Chapter 2
Theoretical Considerations Regarding Embedded Options

In the previous chapter I have introduced the subject of embedded options and guarantees. Now I will specify the terms *embedded option* and *guarantee* and present a systematisation of common examples thereof. Subsequently there will be a literature survey to outline to the reader the actual state of scientific knowledge that is of relevance for the purpose of this thesis.

Before starting, some delimitation are in order. Any kind of embedded option or guarantee is always linked to a particular life insurance contract. Within this thesis, neither non-life nor reinsurance business will be considered. Furthermore, only financial options and financial guarantees are taken into account.

Roughly speaking a *life insurance* is merely an umbrella term for any kind of insurance contract where the benefit is linked to the policyholder's life and risks as survival and death are covered. Note that within this thesis a policyholder of a life insurance is always the insured person as well. In the main there are three life insurance base products.

Term Life Insurance

A term life insurance covers the insured person for a fixed period of time (term) that is shorter than lifetime. If the insured person dies during the term a death benefit is due.

Whole Life Insurance

A whole life insurance covers the insured person for life. A death benefit is due when the insured person dies.

N. Rüfenacht, *Implicit Embedded Options in Life Insurance Contracts*,
Contributions to Management Science, DOI 10.1007/978-3-7908-2843-6_2,
© Springer-Verlag Berlin Heidelberg 2012

Endowment Insurance

An endowment insurance covers the insured person for a fixed period of time (term) that is shorter than lifetime. If the insured person dies during the term a death benefit is due. If the insured person survives the term a survival benefit is due.

For a deeper insight into life insurance methodology and basic life insurance mathematics I recommend Gerber (1997).

There are many possibilities to vary these base products. Different premium payment schemes, increasing death or survival benefits and extended coverage are just a few examples to specify a particular life insurance contract. As we have already seen an insurer might further add certain embedded options and guarantees to its insurance policies. But what are these embedded options and guarantees precisely? In the classical financial market theory an option is defined as follows:

Definition 2 (Call/Put Option). A call/put option is a contract that gives the holder the right, but not the obligation, to buy/sell a specified security (the underlying) by a certain date (the exercise date) for a certain price (the strike price). An American option can be exercised at any time up to the expiration date whereas European options can be exercised only at the expiration date itself.

For a general introduction to option theory and especially option pricing theory I refer the interested reader to Hull (2006). However this is a natural point to define what an embedded option shall be for the purpose of this thesis. With some modifications we get the following:

Definition 3 (Embedded Option in a Life Insurance Contract). An option linked to a life insurance contract is called an embedded option (in a life insurance contract). It gives the policyholder the right, but not the obligation, to actively change his or her life insurance contract by a certain date on terms that are established in advance. European and American type of options are feasible.

2.1 Types of Embedded Options and Guarantees

By way of illustration it is most intuitive to give some tangible examples of embedded options. This selection is non-exhaustive of course but covers the most material embedded options that appear quite frequently in common life insurance products. For this I will follow CEIOPS (2010a).

Surrender Option

A surrender option gives the policyholder the right to fully or partially surrender the policy and receive a pre-defined lump sum amount, the surrender value. That value usually consists of the policy reserve including probable dividends but reduced by a surrender fee. Normally

the surrender option can be exercised by the end of any contract year. Thus, in theory, it is a Bermudan put option on future expected insurance benefits with a strike price equal to the surrender value.[1] In Switzerland for example the surrender option is requested by law for life insurance contracts with a savings character.[2]

Paid-up Policy Option

A paid-up policy option gives the policyholder the right to stop paying premiums. The current policy continues with reduced benefits depending on the policy reserves present at the exercise date. This right can be seen as a put option with a strike price equal to zero on a forward rate agreement. De facto the paid-up policy option exchanges the original contract for a new contract with reduced benefits and no more premium payments. Similar to the surrender option the paid-up option is requested by law in Switzerland, nevertheless the latter option is required for all types of life insurance contracts.[3]

Annuity Conversion Option

An annuity conversion option gives the policyholder the right to convert a lump survival benefit into an annuity at a pre-defined minimum rate of conversion. This option is also known as the GAO that has been introduced in the previous chapter. By converting the policyholder exercises an European call option at a strike price equal to the survival benefit on a lifelong annuity.

Policy Conversion Option

A policy conversion option gives the policyholder the right to convert from the current policy to another at pre-specified terms and conditions. For example a term insurance policy could be converted into an endowment policy. Typical of this option is that it may be exercised without further evidence of insurability.

So far we have seen that most of these embedded options can be described in terms of the classical option theory. Nevertheless, this is rather artificial. However for the purpose of this thesis the crucial aspect is the behaviour of the policyholder since it is his decision whether to exercise any particular option or not.

There exists many more types of embedded options, being in my opinion less relevant for the insurance industry than the ones presented above. However for a

[1] A Bermudan (call or put) option has predetermined, discrete exercise dates. Hence it is something between American and European option type.

[2] VVG (2009) 90 II.

[3] VVG (2009) 90 I.

more extended overview on embedded options I refer the interested reader to Gatzert (2009).

Besides embedded options, life insurance policies are offered containing certain guarantees enabling the policyholder to avert losses or to receive additional benefits. A guarantee is usually applied automatically as defined in the policy terms and conditions and always rebounds to a policyholder's advantage. To remain consistent with the former definition of an embedded option we simply consider guarantees as *implicit* embedded options without the feature of the policyholder's right to actively exercise the option.

To present the most relevant guarantees offered I will follow CEIOPS (2010a) again.

Minimum Interest Rate Guarantee

With a minimum interest rate guarantee savings premiums are accumulated with that predefined minimum interest rate. The minimum interest rate is also termed as the technical interest rate. In Switzerland the federal council dictates the maximum height of the technical interest rate for new business.[4] Currently the technical interest rate is bounded above at 1.75% for policies in Swiss francs, at 2.5% for policies in Euro and at 2.75% for policies in US dollars.[5]

Dividend Option

A dividend option enables policyholders to partake of the profits gained by the insurance company due to positive investment, mortality and expenses results. There are several possibilities of implementing the dividend option in practice. The insurer can either pay out dividends as cash, reduce future premiums or accumulate with interest. Dividends allocated to policyholders are also called surpluses. A life insurance policy containing a dividend option is also called a *participating life insurance policy*.

Guaranteed Minimum Accumulation Benefit

Unit-linked life insurance products "link" the policyholder's benefit to some financial index or fund. Savings premiums are continuously invested in the underlying and so the benefits vary according to its performance. To avoid too low benefits relative to the savings premiums paid due to a weak or even negative performance of the underlying, the insurer may guarantee a minimum accumulation benefit. Similarly a guarantee for a minimum death benefit is possible.

---

[4] Confer VAG (2009) 36 I and AVO (2009) 121.

[5] See http://www.finma.ch/e/beaufsichtigte/versicherungen/lebensversicherungen/Pages/max-tech-zinssatz.aspx. Last checked: 1 June 2011.

There are a few more examples of guarantees within life insurance contracts in Gatzert (2009) again. After having presented the most material embedded options and guarantees, I will proceed by defining those options the focus will be on within the empirical analysis in Chap. 5. Based on that the literature survey will be presented.

Basically I aim to design a fairly general framework that provides an appropriate substructure to handle every kind of embedded option and guarantee. Since there is a focus put on selected options and guarantees the framework finally being presented and applied for modelling purposes in the subsequent chapters will be some special case thereof. But note that this does not affect the requested flexibility of the model itself.

A model is always a simplification of the real world. Nevertheless one should plan for mapping at least the key elements of any model as realistically as possible. With regard to embedded options probably the main key factor is the behaviour of policyholders. This is also recognised by the new solvency and valuation standards and so we find for example the following for the SST.

SAA (2006), Sect. 2.2

> (. . .) It is vital to be consistent when including policyholder and insurer options in solvency calculations. This requires realistic modelling of both policyholder and insurer behaviour. (. . .)

SAA (2006) does not further explain what *realistic modelling of policyholder behaviour* means exactly. For the European Solvency II regime we find quite a similar instruction in Article 79 of the directive EC 138 (2009) that I have cited in Sect. 1.2.2 already. Assumptions made with respect to the likelihood that policyholders will exercise embedded options *shall take account, either explicitly or implicitly, of the impact that future changes in financial and non-financial conditions may have on the exercise of those options*.

In addition CEIOPS (2010a) addresses explicitly to the policyholders' behaviour in TP.2.105–TP.2.111. There are two paragraphs I consider worth noting here.

CEIOPS (2010a), TP2.107

> Assumptions about the likelihood that policy holders will exercise contractual options should be based on analysis of past policyholder behaviour. The analysis should take into account the following:
>
> (a) How beneficial the exercise of the options was or would have been to the policyholders under past circumstances (whether the option is out of or barely in the money or is in the money),
> (b) The influence of past economic conditions,

(c) The impact of past management actions,
(d) Where relevant, how past projections compared to the actual outcome,
(e) Any other circumstances that are likely to influence a decision whether to exercise the option.

CEIOPS (2010a), TP2.109

> In general policyholders' behaviour should not be assumed to be independent of financial markets, of undertaking's treatment of customers or publicly available information unless proper evidence to support the assumption can be observed.

These paragraphs outline the high requirements for an appropriate modelling of policyholder's behaviour. To do so, at least a profound statistical analysis of an insurer's own data would be necessary. Even though the behaviour of policyholders is of vital importance for modelling embedded options it will not be further quantitatively considered within this thesis. That issue is a self-contained one but lies beyond the scope of the thesis. So only guarantees in traditional life insurance products will finally be modelled, interest rate guarantee and dividend option.[6] Note that neither the goals of the thesis nor the flexibility of the model are diminished by omitting policyholder behaviour.

Finally we shall have a look at the MCEV framework. Some respective explanations are written down in the MCEV Principles:

CFO Forum (2009a), G7.2

> Where management discretion exists, has passed through an appropriate approval process and would be applied in ways that impact the time value of financial options and guarantees, the impact of such management discretion may be anticipated in the allowance for financial options and guarantees but should allow for market and policyholders' reaction to such action. (. . .)
> The application of such discretion must consider the environment arising in the future projection which will likely be different from the current environment, but any changes from current decision rules (for example regarding flexible crediting rates or policyholder bonuses) must be supported by appropriate approvals.

CFO Forum (2009a), G7.3

> Dynamic policyholder behaviour should, where material, be in the allowance for the time value of financial options and guarantees.

[6] With *traditional* life insurance products I mean non unit-linked products.

2.2 Literature Survey

The literature survey shall outline for the reader the actual state of the scientific knowledge. Note that all the papers to be discussed will be presented in the nomenclature of the respective authors.

As stated in the previous section, the focus within this thesis will be on the minimum interest rate guarantee and the dividend option, that is de facto a guarantee, too. Both guarantees have been applied for considerable length of time and nowadays these are taken for granted. One of the first papers addressing this issue was written by Briys and de Varenne.[7]

Briys and de Varenne (1997)

The motivation of the paper is based on a brief comparison of banking and insurance industry. It points out the long maturity of insurance liabilities as the main distinguishing feature implying remarkable challenges regarding the insurance industry's asset liability management (ALM). From insurance agents there is a considerable pressure to offer policies at a high guaranteed interest rate for competitive reasons. Keeping the rate high even when market interest rates fall, urges insurance companies to turn away from perfectly matching their long-term liabilities and therefore invest in shorter-term assets.

The model presented is based on Merton's approach to financial intermediaries and only one kind of artificial life insurance policy including interest rate guarantee and dividend option is considered.[8] This policy is comparable to a pure endowment insurance policy, an endowment insurance policy that pays a survival benefit only and no death benefits, paid for a single premium. Within the authors' words *the model enables one to determine the fair interest rate or the fair participation level policyholders should require to fully compensate them for the risks they face.*

Let $[0, T]$ be the planning horizon of any life insurance company. $t = 0$ shall be the time of issuance for a given cohort of life insurance policies maturing in $t = T$. At time $t = 0$ policyholders pay a single premium. Together with paid-in capital E_0 an asset portfolio A_0 is financed.

The life insurance policy shall guarantee a fixed yearly interest rate r_G to the policyholder. On top of this the insurance company shares a δ of its net financial revenues to its policyholders. Since r_G is usually lower than the market rate for a risk free asset of the same maturity as the policy, the participation coefficient δ is an important feature expressing the required risk premium for holding risky life insurance policies.

[7] See Briys and de Varenne (1997).

[8] Confer Merton (1977) and Merton (1990).

At start the company's balance sheet looks as follows (Table 2.1).

Table 2.1 The company's balance sheet at $t = 0$ in the Briys and de Varenne model (Source: Briys and de Varenne (1997))

Assets		Liabilities and equity	
Assets	A_0	Liabilities	$L_0 = \alpha \cdot A_0$
		Equity	$E_0 = (1-\alpha) \cdot A_0$
Total	A_0	Total	A_0

The term $(1-\alpha)$ gives the proportion of the initial assets financed by equity. The asset portfolio is assumed to consist of risky assets like equity, bonds or property only. The time t value of the asset portfolio A_t is then given by a stochastic process capturing both interest rate risk and asset risk.[9]

Over time stakeholder's cash flows occur as contractually defined. Then in $t = T$ three different states are feasible.

1. Total Insolvency

   The time T value of assets A_T is less than the guaranteed benefits and the company is declared bankrupt. The terminal benefit to policyholders is equal to the remaining amount of assets.

$$L_T = A_T \tag{2.1}$$

2. Partial Insolvency

   The company's assets are sufficiently high to pay guaranteed benefits but not to pay any bonus. Let B_T be the financial bonus to policyholders after guaranteed benefits were cashed out. Then

$$\begin{aligned} B_T &= \delta \left[\frac{L_0}{A_0}(A_T - A_0) - (L_T^{\mathrm{g}} - L_0) \right] \\ &= \delta[\alpha A_T - L_T^{\mathrm{g}}], \end{aligned} \tag{2.2}$$

   where δ denotes the level of bonus and $L_T^{\mathrm{g}} = \alpha A_0 \exp(r^{\mathrm{g}} \cdot T)$ is the guaranteed benefit to the policyholders. Obviously in this state the company's assets A_T are greater than the guaranteed benefits L_T^{g}. But if $A_T < L_T^{\mathrm{g}}/\alpha$ there is no bonus since $B_T < 0$.

3. No Insolvency

   The company is able to pay guaranteed benefits and bonus to its policyholders. The total Liabilities at time T are given by

[9] The stochastic process is defined by the differential equation $dA_t/A_t = \mu dt + \sigma_A \cdot [\rho dW_t + \sqrt{1-\rho^2} dZ_t]$ where μ and σ_A denote the instantaneous expected return on assets and their instantaneous volatility respectively. W_t and Z_t are standard Wiener processes.

$$\begin{aligned} L_T &= L_T^{\mathrm{g}} + B_T \\ &= (1-\delta)L_T^{\mathrm{g}} + \delta\alpha A_T \end{aligned} \tag{2.3}$$

Equity has a limited liability feature and thus shareholder's stake is a residual stake. Analogous to the total benefits the total equity payoff is depicted for each of the three different states.

1. Total Insolvency

$$A_T < L_T^{\mathrm{g}} \;\Rightarrow\; E_T = 0 \tag{2.4}$$

2. Partial Insolvency

$$L_T^{\mathrm{g}} \leq A_T < \frac{L_T^{\mathrm{g}}}{\alpha} \;\Rightarrow\; E_T = A_T - L_T^{\mathrm{g}} \tag{2.5}$$

3. No Insolvency

$$A_T \geq \frac{L_T^{\mathrm{g}}}{\alpha} \;\Rightarrow\; E_T = (1-\delta\alpha)A_T - (1-\delta)L_T^{\mathrm{g}} \tag{2.6}$$

Liability and equity payoffs depending on the different states are summarised in Fig. 2.1.

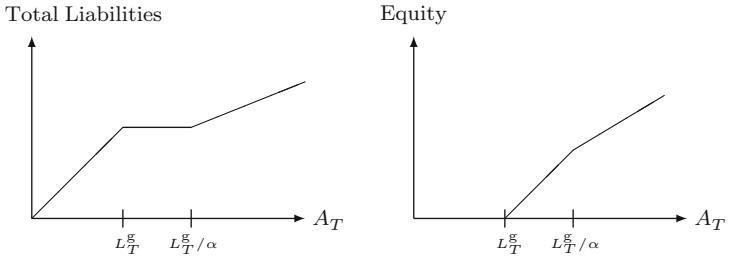


Fig. 2.1 Policyholder's and shareholder's final payoffs in the Briys and de Varenne model (Source: Rüfenacht/Briys and de Varenne (1997))

This framework is the starting point Briys and de Varenne founded their analysis on. They use a contingent claim based methodology to derive a closed-form solution for the pricing of life insurance liabilities. The authors also prove that the model produces more accurate duration and convexity measures since it accounts for stochastic interest rates, default risk and bonus schemes. A significant difference between the effective insurance liability and the traditional Macaulay duration is shown.

There are several suggestions to improve the model. Extensions to more complex life insurance products or taking surrender into account are conceivable for example.

Grosen and Jørgensen (2000)

In their introduction Grosen and Jørgensen expose the importance of a proper valuation and separate balance sheet reporting of embedded options. They mention interest rate guarantee, surrender option and dividend option as common examples embedded in many life insurance policies. Historically insufficient attention has been paid to accurate valuation or reporting issues. Several possible explanations are given for this, such as companies that were not realising or simply not caring about potential financial impacts caused by embedded options.

Motivated by insurance companies having suffered financial distress due to underestimating their embedded options, the authors present a multi-period model providing a smoothing surplus distribution mechanism. Basically that model is an extension of the one presented in Briys and de Varenne (1997). The main contribution of their work lies at a credible modelling of asset returns, a realistic bonus distribution scheme integrating the interest rate guarantee and lastly arbitrage-free valuation of the path-dependent contractual payoffs arising from the applied bonus distribution scheme.

The modelling framework is embedded in a continuous and frictionless economy with perfect financial markets. Thus transaction costs, liquidity squeezes, tax effects and other market imperfections will be ignored. Furthermore any kind of expenses, lapses and mortality are ignored, too.

Let $[0, T]$ be the considered time horizon of the model. In $t = 0$ the policyholder pays a single-sum deposit V_0 to the insurance company and therefor acquires a policy of nominal value P_0. In $t = T$ the policy matures and the policyholder receives a single payment.

When the policy is issued in $t = 0$, V_0 is invested in a portfolio with traded assets at the financial markets, say the *reference portfolio*. Its initial value is denoted by A_0. To illustrate the modelling algorithm a simplified time t balance sheet is used as shown in Table 2.2. Note that this is not a company's balance sheet.

Table 2.2 The policy's time t balance sheet in the Grosen and Jørgensen model (Source: Grosen and Jørgensen (2000))

Assets	Liabilities
$A(t)$	$P(t)$
	$B(t)$
$A(t)$	$A(t)$

$A(t)$ denotes the time t market value of the reference portfolio backing the policy liability. On the liability side we see $P(t)$, the *policyholder's account* or *policy reserve*. $B(t)$ stands for the *bonus reserve* or simply the *buffer*. On the one hand A is clearly a market value whereas P is rather a book value on the other hand. Finally B is defined as a residual quantity given by the difference $A - P$ and thus a hybrid of a market value and a book value. The authors emphasise that equity is not a priori missing in their model but implicitly included as a part of the bonus reserve.

The reference portfolio is assumed to evolve according to the following geometric Brownian motion.

$$dA(t) = \mu A(t)dt + \sigma A(t)dW(t), \qquad A(0) = A_0, \tag{2.7}$$

where μ and σ denote the instantaneous expected return on assets and their instantaneous volatility respectively. $W(t)$ is a standard Wiener process.

The dynamics of the policy reserve P is determined by the sequence $r_P(t)$, the *policy interest rate*, where $t \in I = \{1, 2, \ldots, T\}$. In particular $P(t)$ is defined recursively by

$$P(t) = (1 + r_P(t)) \cdot P(t-1), \qquad t \in I, \tag{2.8}$$

that leads to

$$P(t) = P_0 \prod_{i=1}^{t} (1 + r_P(i)), \qquad t \in I. \tag{2.9}$$

In practice determining r_P is highly subtle influenced by political, legal or strategic considerations. A priori one can decompose the policy interest rate into the sum of the *guaranteed interest rate* r_G and some *bonus interest rate* r_B.

$$r_P(t) = r_G + r_B(t), \qquad t \in I. \tag{2.10}$$

For this model's purposes a simplification of the precise and highly sophisticated process in practice to determine the bonus interest rate r_B and accordingly the policy interest rate r_P is assumed. Here some features are requested. Firstly, realised returns on assets must influence the future policy interest rate r_P. Secondly, r_P should be lowly volatile and yet competitive compared to other market assets of equal riskiness. Thirdly, the insurance company aims for maintaining a certain buffer in order to provide stable returns to its policyholders. So summarising this arguments yields in a policy interest rate r_P that depends on the guaranteed interest rate r_G, the performance of the reference portfolio A and the level of the buffer B.

Translated into mathematical terms the authors present the following term for the policy interest rate:

$$r_P(t) = \max\left\{ r_G, \alpha \cdot \left(\frac{B(t-1)}{P(t-1)} - \beta \right) \right\}, \tag{2.11}$$

where α is the *distribution ratio* and β the *target buffer ratio*. Using Formula 2.10 and 2.11 the bonus interest rate r_B can be expressed as follows.

$$r_B(t) = \max\left\{ 0, \alpha \cdot \left(\frac{B(t-1)}{P(t-1)} - \beta \right) - r_G \right\} \tag{2.12}$$

Recalling Formula 2.8 the evolution of the policyholder's account can be specified.

$$P(t) = P(t-1)\left(1 + \max\left\{r_G, \alpha \cdot \left(\frac{B(t-1)}{P(t-1)} - \beta\right)\right\}\right) \tag{2.13}$$

$$= P(t-1)\left(1 + r_G + \max\left\{0, \alpha \cdot \left(\frac{A(t-1) - P(t-1)}{P(t-1)} - \beta\right) - r_G\right\}\right)$$

At this stage there are two points worth noting. Firstly, the interest rate guarantee implies a floor on the terminal payout given by $P_{\min}(T) = (1+r_G)^T \cdot P_0$. It becomes effective if there is never any bonus distributed to the policyholders. Besides, the floor is lifted as soon as there is any bonus distributed since bonus is allocated to the policy account. Secondly, Eq. 2.13 implies that the bonus mechanism is indeed an embedded option.

To give the reader an impression of the model's behaviour two sample plots are in order. Figure 2.2 clearly demonstrates the smoothing effect on the policy interest rate r_P. The return on the reference portfolio A is quite volatile compared to r_P that is more or less stable.

In Fig. 2.3 the evolution of all relevant accounts modelled are plotted. Again it is a sample simulation with arbitrary set parameters but anyhow contains a lot of information on the model. Due to the interest rate guarantee the policy account P is strictly increasing whereas for the reference portfolio A decreasing values can also occur. Considering the buffer B one observes that even negative account values are possible. The policy finally seems to be less risky than the corresponding asset portfolio.

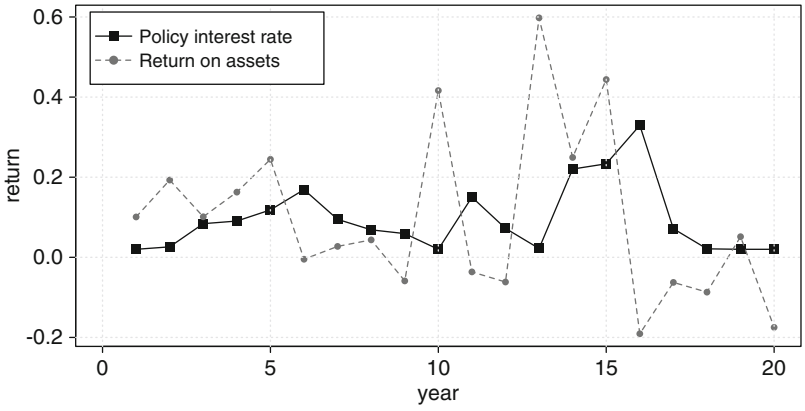


Fig. 2.2 A sample simulation of the policy interest rate $r_P(t)$ and the return on the reference portfolio $A(t)$ with parameters $r_G = 0.02, \mu = 0.05, \sigma = 0.2, \alpha = 0.3, \beta = 0.1, P_0 = 100$, $B_0 = 10$

Based on the model introduced above the authors organise their valuation of specific but artificial policies. In the concluding section they emphasise once more that life insurance companies have not paid serious attention to a proper valuation of

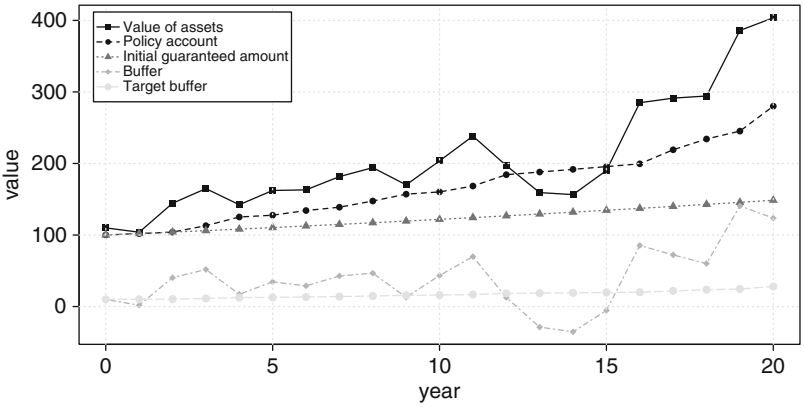


Fig. 2.3 A sample simulation of the asset value $A(t)$, the policy account $P(t)$, the initial guaranteed amount $P_0 \cdot (1 + r_G)^t$, the buffer $B(t)$ and the target buffer $\beta \cdot P(t)$ with parameters $r_G = 0.02, \mu = 0.05, \sigma = 0.2, \alpha = 0.3, \beta = 0.1, P_0 = 100, B_0 = 10$

embedded options in the past. It is shown that their modelled life insurance policy can be decomposed into three basic elements: A risk free bond, a dividend option and a surrender option.

To explore the properties of the model a quantitative analysis is presented in Grosen and Jørgensen (2000). They observe that the assumed bonus policy and the spread between the assumed expected return on assets μ and the interest rate guarantee r_G are key policy value drivers. Furthermore they show that in case of realistic parameter choice default probabilities of substantial size result.

An outlook for further research suggests applying *market value accounting* regarding the balance sheet as well as to the bonus policy. Additionally, they give examples on how to specify the model more realistically. One could take into account common features like periodic premiums, mortality, lapses and various expenses.

Jensen et al. (2001)

The paper Jensen et al. (2001) is co-written by P. L. Jørgensen and A. Grosen and basically an extension of their earlier published paper Grosen and Jørgensen (2000). The model developed therein shall be their starting point.

Mainly, the current paper presents two improvements. On the one hand they enhance the numerical procedure to calculate a policy value by reducing the dimensionality of the underlying algorithm. The authors denote that issue as the core contribution. On the other hand the former model becomes more realistic as mortality is taken into account.

In the concluding section they confirm that the enhancements worked out in the current paper support the results created in the former paper and are indeed an improvement.

Grosen and Jørgensen (2002a)

This is the third and last pure academic paper written or co-written be A. Grosen and P. L. Jørgensen which I will present here. In Grosen and Jørgensen (2000) a multi-period model providing a smoothing surplus distribution mechanism is presented. This model is basically an extension of the one developed in Briys and de Varenne (1997). Further improvements were applied in Jensen et al. (2001) by enhancing some numerical procedures and by taking mortality into account to give the model slightly more realism.

Now the current paper makes use of formerly gained insights and improves the modelling approach by E. Bryis and F. de Varenne again by focussing on the market valuation of both, the equity and the liabilities. Additionally some kind of regulatory intervention mechanism is implemented in the model. The model basics were outlined when discussing Briys and de Varenne (1997) on pages 23ff. To describe the extensions presented in Grosen and Jørgensen (2002a) a lot of mathematical theory is necessary. Since this lies beyond the scope of that thesis I refer the interested reader, therefore, to the original paper.

There is a detailed discussion on where to use this valuation framework whereas two issues are highlighted. As one possible area of application the determination of model parameters characterising initially fair premiums is suggested. Another application possibility is the fair market valuation of a company's equity and liabilities after the inception of the policies taking into account changes in market conditions.

In the concluding section the authors make mention of one important issue for further research. So far their models always considered only one homogenous class of policies with one fixed interest rate guaranteed, paid for a single premium and neglecting mortality.[10] To get a real practical model more flexibility towards contractual specifications is absolutely necessary.

Grosen and Jørgensen (2002b)

This paper is an attempt by A. Grosen and P. L. Jørgensen to apply their earlier developed model on real existing data from a handful of Danish pension and life insurance (P&L) companies.[11] Specifically, its objective is to *test empirically*

[10]Mortality is considered in Jensen et al. (2001).

[11]Confer Grosen and Jørgensen (2000).

whether (the) proposed model is a good description of the actual interest rate crediting behavior of the six largest Danish P&L companies during the period 1991–2000.[12]

These six P&L companies are AP, Codan, Danica, PFA, Topdanmark and Tryg. Since these companies do not provide detailed information on bonus distribution decisions the authors serve themselves with publicly available data consisting of policy interest rates and solvency ratios from the sample insurance companies. Their data set covers a 10 year period from 1991–2000. When the paper was written the six sample insurance companies represented around 55% of the Danish P&L savings market.

I have introduced the underlying model when presenting Grosen and Jørgensen (2000) on pages 26 ff. To ease further reading I will recall the relevant variables. $r_P(t)$ denotes the time t policy interest rate consisting of a guaranteed rate r_G and a non-guaranteed, variable bonus interest rate r_B that is greater or equal zero. The policy interest rate is the interest rate actually credited to the policyholder. $B(t)$ and $P(t)$ stand for the time t value of the buffer and the policy reserve respectively. The ratio B/P shall be the measure for the solvency ratio as mentioned above.

Given the base formula for the policy interest rate one observes that r_P is specified by two variables, the distribution ratio α and the target buffer ratio β.[13] To describe their statistical model a slightly modified version of the base formula is taken. Define $\psi = \beta + r_G/\alpha$, then

$$r_P^i(t) \;=\; r_G + \max\left\{0, \alpha_1^i \frac{B^i(t-1)}{P^i(t-1)} - \alpha_1^i \psi^i\right\}, \tag{2.14}$$

where the subscript $i,\ i \in \{1, 2, \ldots 6\}$ indexes the different companies considered. Note that for simplifying matters r_G is assumed to be fixed at 4.5% by the authors throughout their analysis.[14] As explained the available data consists of a time series of the policy interest rate $r_P^i(t)$ and the relation $\frac{B^i(t-1)}{P^i(t-1)}$ for each i. With

$$y_t^i = r_P^i(t) - r_G, \tag{2.15}$$

$$x_t^i = \frac{B^i(t-1)}{P^i(t-1)} \tag{2.16}$$

and

$$\alpha_0^i = -\alpha_1^i \psi^i, \tag{2.17}$$

[12] See Grosen and Jørgensen (2002b).

[13] $r_P(t) \;=\; \max\left\{r_G, \alpha \cdot \left(\frac{B(t-1)}{P(t-1)} - \beta\right)\right\}$

[14] See the respective explanations in Grosen and Jørgensen (2002b).

their statistical model corresponding to Eq. 2.14 is given by the following expression.

$$y_t^i = \max\left\{0, \alpha_0^i + \alpha_1^i x_t^i + \epsilon_t^i\right\}, \qquad \epsilon_t^i \sim \mathcal{N}(0, \sigma_i^2) \text{ i.i.d} \tag{2.18}$$

To estimate the parameters α_0^i, α_1^i and σ_i^2 a maximum likelihood (ML) approach of Amemiya (1973) is used. To give the reader an impression of the results the average values for the ML estimators are plotted in Table 2.3. For a more detailed reporting I refer to the original paper.

Table 2.3 Average values of the estimated parameters $\hat{\alpha}^i$ and $\hat{\Psi}^i$ for the Grosen and Jørgensen model (Source: Grosen and Jørgensen (2002b))

Average ML estimators		Average fit measure
$\hat{\alpha}$	$\hat{\Psi}$	R^2
0.1686	0.0670	0.455

The authors conclude that, even though their sample was rather poor, the model enables a good fit of the observed data. Regarding the definition of bonus policy in practice they state to have gained some valuable insights from the results. Certainly this model may be of interest for other markets than the Danish.

Further improvements are suggested by additional explanatory variables like mortality, surrender or other parameters affecting the determination of premiums. A short but interesting comment is the author's forecast that moving forward to *market value based valuation* will become a standard and also mean a great challenge to the P&L companies. As we know today this has become perfectly true.

A. Grosen and P. L. Jørgensen close their paper by mentioning that the estimated bonus mechanism parameters could also be viewed as a ranking and comparison measure for particular insurance policies.

Hansen and Miltersen (2002)

The authors of Hansen and Miltersen (2002) present a further modelling framework to analyse minimum interest rate guarantee and dividend option. For modelling purposes they propose to make use of the smoothing surplus distribution mechanism used by most Danish life insurance companies and pension companies, basically an extension of the model presented in Grosen and Jørgensen (2000).

Their model considers a pure endowment insurance policy containing a dividend option. Bonuses are assumed to be allocated on a yearly basis from some bonus reserve and finally paid out at maturity T. On top of, that the policyholder might receive a terminal bonus if the bonus reserve is positive at the end of its contract. If the bonus reserve is negative then, the deficit will be covered by the company. De facto this is equivalent to issuing a put option on the bonus reserve.

Within their paper two different cases are considered. Firstly, there is only one policyholder. Here the authors characterise fair contracts between the policyholder and the company. Secondly, they investigate two policyholders that can differ with respect to the minimum interest rate guarantee, entry dates or exit dates. For this setting different distribution mechanisms for the terminal bonus are proposed.

For the one policyholder case their model is based on a simplified balance sheet approach similar to the approaches suggested by Briys and de Varenne (1997) or Grosen and Jørgensen (2000). On the asset side, X denotes the company's total assets. Initially the policyholder pays that amount X and the company invests it in some reference portfolio. On the liability side, there are the *policyholder's account* A, the *bonus reserve* B and the *company's account* C. See Table 2.4 therefore:

Table 2.4 The company's initial balance sheet in the Hansen and Miltersen model (Source: Hansen and Miltersen (2002))

Assets	Liabilities
X	A
	B
	C
X	X

At start the initial deposit paid by the policyholder is credited to the policyholder's account A, hence $A(0) = X(0) = X$. To model the reference portfolio X a geometric Brownian motion is used. The time t value of the reference portfolio, $X(t)$ is then described by the following stochastic differential equation.

$$dX(t) = rX(t)dt + \sigma X(t)dW(t), \qquad X(0) = X, \tag{2.19}$$

under the equivalent martingale measure Q. r and σ denote the instantaneous risk free interest rate and the instantaneous volatility of the reference portfolio respectively. W is a standard Wiener process under Q. For further information on equivalent martingale measures the interested reader is referred to Harrison and Kreps (1979) and Harrison and Pliska (1981).

Due to the interest rate guarantee at least an interest rate of g is credited to the policyholder's account A at the end of each year. To understand the bonus distribution mechanism it is easiest to describe first the bonus allocation to both, the policyholder's account A and the company's account C. In the latter the company collects payments for issuing and guaranteeing contractual benefits.

Primarily depending on the ratio $B/(A+C)$ a possible extra amount on top of the minimum interest rate guaranteed g is determined. So the sum $A(t-1) + C(t-1)$ shall be continuously compounded at the rate $r_{AC}(t)$, where

$$r_{AC}(t) = \max\left\{g, \ln\left(1 + (\alpha + \rho)\left(\frac{B(t-1)}{A(t-1) + C(t-1)} - \gamma\right)\right)\right\}, \tag{2.20}$$

with $\alpha + \rho \in [0, 1]$ and $t \in [1, 2, \ldots, T]$. The evolution of the sum $A + C$ is then given by

$$(A(t) + C(t)) \;=\; (A(t-1) + C(t-1))\, e^{\max\{g,\lambda\}}, \tag{2.21}$$

with $\lambda = \ln\left(1 + (\alpha + \rho)\left(\frac{B(t-1)}{A(t-1)+C(t-1)} - \gamma\right)\right)$, $\alpha, \rho \in [0, 1]$ and $\alpha + \rho \in [0, 1]$. To split the evolution of A and C a percentage payment fee $\xi \in [0, 1]$ is introduced. Modifying Eq. 2.21 leads to

$$A(t) \;=\; A(t-1) e^{\max\left\{g, \ln\left(1+\alpha\left(\frac{B(t-1)}{A(t-1)+C(t-1)} - \gamma\right)\right)\right\} - \xi} \tag{2.22}$$

and

$$C(t) \;=\; (A(t) + C(t)) - A(t). \tag{2.23}$$

Considering Formula 2.21 and 2.23 we see two different ways the company can collect payment into its account C for issuing the minimum interest rate guarantee. Either we have $\xi > 0$ and $\rho = 0$, called the *direct* method for collecting payments, or the opposite case with $\rho > 0$ and $\xi = 0$, called the *indirect* method for collecting payments. In the subsequent the authors focus on the direct method.

The bonus reserve B is a residual quantity and evolves according to the following equation. With $B(0) = B = 0$ we have

$$\begin{aligned} B(t) \;=\; & B(t-1) + X(t) - X(t-1) - (A(t) + C(t)) \\ & + (A(t-1) + C(t-1)). \end{aligned} \tag{2.24}$$

At maturity date T the policyholder receives the amount of its account $A(T)$ plus – if positive – the terminal value of the bonus account $B(T)$. In the opposite case where $B(T)$ is negative the company covers the deficit with its account $C(T)$.

To determine a fair set of contract parameters like g, α, ρ, ξ and γ, M. Hansen and K. R. Miltersen define a fairness condition. Let

$$V_t\,(Z(T)) \;=\; e^{-r(T-t)} E_t^Q [Z(T)] \tag{2.25}$$

be the time t *market value operator* with $E_t^Q[\cdot]$ the conditional expectation under the equivalent martingale measure Q and $Z(T)$ some stochastic payoff at date T. Then the fairness condition is given by

$$X = V_0(A(T) + B^+(T)) \;\Leftrightarrow\; 1 = V_0\left(\frac{A(T)}{X}\right) + V_0\left(\frac{B^+(T)}{X}\right), \tag{2.26}$$

where B^+ denotes positive values of the bonus account.

I wish to refer to the second purpose of this paper, investigating the two policyholder case. Modelling two (or several) policyholders instead of just one means deciding whether to pool bonus reserves or not. Using the same framework as introduced above, the authors analyse various contractual specifications for two policyholders and terminal bonus distribution schemes with respect to pooled and non-pooled bonus reserves.

The authors conclude that their model is able to price life insurance contracts containing a minimum interest rate guarantee and a dividend option using the principle of fair valuation. Due to this there is no need for an up-front premium. The policyholder pays for the guarantee via the annual payment fee (ξ).

Analysing the two policyholder case shows that when pooling inhomogeneous policyholders, it is more favourable from the company's point of view to leave at least one of the policyholders worse off. Furthermore the modelling results indicate that old policyholders with a higher interest rate guarantee "cheat" newer policyholders with a lower guarantee with respect to a fair distribution of the pooled bonuses.

M. Hansen and K. R. Miltersen finalise with the remark that life insurance companies should not pool the policyholder's bonus reserve, implying the calculation of an individual payment fee for each policyholder.

Haberman et al. (2003)

This paper is motivated by a handful of reasons for public concern about the financial health of life insurance companies. Whilst former presented papers identified fallen interest rates, fallen equity markets and deliberate mismatching in favour of equities as the main risk drivers, Haberman et al. (2003) mention two new and important issues on that matter: The move towards more transparency with the customers and the international move towards market based accountancy standards.

Based on these insights the purpose of the current paper is to develop a suitable valuation framework for participating life insurance policies. Conceptually that valuation framework is derived from Grosen and Jørgensen (2000). Again they assume the policy reserve P evolves according to the following formula.

$$P(t) \;=\; (1 + r_P(t)) \cdot P(t-1), \qquad t \in I, \tag{2.27}$$

where $I = \{1, 2, \ldots, T\}$. Instead of just one definition for the policy interest rate r_P three different smoothing schemes are suggested to define r_P.[15] Note that these schemes are not imaginary but based on evidence from Needleman and Roff (1995), Chadburn (1998) and Tillinghast (2001).

[15] Confer Formula 2.11.

Scheme I

Let $\beta \in (0,1)$ be the *participating rate* and $n = \min(t, \tau)$, $\tau < T$, the length of the smoothing period. Then

$$r_P(t) \;=\; \max\left\{r_G, \frac{\beta}{n}\left(\frac{A(t)}{A(t-1)} + \ldots + \frac{A(t-n+1)}{A(t-n)} - n\right)\right\}. \qquad (2.28)$$

Scheme II

Let β and n be defined as for scheme I. Then

$$r_P(t) \;=\; \max\left\{r_G, \beta\left(\sqrt[n]{\frac{A(t)}{A(t-n)}} - 1\right)\right\}. \qquad (2.29)$$

Scheme III

Let P^1 be the unsmoothed asset share defined as

$$P^1(t) = P^1(t-1)(1 + r_P(t)) \qquad (2.30)$$

and

$$r_P(t) = \max\left\{r_G, \beta\frac{A(t) - A(t-1)}{A(t-1)}\right\}. \qquad (2.31)$$

Then the policy reserve P is defined as follows

$$P(t) = \alpha P^1(t) + (1-\alpha)P(t-1), \qquad (2.32)$$

with $\alpha \in (0,1)$ playing the role of the smoothing parameter.

At maturity date T the policyholder receives the terminal value of its policy reserve $P(T)$ plus a fraction $\gamma R(T)$, $\gamma \in (0,1)$ of a possible surplus generated by the reference portfolio A denoted by

$$R(T) = \max(0, A(T) - P(T)). \qquad (2.33)$$

On the other side the insurance company gets what is left. This can be either a gain if $A(T) \geq P(T)$ or a loss if $A(T) < P(T)$. A potential loss shall be denoted by

$$D(T) = \max(0, P(T) - A(T)). \qquad (2.34)$$

S. Haberman, L. Ballotta and N. Wang conclude with a numerical analysis of the three different smoothing schemes. Using the risk neutral valuation they focus on the following quantities.

$$V_P(0) = E^Q[e^{-rT} P(T)], \tag{2.35}$$

$$V_R(0) = E^Q[e^{-rT} R(T)] \tag{2.36}$$

and

$$V_D(0) = E^Q[e^{-rT} D(T)], \tag{2.37}$$

with r the instantaneous risk free interest rate. By using different smoothing schemes, the model becomes slightly more realistic and enables a comparison in this regard. But since their analysis is based on artificial life insurance contracts, only the numerical results do not really provide new and valuable insights compared to earlier works.

Cummins et al. (2004)

This paper investigates various implementations of interest rate guarantees and dividend options across different countries. The authors focus on modelling common practice in Denmark, Germany, Norway, the United Kingdom and the United States.

The basic modelling set-up formulated in Cummins et al. (2004) is comparable to the balance sheet approaches presented in Grosen and Jørgensen (2000) or in Hansen and Miltersen (2002), whereas the surplus distribution schemes naturally differ from country to country. Insurance specific factors like mortality or periodic premiums are as usually neglected. Furthermore only one asset investment opportunity, called the *market index*, is taken into account.

J. D. Cummins, K. R. Miltersen and S.-A. Persson remark that in European contracts the surplus distribution mechanism is usually designed more sophisticated than in the US equivalent where, for example, no bonus account is modelled. However, given the objective is to provide the policyholder a future benefit with low risk, their numerical analysis indicates that United States' policies outperform European policies.

They further observe that the Danish, German and the United Kingdom contracts including more sophisticated smoothing mechanisms basically do not perform significantly differently than the assumed market index. This means that the minimum interest rate guarantee and the dividend option have virtually no valuable impact on the policyholder's benefits.

Kling et al. (2007)

In the papers discussed so far various ways to give interest rate guarantees are considered. Remember the point-to-point scheme suggested in Briys and de Varenne (1997). Hence the guarantee is only relevant at maturity. As opposed to this we know year-by-year (or cliquet-style) guarantees as presented e.g. in Grosen and Jørgensen (2000) or Hansen and Miltersen (2002), where the interest rate guarantee is credited yearly. I have further presented papers with different mechanisms *defining* a surplus amount. Therefore see Haberman et al. (2003), for example.

Lastly Kling et al. (2007) consider mechanisms *distributing* annual surpluses to policyholders. For their purposes they present three different possibilities.

1. Surpluses may be allocated to the policy account. Thus allocated surpluses are charged with the same interest rate as the policy account.
2. Surpluses may be allocated to a separate account. That accountÊis owned by the policyholder but there is no interest credited on it.
3. Surplus may be allocated to a terminal bonus account. The policyholder will receive a possible terminal bonus only at maturity or, depending on contractual specifications, a reduced amount thereof when surrendering. As long as the policy is in force, the insurance company may use that account to pay the minimum interest rate guarantee if the return on their assets does not yield the minimum interest rate guaranteed in any particular year.

Thus, the purpose of the current paper is to analyse numerical impacts on the risk exposure of a life insurance company due the different surplus distribution mechanisms presented above. For this the authors use their own modelling framework based on a balance sheet approach enabling the consideration of year-by-year as well as point-to-point guarantees. The focus is, however, on the different surplus distribution mechanisms.

The numerical analysis shows clearly that the third distribution mechanism significantly differs from mechanism one and two. The third mechanism faces a much lower shortfall risk than the other two. Thus, a company applying that mechanism may invest its funds in riskier asset classes promising higher expected returns and also provide a higher interest rate guarantee for this.

A. Kling, A. Richter and J. Russ notice that the third mechanism is severely restricted by regulators in some European countries whilst it is not in some other European countries. This may lead to a distortion of competition since the current analysis has indicated some strong pros of this distribution mechanism. They conclude that severe regulations are not always advantageous for the insurance industry and could even be counterproductive as shown by this example.

2.3 Conclusions for the Modelling Framework

In this chapter I have defined basic types of life insurance products and introduced common options and guarantees embedded therein. I have further appointed the most relevant ones for the purpose of this thesis: Minimum interest rate guarantee and dividend option. Based on that criterion, a literature survey outlining the reader the actual state of the scientific knowledge was presented.

Remember the objectives formulated in Sect. 1.3 where I have requested a quantitative modelling approach to handle embedded options and guarantees based on modern valuation techniques with respect to current regulatory and accounting standards. Taking the literature survey we have seen several different approaches as to how such a model could be designed. But in order to achieve these objectives I definitely identify some important issues that have not or have just insufficiently been taken into account so far. Thus to finalise the current chapter, I aim to outline theoretical characteristics for the modelling framework I will formulate in the upcoming chapter. Hereby, certain features from existing works are adopted and my own extensions are presented.

- As previously announced I basically aim to design a fairly general and transparent framework that provides an appropriate substructure to handle every kind of embedded option and guarantee. In the end the focus will be on minimum interest rate guarantee and dividend option and so the framework finally being presented and applied for modelling purposes in the subsequent chapters will be some special case thereof. But note that this does not affect the requested flexibility of the model itself. Having said this, a year-by-year balance sheet approach as suggested by Grosen and Jørgensen (2000) seems to be a good starting point. Take notice of the fact that this model is more than just a theoretical construct but also widely accepted in practice.[16]
- According to Sect. 1.3 the central question I aim to answer with that model is about the value the considered embedded options and guarantees do have, absolutely and relative to the current policy reserves. Also remember that market consistency is the basic prerequisite for valuation methodologies applied nowadays and so this model must be set up. For this reason it shall be calibrated consistently to true market data on the asset side and allow for considering real, existing and in-force policy data on the liability side. Except Grosen and Jørgensen (2002b), discussed papers in the literature survey consequently set up their analysis on artificial insurance policies decreasing practical benefits from the respective conclusions.
- Referring to the selected balance sheet approach, the asset side shall be modelled stochastically. As stated above the underlying stochastic process will be calibrated to true market data. But with a simple Brownian motion, as applied in most of the papers that have been discussed in the literature survey,

[16] Confer Hansen and Miltersen (2002).

one does not achieve an accurate mapping of an insurance company's asset portfolio. Therefore, considering the asset portfolio of a real existing insurance company leads to a joint stochastic modelling of three asset types: Two correlated processes for an equity and a property price index and *zero coupon bond* (ZCB) prices.

- Most of the innovation will happen on the liability side. First of all, a set of approximately 110,000 real existing life insurance policies will provide the data basis for the model. Since there is no market for these in-force policies a market consistent value needs to be determined. Hence mortality and expenses are to be taken into account, too. Note that the only paper partially considering mortality was Jensen et al. (2001). Explicitly mentioning the importance of market consistent valuation of both, the equity and the liabilities, could be found in Grosen and Jørgensen (2002a) only.
- Haberman et al. (2003) addresses different mechanisms to determine a possible surplus. Indeed this has significant impacts on the entire model in general and on the dividend option in particular. Thus, I will make allowance for handling that issue in detail.
- With Kling et al. (2007) I have discussed one paper investigating different mechanisms to distribute a possible surplus, another crucial element regarding the value of the dividend option. Since that distribution mechanism is contractually specified for the given policy data, only performing a "what-if analysis" makes sense.
- A last point worth noting here concerns a company's shareholders. Even though the model by Briys and de Varenne (1997) incorporates some equity component, the shareholders interests are not accurately incorporated there in my opinion.

In the following Chaps. 3 and 4 I will formulate the entire modelling framework based on the theoretical outline given above and present all market and insurance specific data needed therefore.



http://www.springer.com/978-3-7908-2842-9

Implicit Embedded Options in Life Insurance Contracts
A Market Consistent Valuation Framework
Rüfenacht, N.
2012, XXVI, 170 p., Hardcover
ISBN: 978-3-7908-2842-9
A product of Physica-Verlag Heidelberg