_t}{2}\lambda^t_x + \lambda^t_{xz} + \frac{h_t}{2}\overline{n_{xt}} = 0 \\
&-P^t_{xx,x} - P^t_{xy,y} + 2S^t_{xz} + \frac{h_t^2}{4}\lambda^t_x - h_t\lambda^t_{xz} + \lambda^{tc}_{xz} - \frac{h_t^2}{4}\overline{n_{xt}} = 0 \\
&-R^t_{xx,x} - R^t_{xy,y} + 3T^t_{xz} + \frac{h_t^3}{8}\lambda^t_x + \frac{3h_t^2}{4}\lambda^t_{xz} + \frac{h_t^3}{8}\overline{n_{xt}} = 0 \\
&-N^t_{yy,y} - N^t_{xy,x} + \lambda^t_y - \overline{n_{yt}} = 0 \\
&-M^t_{yy,y} - M^t_{xy,x} + Q^t_{yz} + \frac{h_t}{2}\lambda^t_y + \lambda^t_{yz} + \frac{h_t}{2}\overline{n_{yt}} = 0 \\
&-P^t_{yy,y} - P^t_{xy,x} + 2S^t_{yz} + \frac{h_t^2}{4}\lambda^t_y - h_t\lambda^t_{yz} + \lambda^{tc}_{yz} - \frac{h_t^2}{4}\overline{n_{yt}} = 0 \\
&-R^t_{yy,y} - R^t_{xy,x} + 3T^t_{yz} + \frac{h_t^3}{8}\lambda^t_y + \frac{3h_t^2}{4}\lambda^t_{yz} + \frac{h_t^3}{8}\overline{n_{yt}} = 0 \\
&-Q^t_{xz,x} - Q^t_{yz,y} - \mathcal{N}(w_{0t}) + \lambda^t_z - \lambda^t_{xz,x} - \lambda^t_{yz,y} - q_t = 0
\end{aligned} \tag{19}$$

and for the bottom face sheet as:

$$\begin{aligned}
&-N^b_{xx,x} - N^b_{xy,y} + \lambda^b_x - \overline{n_{xb}} = 0 \\
&-M^b_{xx,x} - M^b_{xy,y} + Q^b_{xz} - \frac{h_b}{2}\lambda^b_x + \lambda^b_{xz} - \frac{h_b}{2}\overline{n_{xb}} = 0 \\
&-P^b_{xx,x} - P^b_{xy,y} + 2S^b_{xz} + \frac{h_b^2}{4}\lambda^b_x + h_b\lambda^b_{xz} + \lambda^{bc}_{xz} - \frac{h_b^2}{4}\overline{n_{xb}} = 0 \\
&-R^b_{xx,x} - R^b_{xy,y} + 3T^b_{xz} - \frac{h_b^3}{8}\lambda^b_x + \frac{3h_b^2}{4}\lambda^b_{xz} - \frac{h_b^3}{8}\overline{n_{xb}} = 0 \\
&-N^b_{yy,y} - N^b_{xy,x} + \lambda^b_y - \overline{n_{yb}} = 0 \\
&-M^b_{yy,y} - M^b_{xy,x} + Q^b_{yz} - \frac{h_b}{2}\lambda^b_y + \lambda^b_{yz} - \frac{h_b}{2}\overline{n_{yb}} = 0 \\
&-P^b_{yy,y} - P^b_{xy,x} + 2S^b_{yz} + \frac{h_b^2}{4}\lambda^b_y + h_b\lambda^b_{yz} + \lambda^{bc}_{yz} - \frac{h_b^2}{4}\overline{n_{yb}} = 0 \\
&-R^b_{yy,y} - R^b_{xy,x} + 3T^b_{yz} - \frac{h_b^3}{8}\lambda^b_y + \frac{3h_b^2}{4}\lambda^b_{yz} - \frac{h_b^3}{8}\overline{n_{yb}} = 0 \\
&-Q^b_{xz,x} - Q^b_{yz,y} - \mathcal{N}(w_{0b}) + \lambda^b_z - \lambda^b_{xz,x} - \lambda^b_{yz,y} - q_b = 0
\end{aligned} \tag{20}$$

and also for the core as:

$$
\begin{aligned}
&-N^c_{xx,x} - N^c_{xy,y} - \lambda^t_x - \lambda^b_x = 0\\
&-M^c_{xx,x} - M^c_{xy,y} + Q^c_{xz} + \frac{h_c}{2}\lambda^t_x - \frac{h_c}{2}\lambda^b_x - \frac{G^c_{xz}}{2h_t Q^t_{55}}\lambda^{tc}_{xz} + \frac{G^c_{xz}}{2h_b Q^b_{55}}\lambda^{bc}_{xz} = 0\\
&-P^c_{xx,x} - P^c_{xy,y} + 2S^c_{xz} - \frac{h_c^2}{4}\lambda^t_x - \frac{h_c^2}{4}\lambda^b_x + \frac{G^c_{xz}h_c}{2h_t Q^t_{55}}\lambda^{tc}_{xz} + \frac{G^c_{xz}h_c}{2h_b Q^b_{55}}\lambda^{bc}_{xz} = 0\\
&-R^c_{xx,x} - R^c_{xy,y} + 3T^c_{xz} + \frac{h_c^3}{8}\lambda^t_x - \frac{h_c^3}{8}\lambda^b_x - \frac{3G^c_{xz}h_c^2}{8h_t Q^t_{55}}\lambda^{tc}_{xz} + \frac{3G^c_{xz}h_c^2}{8h_b Q^b_{55}}\lambda^{bc}_{xz} = 0\\
&-N^c_{yy,y} - N^c_{xy,x} - \lambda^t_y - \lambda^b_y = 0\\
&-M^c_{yy,y} - M^c_{xy,x} + Q^c_{yz} + \frac{h_c}{2}\lambda^t_y - \frac{h_c}{2}\lambda^b_y - \frac{G^c_{yz}}{2h_t Q^t_{44}}\lambda^{tc}_{yz} + \frac{G^c_{yz}}{2h_b Q^b_{44}}\lambda^{bc}_{yz} = 0\\
&-P^c_{yy,y} - P^c_{xy,x} + 2S^c_{yz} - \frac{h_c^2}{4}\lambda^t_y - \frac{h_c^2}{4}\lambda^b_y + \frac{G^c_{yz}h_c}{2h_t Q^t_{44}}\lambda^{tc}_{yz} + \frac{G^c_{yz}h_c}{2h_b Q^b_{44}}\lambda^{bc}_{yz} = 0\\
&-R^c_{yy,y} - R^c_{xy,x} + 3T^c_{yz} + \frac{h_c^3}{8}\lambda^t_y - \frac{h_c^3}{8}\lambda^b_y - \frac{3G^c_{yz}h_c^2}{8h_t Q^t_{44}}\lambda^{tc}_{yz} + \frac{3G^c_{yz}h_c^2}{8h_b Q^b_{44}}\lambda^{bc}_{yz} = 0\\
&-Q^c_{xz,x} - Q^c_{yz,y} - \mathcal{N}(w_{0c}) - \lambda^t_z - \lambda^b_z + \frac{G^c_{xz}}{2h_t Q^t_{55}}\lambda^{tc}_{xz,x} + \frac{G^c_{yz}}{2h_t Q^t_{44}}\lambda^{tc}_{yz,y}\\
&\qquad - \frac{G^c_{xz}}{2h_b Q^b_{55}}\lambda^{bc}_{xz,x} - \frac{G^c_{yz}}{2h_b Q^b_{44}}\lambda^{bc}_{yz,y} = 0\\
&-S^c_{xz,x} - S^c_{yz,y} + N^c_{zz} + \frac{h_c}{2}\lambda^t_z - \frac{h_c}{2}\lambda^b_z - \frac{G^c_{xz}h_c}{4h_t Q^t_{55}}\lambda^{tc}_{xz,x} - \frac{G^c_{yz}h_c}{4h_t Q^t_{44}}\lambda^{tc}_{yz,y}\\
&\qquad - \frac{G^c_{xz}h_c}{4h_b Q^b_{55}}\lambda^{bc}_{xz,x} - \frac{G^c_{yz}h_c}{4h_b Q^b_{44}}\lambda^{bc}_{yz,y} = 0\\
&-T^c_{xz,x} - T^c_{yz,y} + 2M^c_{zz} - \frac{h_c^2}{4}\lambda^t_z - \frac{h_c^2}{4}\lambda^b_z + \frac{G^c_{xz}h_c^2}{8h_t Q^t_{55}}\lambda^{tc}_{xz,x} + \frac{G^c_{yz}h^2_{c\,c}}{8h_t Q^t_{44}}\lambda^{tc}_{yz,y}\\
&\qquad - \frac{G^c_{xz}h_c^2}{8h_b Q^b_{55}}\lambda^{bc}_{xz,x} - \frac{G^c_{yz}h_c^2}{8h_b Q^b_{44}}\lambda^{bc}_{yz,y} = 0
\end{aligned}
\tag{21}
$$

where for $j = t, b, c$:

$$
\mathcal{N}\left(w_{0j}\right) = \frac{\partial}{\partial x}\left(w_{0j,x}N^j_{xx} + w_{0i,y}N^i_{xy}\right) + \frac{\partial}{\partial y}\left(w_{0j,y}N^j_{yy} + w_{0j,x}N^j_{xy}\right) \tag{22}
$$

The resultants in the Eqs. (19)–(21) can be related to the total strains by the following equations. For each face sheet $(i = t, b)$:

$$
\begin{aligned}
\begin{Bmatrix} N_{xx}^i \\ M_{xx}^i \\ P_{xx}^i \\ R_{xx}^i \end{Bmatrix} &= \begin{bmatrix} K_{i,11}^0 & K_{i,11}^1 & K_{i,11}^2 & K_{i,11}^3 \\ K_{i,11}^1 & K_{i,11}^2 & K_{i,11}^3 & K_{i,11}^4 \\ K_{i,11}^2 & K_{i,11}^3 & K_{i,11}^4 & K_{i,11}^5 \\ K_{i,11}^3 & K_{i,11}^4 & K_{i,11}^5 & K_{i,11}^6 \end{bmatrix} \begin{Bmatrix} u_{0i,x} + \frac{1}{2}\left(w_{0i,x}\right)^2 \\ u_{1i,x} \\ u_{2i,x} \\ u_{3i,x} \end{Bmatrix} \\
&+ \begin{bmatrix} K_{i,12}^0 & K_{i,12}^1 & K_{i,12}^2 & K_{i,12}^3 \\ K_{i,12}^1 & K_{i,12}^2 & K_{i,12}^3 & K_{i,12}^4 \\ K_{i,12}^2 & K_{i,12}^3 & K_{i,12}^4 & K_{i,12}^5 \\ K_{i,12}^3 & K_{i,12}^4 & K_{i,12}^5 & K_{i,12}^6 \end{bmatrix} \begin{Bmatrix} v_{0i,y} + \frac{1}{2}\left(w_{0i,y}\right)^2 \\ v_{1i,y} \\ v_{2i,y} \\ v_{3i,y} \end{Bmatrix} \\
\begin{Bmatrix} N_{yy}^i \\ M_{yy}^i \\ P_{yy}^i \\ R_{yy}^i \end{Bmatrix} &= \begin{bmatrix} K_{i,21}^0 & K_{i,21}^1 & K_{i,21}^2 & K_{i,21}^3 \\ K_{i,21}^1 & K_{i,21}^2 & K_{i,21}^3 & K_{i,21}^4 \\ K_{i,21}^2 & K_{i,21}^3 & K_{i,21}^4 & K_{i,21}^5 \\ K_{i,21}^3 & K_{i,21}^4 & K_{i,21}^5 & K_{i,21}^6 \end{bmatrix} \begin{Bmatrix} u_{0i,x} + \frac{1}{2}\left(w_{0i,x}\right)^2 \\ u_{1i,x} \\ u_{2i,x} \\ u_{3i,x} \end{Bmatrix} \\
&+ \begin{bmatrix} K_{i,22}^0 & K_{i,22}^1 & K_{i,22}^2 & K_{i,22}^3 \\ K_{i,22}^1 & K_{i,22}^2 & K_{i,22}^3 & K_{i,22}^4 \\ K_{i,22}^2 & K_{i,22}^3 & K_{i,22}^4 & K_{i,22}^5 \\ K_{i,22}^3 & K_{i,22}^4 & K_{i,12}^5 & K_{i,22}^6 \end{bmatrix} \begin{Bmatrix} v_{0i,y} + \frac{1}{2}\left(w_{0i,y}\right)^2 \\ v_{1i,y} \\ v_{2i,y} \\ v_{3i,y} \end{Bmatrix} \\
\begin{Bmatrix} N_{xy}^i \\ M_{xy}^i \\ P_{xy}^i \\ R_{xy}^i \end{Bmatrix} &= \begin{bmatrix} K_{i,66}^0 & K_{i,66}^1 & K_{i,66}^2 & K_{i,66}^3 \\ K_{i,66}^1 & K_{i,66}^2 & K_{i,66}^3 & K_{i,66}^4 \\ K_{i,66}^2 & K_{i,66}^3 & K_{i,66}^4 & K_{i,66}^5 \\ K_{i,66}^3 & K_{i,66}^4 & K_{i,66}^5 & K_{i,66}^6 \end{bmatrix} \begin{Bmatrix} u_{0i,y} + v_{0i,x} + w_{0i,x}w_{0i,y} \\ u_{1i,y} + v_{1i,x} \\ u_{2i,y} + v_{2i,x} \\ u_{3i,y} + v_{3i,x} \end{Bmatrix} \\
\begin{Bmatrix} Q_{xz}^i \\ S_{xz}^i \\ T_{xz}^i \end{Bmatrix} &= \begin{bmatrix} K_{i,55}^0 & 2K_{i,55}^1 & 3K_{i,55}^2 \\ K_{i,55}^1 & 2K_{i,55}^2 & 3K_{i,55}^3 \\ K_{i,55}^2 & 2K_{i,55}^3 & 3K_{i,55}^4 \end{bmatrix} \begin{Bmatrix} u_{1i} + w_{0i,x} \\ u_{2i} \\ u_{3i} \end{Bmatrix} \\
\begin{Bmatrix} Q_{yz}^i \\ S_{yz}^i \\ T_{yz}^i \end{Bmatrix} &= \begin{bmatrix} K_{i,44}^0 & 2K_{i,44}^1 & 3K_{i,44}^2 \\ K_{i,44}^1 & 2K_{i,44}^2 & 3K_{i,44}^3 \\ K_{i,44}^2 & 2K_{i,44}^3 & 3K_{i,44}^4 \end{bmatrix} \begin{Bmatrix} v_{1i} + w_{0i,y} \\ v_{2i} \\ v_{3i} \end{Bmatrix}
\end{aligned}
\tag{23}
$$

and for the core:

$$\begin{Bmatrix} N_{xx}^c \\ M_{xx}^c \\ P_{xx}^c \\ R_{xx}^c \end{Bmatrix} = C_{11}^c \begin{bmatrix} K_c^0 & K_c^1 & K_c^2 & K_c^3 \\ K_c^1 & K_c^2 & K_c^3 & K_c^4 \\ K_c^2 & K_c^3 & K_c^4 & K_c^5 \\ K_c^3 & K_c^4 & K_c^5 & K_c^6 \end{bmatrix} \begin{Bmatrix} u_{0c,x} + \frac{1}{2}\left(w_{0c,x}\right)^2 \\ u_{1c,x} \\ u_{2c,x} \\ u_{3c,x} \end{Bmatrix}$$

$$+ C_{12}^c \begin{bmatrix} K_c^0 & K_c^1 & K_c^2 & K_c^3 \\ K_c^1 & K_c^2 & K_c^3 & K_c^4 \\ K_c^2 & K_c^3 & K_c^4 & K_c^5 \\ K_c^3 & K_c^4 & K_c^5 & K_c^6 \end{bmatrix} \begin{Bmatrix} v_{0c,y} + \frac{1}{2}\left(w_{0c,y}\right)^2 \\ v_{1c,y} \\ v_{2c,y} \\ v_{3c,y} \end{Bmatrix} + C_{13}^c \begin{bmatrix} K_c^0 & 2K_c^1 \\ K_c^1 & 2K_c^2 \\ K_c^2 & 2K_c^3 \\ K_c^3 & 2K_c^4 \end{bmatrix} \begin{Bmatrix} w_{1c} \\ w_{2c} \end{Bmatrix}$$

$$\begin{Bmatrix} N_{yy}^c \\ M_{yy}^c \\ P_{yy}^c \\ R_{yy}^c \end{Bmatrix} = C_{21}^c \begin{bmatrix} K_c^0 & K_c^1 & K_c^2 & K_c^3 \\ K_c^1 & K_c^2 & K_c^3 & K_c^4 \\ K_c^2 & K_c^3 & K_c^4 & K_c^5 \\ K_c^3 & K_c^4 & K_c^5 & K_c^6 \end{bmatrix} \begin{Bmatrix} u_{0c,x} + \frac{1}{2}\left(w_{0c,x}\right)^2 \\ u_{1c,x} \\ u_{2c,x} \\ u_{3c,x} \end{Bmatrix}$$

$$+ C_{22}^c \begin{bmatrix} K_c^0 & K_c^1 & K_c^2 & K_c^3 \\ K_c^1 & K_c^2 & K_c^3 & K_c^4 \\ K_c^2 & K_c^3 & K_c^4 & K_c^5 \\ K_c^3 & K_c^4 & K_c^5 & K_c^6 \end{bmatrix} \begin{Bmatrix} v_{0c,y} + \frac{1}{2}\left(w_{0c,y}\right)^2 \\ v_{1c,y} \\ v_{2c,y} \\ v_{3c,y} \end{Bmatrix} + C_{23}^c \begin{bmatrix} K_c^0 & 2K_c^1 \\ K_c^1 & 2K_c^2 \\ K_c^2 & 2K_c^3 \\ K_c^3 & 2K_c^4 \end{bmatrix} \begin{Bmatrix} w_{1c} \\ w_{2c} \end{Bmatrix}$$

$$\begin{Bmatrix} N_{zz}^c \\ M_{zz}^c \end{Bmatrix} = C_{31}^c \begin{bmatrix} K_c^0 & K_c^1 & K_c^2 & K_c^3 \\ K_c^1 & K_c^2 & K_c^3 & K_c^4 \end{bmatrix} \begin{Bmatrix} u_{0c,x} \\ u_{1c,x} \\ u_{2c,x} \\ u_{3c,x} \end{Bmatrix}$$

$$+ C_{32}^c \begin{bmatrix} K_c^0 & K_c^1 & K_c^2 & K_c^3 \\ K_c^1 & K_c^2 & K_c^3 & K_c^4 \end{bmatrix} \begin{Bmatrix} v_{0c,y} \\ v_{1c,y} \\ v_{2c,y} \\ v_{3c,y} \end{Bmatrix} + C_{33}^c \begin{bmatrix} K_c^0 & 2K_c^1 \\ K_c^1 & 2K_c^2 \end{bmatrix} \begin{Bmatrix} w_{1c} \\ w_{2c} \end{Bmatrix}$$

$$\begin{Bmatrix} N_{xy}^c \\ M_{xy}^c \\ P_{xy}^c \\ R_{xy}^c \end{Bmatrix} = G_{xy}^c \begin{bmatrix} K_c^0 & K_c^1 & K_c^2 & K_c^3 \\ K_c^1 & K_c^2 & K_c^3 & K_c^4 \\ K_c^2 & K_c^3 & K_c^4 & K_c^5 \\ K_c^3 & K_c^4 & K_c^5 & K_c^6 \end{bmatrix} \begin{Bmatrix} u_{0c,y} + v_{0c,x} + w_{0c,x} w_{0c,y} \\ u_{1c,y} + v_{1c,x} \\ u_{2c,y} + v_{2c,x} \\ u_{3c,y} + v_{3c,x} \end{Bmatrix}$$

$$\begin{Bmatrix} Q_{xz}^c \\ S_{xz}^c \\ T_{xz}^c \end{Bmatrix} = G_{xz}^c \begin{bmatrix} K_c^0 & K_c^1 & K_c^2 \\ K_c^1 & K_c^2 & K_c^3 \\ K_c^2 & K_c^3 & K_c^4 \end{bmatrix} \begin{Bmatrix} u_{1c} + w_{0c,x} \\ 2u_{2c} + w_{1c,x} \\ 3u_{3c} + w_{2c,x} \end{Bmatrix}$$

$$\begin{Bmatrix} Q_{yz}^c \\ S_{yz}^c \\ T_{yz}^c \end{Bmatrix} = G_{yz}^c \begin{bmatrix} K_c^0 & K_c^1 & K_c^2 \\ K_c^1 & K_c^2 & K_c^3 \\ K_c^2 & K_c^3 & K_c^4 \end{bmatrix} \begin{Bmatrix} v_{1c} + w_{0c,y} \\ 2v_{2c} + w_{1c,y} \\ 3v_{3c} + w_{2c,y} \end{Bmatrix} \tag{24}$$

The coefficients $K_{i,jk}^n$ and K_c^n can be defined as:

$$K^n_{i,jk} = \sum_{l=1}^{l_i} Q^l_{jk} \int_{z_i^{l-1}}^{z_i^l} z_i^n dz_i \tag{25}$$

$$K^n_c = \int_{\frac{-h_c}{2}}^{\frac{h_c}{2}} z_c^n dz_c = \begin{cases} 0 & n = even \\ \frac{h_c^{n+1}}{(n+1)2^n} & n = odd \end{cases} \tag{26}$$

where $l_i (i = t, b)$ are the number of composite layers in each face sheet and Q^l_{jk} are the reduced stiffness coefficients of the lth composite layer of each face sheet. Also, z_i^{l-1} and z_i^l are the upper and lower vertical distance of the lth composite layer from the mid-plane of each face sheet.

3 Analytical Solution

Exact analytical solutions of Eqs. (19)–(21) exist for a simply-supported rectangular sandwich plate with cross-ply face sheets. Both face sheets are considered as a cross-ply laminated composite.

For simply-supported plates, the tangential displacements on the boundary are admissible, but the transverse displacements are not as such. Therefore, the boundary conditions of simply-supported plates can be expressed as:

At edges $x = 0$ and $x = a$;

$$\begin{aligned} v_{0j} = 0, v_{1j} = 0, v_{2j} = 0, v_{3j} = 0, j = t, b, c \\ w_{0t} = 0, w_{0b} = 0, w_{0c} = 0, w_{1c} = 0, w_{2c} = 0 \end{aligned} \tag{27}$$

At edges $y = 0$ and $y = b$;

$$\begin{aligned} u_{0j} = 0, u_{1j} = 0, u_{2j} = 0, u_{3j} = 0, j = t, b, c \\ w_{0t} = 0, w_{0b} = 0, w_{0c} = 0, w_{1c} = 0, w_{2c} = 0 \end{aligned} \tag{28}$$

Equation (22) can be rewritten as:

$$\mathcal{N}\left(w_{0j}\right) = \frac{\partial}{\partial x}\left(w_{0j,x}\hat{N}^j_{xx} + w_{0i,y}\hat{N}^j_{xy}\right) + \frac{\partial}{\partial y}\left(w_{0j,y}\hat{N}^j_{yy} + w_{0j,x}\hat{N}^j_{xy}\right) \tag{29}$$

where $\hat{N}^{\mathrm{j}}_{\mathrm{xx}}, \hat{N}^{\mathrm{j}}_{\mathrm{yy}}, \hat{N}^{\mathrm{j}}_{\mathrm{xy}}$ ($j = t$, b and c) are external in-plane loads exerted to the top and bottom face sheets and the core. Therefore:

$$\mathcal{N}\left(w_{0j}\right) = w_{0j,xx}\hat{N}^j_{xx} + 2w_{0j,xy}\hat{N}^j_{xy} + w_{0j,yy}\hat{N}^j_{yy} \tag{30}$$

For sandwich plate subjected to uniaxial compressive loading:

$$\hat{N}_{xx} = -\hat{N}_0, \hat{N}_{yy} = \hat{N}_{xy} = \overline{n_{xt}} = \overline{n_{yt}} = \overline{n_{xb}} = \overline{n_{yb}} = q_t = q_b = 0 \tag{31}$$

The in-plane compressive loads at edges $x = 0$ or $x = a$ are illustrated in Fig. 2. At these edges, the equilibrium equations can be defined as:

$$\begin{aligned} \hat{N}_{xx}^t + \hat{N}_{xx}^b + \hat{N}_{xx}^c &= -\hat{N}_0 \\ \hat{N}_{xx}^t \left(\frac{h_t + h_c}{2} \right) &= \hat{N}_{xx}^b \left(\frac{h_b + h_c}{2} \right) \end{aligned} \tag{32}$$

where $\hat{N}_{xx}^t, \hat{N}_{xx}^b$ and $\hat{N}_{xx}^c$ are the parts of total load which are exerted to the top face sheet, bottom face sheet and the core along x-direction, respectively. Thus, by setting the nonlinear terms of strains to zero and $z_t = z_b = z_c = 0$ at the mid-plane of the top and bottom face sheets and the core:

$$\begin{aligned} \hat{N}_{xx}^t &= K_{t,11}^0 u_{0t,x} + K_{t,12}^0 v_{0t,y} \\ \hat{N}_{xx}^b &= K_{b,11}^0 u_{0b,x} + K_{b,12}^0 v_{0b,y} \\ \hat{N}_{xx}^c &= C_{11}^c K_c^0 u_{0c,x} + C_{12}^c K_c^0 v_{0c,y} \end{aligned} \tag{33}$$

The sandwich plates can be analyzed for the following two loading conditions [16]:

Case I: uniform state of stress in which all the layers are subjected to equal edge stresses.
Case II: uniform state of strain in which the individual layers are subjected to stresses in proportion to their elastic modulus.

In the buckling analysis, if a uniform state of strain is assumed, the relative edge stresses in the individual layers are proportional to the respective elastic modulus. The in-plane flexural rigidity of the soft cores is comparatively very small and hence the condition of uniform strain state is more realistic for sandwich plates [16]. Therefore, in this analysis the uniform strain state is assumed. Hence, the external in-plane loads exerted to the top and bottom face sheets and the core along x-direction can be defined as:

$$\begin{aligned} \hat{N}_{xx}^t &= \frac{\hat{N}_0 (h_b + h_c) \left(K_{t,12}^0 K_{b,11}^0 - K_{t,11}^0 K_{b,12}^0 \right)}{(h_t + h_b + 2h_c) \left(K_{t,12}^0 K_{b,11}^0 - K_{t,11}^0 K_{b,12}^0 \right) + K_c^0 (h_b + h_c) \left(K_{b,11}^0 C_{12}^c - K_{b,12}^0 C_{11}^c \right) + K_c^0 (h_t + h_c) \left(K_{t,12}^0 C_{11}^c - K_{t,11}^0 C_{12}^c \right)} \\ \hat{N}_{xx}^b &= \frac{\hat{N}_0 (h_t + h_c) \left(K_{t,12}^0 K_{b,11}^0 - K_{t,11}^0 K_{b,12}^0 \right)}{(h_t + h_b + 2h_c) \left(K_{t,12}^0 K_{b,11}^0 - K_{t,11}^0 K_{b,12}^0 \right) + K_c^0 (h_b + h_c) \left(K_{b,11}^0 C_{12}^c - K_{b,12}^0 C_{11}^c \right) + K_c^0 (h_t + h_c) \left(K_{t,12}^0 C_{11}^c - K_{t,11}^0 C_{12}^c \right)} \\ \hat{N}_{xx}^c &= \frac{K_c^0 \hat{N}_0 \left[(h_b + h_c) \left(K_{b,11}^0 C_{12}^c - K_{b,12}^0 C_{11}^c \right) + (h_t + h_c) \left(K_{t,12}^0 C_{11}^c - K_{t,11}^0 C_{12}^c \right) \right]}{(h_t + h_b + 2h_c) \left(K_{t,12}^0 K_{b,11}^0 - K_{t,11}^0 K_{b,12}^0 \right) + K_c^0 (h_b + h_c) \left(K_{b,11}^0 C_{12}^c - K_{b,12}^0 C_{11}^c \right) + K_c^0 (h_t + h_c) \left(K_{t,12}^0 C_{11}^c - K_{t,11}^0 C_{12}^c \right)} \end{aligned} \tag{34}$$

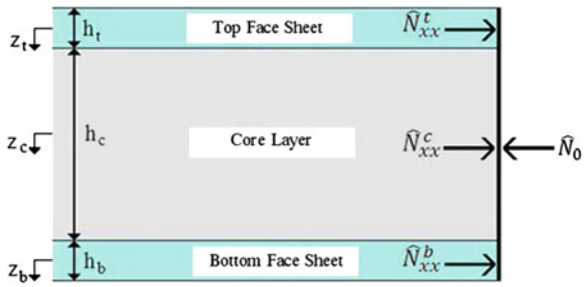


Fig. 2 Distributions of external in-plane loads exerted to the top and bottom face sheets and the core

The partial loads along y-direction can be calculated using the procedure presented along Eqs. (32)–(34). Therefore, Eq. (30) can be rewritten as:

$$\mathcal{N}(w_{0i}) = w_{0i,xx}\hat{N}_{xx}^{i} \tag{35}$$

Using Navier's procedure, the solution of the displacement variables satisfying the above boundary conditions can be expressed in the following forms:

$$\begin{aligned}
u_{ij}(x,y) &= \sum_{n=1}^{N}\sum_{m=1}^{M} U_{ij}^{mn} cos(\alpha_m x)\cdot sin(\beta_n y)\\
v_{ij}(x,y) &= \sum_{n=1}^{N}\sum_{m=1}^{M} V_{ij}^{mn} sin(\alpha_m x)\cdot cos(\beta_n y)\\
w_{0j}(x,y) &= \sum_{n=1}^{N}\sum_{m=1}^{M} W_{0j}^{mn} sin(\alpha_m x)\cdot sin(\beta_n y)\\
u_{ic}(x,y) &= \sum_{n=1}^{N}\sum_{m=1}^{M} U_{ic}^{mn} cos(\alpha_m x)\cdot sin(\beta_n y)\\
v_{ic}(x,y) &= \sum_{n=1}^{N}\sum_{m=1}^{M} V_{ic}^{mn} sin(\alpha_m x)\cdot cos(\beta_n y)\\
w_{kc}(x,y) &= \sum_{n=1}^{N}\sum_{m=1}^{M} W_{kc}^{mn} sin(\alpha_m x)\cdot sin(\beta_n y)
\end{aligned} \tag{36}$$

By using Navier's procedure, the Lagrange multipliers can be expressed in the following forms:

$$\begin{aligned}
\lambda_x^j(x,y) &= \sum_{n=1}^{N}\sum_{m=1}^{M} X_j^{mn} cos(\alpha_m x)\cdot sin(\beta_n y)\\
\lambda_y^j(x,y) &= \sum_{n=1}^{N}\sum_{m=1}^{M} Y_j^{mn} sin(\alpha_m x)\cdot cos(\beta_n y)\\
\lambda_z^j(x,y) &= \sum_{n=1}^{N}\sum_{m=1}^{M} Z_j^{mn} sin(\alpha_m x)\cdot sin(\beta_n y)\\
\lambda_{xz}^j(x,y) &= \sum_{n=1}^{N}\sum_{m=1}^{M} L_{xj}^{mn} cos(\alpha_m x)\cdot sin(\beta_n y)\\
\lambda_{yz}^j(x,y) &= \sum_{n=1}^{N}\sum_{m=1}^{M} L_{yj}^{mn} sin(\alpha_m x)\cdot cos(\beta_n y)\\
\lambda_{xz}^{jc}(x,y) &= \sum_{n=1}^{N}\sum_{m=1}^{M} L_{xjc}^{mn} cos(\alpha_m x)\cdot sin(\beta_n y)\\
\lambda_{yz}^{jc}(x,y) &= \sum_{n=1}^{N}\sum_{m=1}^{M} L_{yjc}^{mn} sin(\alpha_m x)\cdot cos(\beta_n y)
\end{aligned} \tag{37}$$

where $= t, b,\ i = 0,\ 1,\ 2,\ 3,\quad k = 0,\ 1,\ 2$ and $\alpha_m = \frac{m\pi}{a}, \beta_n = \frac{n\pi}{b}$ in which m and n are the wave numbers. By substituting Eqs. (34), (35), (36) and (37) into (19), (20) and (21), the final equations of motion in the matrix form can be determined as:

$$[A]_{43*43}\{X\}_{43*1} = \{0\}_{43*1} \tag{38}$$

where $[A]$ is the coefficients matrix and $\{X\}$ is defined as:

$$\begin{aligned}
\{X\} = \{ & U_{0t}^{mn}\ U_{1t}^{mn}\ U_{2t}^{mn}\ U_{3t}^{mn}\ V_{0t}^{mn}\ V_{1t}^{mn}\ V_{2t}^{mn}\ V_{3t}^{mn}\ W_{0t}^{mn}\ X_t^{mn}\ Y_t^{mn}\ Z_t^{mn}\\
& L_{xt}^{mn}\ L_{yt}^{mn}\ L_{xtc}^{mn}\ L_{ytc}^{mn}\ U_{0c}^{mn}\ U_{1c}^{mn}\ U_{2c}^{mn}\ U_{3c}^{mn}\ V_{0c}^{mn}\ V_{1c}^{mn}\ V_{2c}^{mn}\ V_{3c}^{mn}\\
& W_{0c}^{mn}\ W_{1c}^{mn}\ W_{2c}^{mn}\ X_b^{mn}\ Y_b^{mn}\ Z_b^{mn}\ L_{xb}^{mn}\ L_{yb}^{mn}\ L_{xbc}^{mn}\ L_{ybc}^{mn}\\
& U_{0b}^{mn}\ U_{1b}^{mn}\ U_{2b}^{mn}\ U_{3b}^{mn}\ V_{0b}^{mn}\ V_{1b}^{mn}\ V_{2b}^{mn}\ V_{3b}^{mn}\ W_{0b}^{mn}\ \}
\end{aligned} \tag{39}$$

The nonzero result and buckling load is obtained when the determinant of $[A]$ is set to be zero.

4 Numerical Results and Discussion

In this section, several examples of the overall buckling and face wrinkling problems of the sandwich plates are studied to verify the accuracy and applicability of the present higher order theory. The results obtained by present theory are

Table 1 Dimensionless overall buckling load for symmetric square sandwich plate $[(0/90)_5/\text{Core}/(90/0)_5]$

h_t/h	a/h	Elasticity [31]	Present	GLPT [26]	MLW [16]	ESL [16]
0.025	20	2.5543	2.5658	2.5391	2.5390	2.6386
	10	2.2376	2.2621	2.1914	2.1904	2.2942
	20/3	1.8438	1.8882	1.7961	1.7952	1.8980
	5	1.5027	1.5316	1.4449	1.4427	1.5393
0.05	20	4.6590	4.6804	4.6387	4.6386	4.7857
	10	3.7375	3.7611	3.6770	3.6759	3.8475
	20/3	2.7911	2.8320	2.7509	2.7506	2.9222
	5	2.0816	2.1017	2.0431	2.0426	2.1977
0.075	20	6.4224	6.4414	6.3915	6.3914	6.5644
	10	4.7637	4.8256	4.7432	4.7433	4.9580
	20/3	3.3729	3.4026	3.3387	3.3385	3.5466
	5	2.3973	2.4067	2.3674	2.3672	2.5461
0.1	20	7.8969	7.9171	7.8632	7.8631	8.0544
	10	5.6081	5.6215	5.5471	5.5463	5.7946
	20/3	3.7883	3.7931	3.7430	3.7424	3.9752
	5	2.6051	2.6077	2.5791	2.5789	2.7719

compared with the results in the previous literature. The following dimensionless buckling load used in the present analysis is defined as [16, 26]:

$$\bar{N} = \frac{a^2 \hat{N}_0}{E_2 h^3} \tag{40}$$

where E_2 is the transverse elastic modulus of the face sheets. Two types of buckling modes are studied for sandwich plates: overall buckling and wrinkling modes. Generally, the overall buckling load corresponds to both wave numbers equal to unity ($m = n = 1$). If the buckling load of a higher wave number is less than the overall buckling load, the sandwich plate fails in the wrinkling mode, although, it is not a general case. For assessing the wrinkling possibility, the wave number m should be increased in steps of one, when the wave number n is considered to be unity.

Example 1*: Overall buckling of a square sandwich plate*

A square symmetric sandwich plate with stack-up sequence of $\left[\left(0^\circ/90^\circ\right)_5/\text{Core}/\left(90^\circ/0^\circ\right)_5\right]$ with a total thickness of h is considered. The sandwich plate consists of equal thickness cross-ply laminated face sheets with 10 layers and a soft orthotropic core. The analysis is performed for different thickness ratios ($a/h = 20$, 10, 20/3 and 5) and different face sheet thickness ratios ($h_t/h = 0.025$, 0.05, 0.075 and 0.1). The dimensionless overall buckling loads obtained by 3D elasticity solution [31], global high-order equivalent single layer theory (ESL) [16], higher-order global–local plate theory (GLPT) [26], mixed layer-wise (MLW) theory [16] and the present high-order analytical theory are given in Table 1. All results are presented for case (II) loading which is uniform state of strain. The material constants used in this example are assumed as follows:

For each composite layer of the face sheets [16]:

$$E_1 = 19E,\ E_2 = E_3 = E,\ G_{12} = G_{13} = 0.52E,\ G_{23} = 0.338E, \\ \nu_{12} = \nu_{13} = 0.32,\ \nu_{23} = 0.49 \tag{41}$$

For the orthotropic core [16]:

$$E_x = 3.2 \times 10^{-5}\,E,\ E_y = 2.9 \times 10^{-5}\,E,\ E_z = 0.4E,\ G_{xy} = 2.4 \times 10^{-3}\,E \\ G_{yz} = 6.6 \times 10^{-2}\,E,\ G_{xz} = 7.9 \times 10^{-2}\,E,\ \nu_{xy} = 0.99,\ \nu_{xz} = \nu_{yz} = 3 \times 10^{-5} \tag{42}$$

It can be seen that the present results are in good agreement with 3D elasticity solutions [31]. Also, it can be concluded that the results of the ESL theory are very far from the 3D elasticity solutions and are not accurate.

Example 2: Wrinkling of a square sandwich plate

The dimensionless wrinkling loads of the above square symmetric sandwich plate with stacking sequence of $\left[\left(0^\circ/90^\circ\right)_5/\mathrm{Core}/\left(90^\circ/0^\circ\right)_5\right]$ are obtained by the present higher-order theory. All geometrical parameters and material properties are the same as the Example 1. The results obtained by global high-order equivalent single layer theory (ESL) [16], high-order global–local plate theory (GLPT) [26], mixed layer-wise theory (MLW) [16] and the present high-order analytical theory are presented and compared in Table 2. Also, if wrinkling mode is possible, the mode number of wrinkling loads (m) is presented in parenthesis. A 3D elasticity solution for wrinkling loads of sandwich plates was not presented by Noor et al. [31]. Kardomateas [32] presented a 2D elasticity solution for the wrinkling analysis of sandwich beams or wide sandwich panels which is not applicable for this 3D example.

The results presented in Table 2 indicate that based on all the theories, for thin sandwich plates ($a/h = 20$), wrinkling behavior does not occur. It can be seen that the present results are in good agreement with GLPT [26] and MLW [16] results, but the ESL theory [16] could not predict the wrinkling modes as well as the overall buckling, accurately. For a constant thickness ratio (a/h), the wrinkling loads increase with an increase in face sheet thickness ratio (h_t/h), because the stiffness of the face sheets is much greater than the stiffness of the core.

Example 3: Wrinkling of a rectangular sandwich plate

A rectangular symmetric sandwich plate with stack-up sequence of $\left[0^\circ/90^\circ/\mathrm{Core}/90^\circ/0^\circ\right]$ and equal thickness cross-ply laminated face sheets and a soft orthotropic core is considered. The analysis is performed for different thickness ratios ($a/h = 20$, 10, 20/3 and 5) and different aspect ratios ($a/b = 0.5$, 1, 2 and 5). The face sheet thickness ratio is considered constant and equal to $h_t/h = 0.05$. The dimensionless overall buckling and wrinkling loads obtained by the

Table 2 Dimensionless wrinkling load for symmetric square sandwich plate [$(0/90)_5$/Core/$(90/0)_5$]

h_t/h	a/h	Present	GLPT [26]	MLW [16]	ESL [16]
0.025	20	-[a]	–	–	–
	10	1.3350 (54)	1.3601	1.2766 (57)	1.4395 (54)
	20/3	0.5942 (36)	0.5837	0.5680 (38)	0.6407 (36)
	5	0.3348 (27)	0.3486	0.3200 (28)	0.3611 (27)
0.05	20	–	–	–	–
	10	2.9358 (42)	2.9653	2.8002 (43)	3.4713 (37)
	20/3	1.3059 (28)	1.2826	1.2456 (29)	1.5440 (25)
	5	0.7355 (21)	0.7305	0.7016 (22)	0.8697 (19)
0.075	20	–	–	–	–
	10	–	4.6843	4.6321 (39)	–
	20/3	2.1843 (25)	2.1304	2.0595 (26)	2.6357 (21)
	5	1.2301 (19)	1.2163	1.1597 (19)	1.4839 (15)
0.1	20	–	–	–	–
	10	–	–	–	–
	20/3	3.1418 (24)	2.8792	2.9284 (25)	–
	5	1.7686 (18)	1.6249	1.6483 (19)	2.1324 (14)

[a] Wrinkling mode is not possible

Table 3 Dimensionless overall buckling and wrinkling load for symmetric rectangular sandwich plate [0/90/Core/90/0] ($h_t/h = 0.05$)

a/b	a/h	Overall buckling	Wrinkling
0.5	20	9.8395	–[a]
	10	8.2161	–
	20/3	6.4504	3.9533 (29)
	5	4.9626	2.2253 (22)
1	20	4.6808	–
	10	3.7619	2.2247 (43)
	20/3	2.8328	0.9899 (29)
	5	2.1021	0.5579 (22)
2	20	7.3479	–
	10	4.2763	0.5577 (43)
	20/3	2.5438	0.2490 (29)
	5	1.6267	0.1410 (22)
5	20	17.3411	–
	10	5.6626	0.0910 (43)
	20/3	2.6151	0.0416 (29)
	5	1.4518	0.0243 (22)

[a] Wrinkling mode is not possible

present high-order analytical theory are given in Table 3. Also, if the wrinkling mode is possible, the mode numbers of wrinkling loads (m) is presented in parenthesis. All material constants are the same as the Example 1.

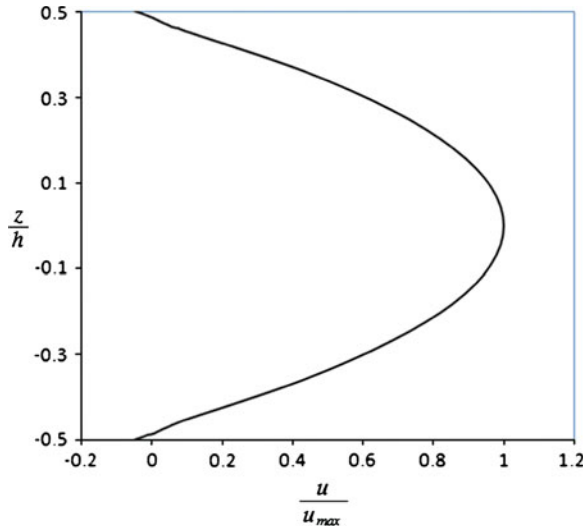


Fig. 3 Distributions of in-plane displacement through the thickness of symmetric sandwich plate [0/90/Core/90/0] subjected to wrinkling load ($a/b = 2$, $a/h = 5$, $h_t/h = 0.05$)

Results show that for a constant aspect ratio (a/b), the overall buckling and wrinkling loads increase with increase in thickness ratio (a/h). Also, it can be seen that for thin sandwich plates ($a/h = 20$), the wrinkling behavior does not occur.

In Table 3, it can be seen that the symmetric rectangular sandwich plate with $a/b = 2$, $h_t/h = 0.05$ and $a/h = 5$ wrinkled in mode number $m = 22$. The through-thickness distributions of some dimensionless parameters for this symmetric sandwich plate subjected to wrinkling load ($m = 22$) are plotted in Figs. 3–6. Figs. 3 and 4 show the distributions of in-plane and transverse displacements through the thickness of the points ($x = 0$, $y = b/2$) and ($x = a/2$, $y = b/2$) of the sandwich plate, respectively. As shown in these figures, the displacements continuity between each face sheet-core interface is satisfied. The distribution of in-plane normal stress through the thickness of the mid-point ($x = a/2$, $y = b/2$) of the sandwich plate is shown in Fig. 5. This figure shows that the in-plane stress is discontinue at the face sheet-core interfaces. Also, it can be observed that the in-plane stresses in the core are very small in comparison with those obtained in the face sheets. Figure 6 shows the through thickness distribution of transverse shear stress τ_{yz} of the edge-point ($x = a/2$, $y = 0$) for the sandwich plate. As shown in this figure, the transverse shear stress distribution is continues in the face sheet-core interfaces and is zero on the upper and lower surfaces of the sandwich plate.

5 Conclusions

In this chapter, a new improved higher-order theory was presented for overall buckling and wrinkling analysis of soft-core sandwich plates. An analytical solution for buckling and wrinkling analysis of simply supported sandwich plates

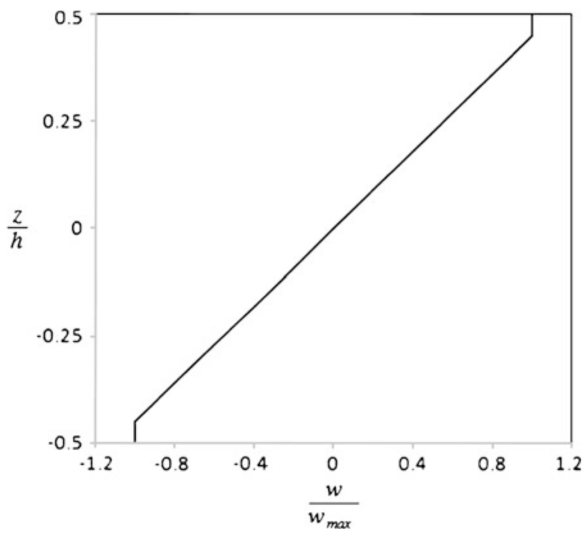


Fig. 4 Distributions of transverse displacement through the thickness of symmetric sandwich plate [0/90/Core/90/0] subjected to wrinkling load ($a/b = 2$, $a/h = 5$, $h_t/h = 0.05$)

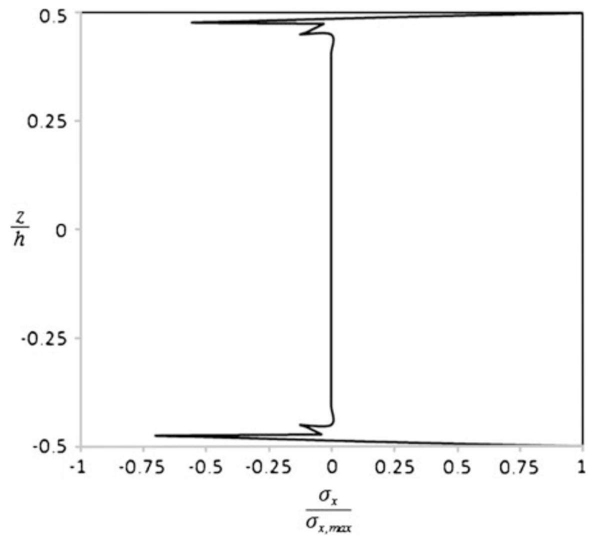


Fig. 5 Distributions of in-plane normal stresses through the thickness of symmetric sandwich plate [0/90/Core/90/0] subjected to wrinkling load ($a/b = 2$, $a/h = 5$, $h_t/h = 0.05$)

under various in-plane compressive loads were presented using Navier's solution. The presented theory satisfies the continuity conditions of transverse shear stresses at the interfaces as well as the conditions of zero transverse shear stresses on the upper and lower surfaces of the plate. The nonlinear von Kármán type relations were used to obtain strains and also transverse flexibility and transverse normal strain and stress of the core considered in the analysis. It can be concluded from the results that the overall buckling loads obtained by the present theory are in good agreement with elasticity solutions and other accurate numerical results. The

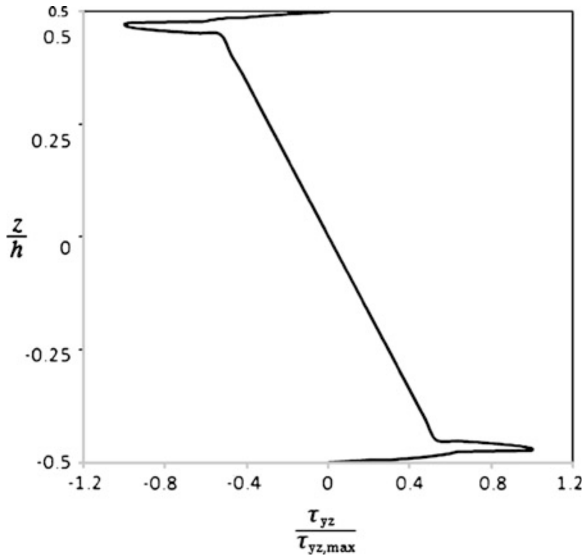


Fig. 6 Distributions of transverse shear stress through the thickness of symmetric sandwich plate [0/90/Core/90/0] subjected to wrinkling load ($a/b = 2$, $a/h = 5$, $h_f/h = 0.05$)

in-plane flexural rigidity of the soft cores is comparatively very small and hence the condition of uniform strain state is more realistic for sandwich plates.

Also, it can be concluded that the present theory can estimate the wrinkling loads as well as the mode number accurately. The overall buckling loads calculated by ESL theories are higher than that of the present analysis.

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