
Preface

Lagrange, in the *Mécanique analytique* of 1788, identified three programs of research, or paradigms, in the history of statics: the lever, the composition of forces, and the principle of virtual work. The paradigm of the lever would have been in force from antiquity until the early XVIII century, when Varignon was asserting the parallelogram law for composition and decomposition of forces. The principle of virtual work would have become dominant since the XIX century. This picture is in my opinion quite realistic, although the final formulation predicted by Lagrange was never fully realized because the principle of virtual work has never replaced the rule of the composition of forces, but at most has outflanked it. Also the picture is too schematic. In fact, some form of law of virtual work has always existed in mechanics, always however with limited applications.

The law of virtual work, as usually presented in modern textbooks of mechanics, says that there is equilibrium for one or more bodies subjected to a system of forces if and only if the total virtual work is zero for any virtual displacement. In Chapter 2 of this book the meaning of the terms work and virtual is described in some detail; here I will only mention that, since Lagrange in the second half of the XVIII century, the law of virtual work had no appreciable changes in its formulation. The view on its role in mechanics is instead still varying, passing from the enthusiasm of the XIX century to a modest presence in modern rational mechanics as well as, all considered, in the engineering field, albeit with some important exceptions.

The present book starts from the first documented formulations of laws of virtual work. They usually have only a vague analogy to the modern ones and only mathematically. Attention is paid to Arabic and Latin mechanics of the Middle Ages. With the Renaissance there began to appear slightly different wordings of the law, which were often proposed as unique principles of statics. With Bernoulli and Lagrange the process reached its apex. The book ends with some chapters dealing with the discussions that took place in the French school on the role of the Lagrangian law of virtual work and its applications to continuum mechanics.

Even though the book takes a particular point of view, it presents an important slice of history of mechanics. Essential reference is made to primary sources; secondary literature is mainly used to frame the contributions of the scientists consid-

ered in their times. To allow a better understanding of the ideas of the authors studied, English translations are always accompanied by original quotations (Appendix). No pre-conceived historical hypotheses have been explicitly assumed though. The mere existence of the book suggests that I have in mind a continuous chain connecting concepts from antiquity up to now. However the nature of the chain is complex and I leave it to the reader to unveil it.

The book is the result of a twenty year study of mechanics and its history and should be of interest to historians of mathematics and physics. It should also arouse interest among engineers who are now perhaps the most important witnesses of classical mechanics, and with it, of the law of virtual work.

I want to acknowledge Giuseppe Ruta, Romano Gatto, Antonino Drago for contributing comments and suggestions to specific parts. Cesare Tocci for suggestions regarding the whole book, and finally I want to acknowledge Raffaele Pisano for his reading and the debates we have had.

Editorial considerations

Figures related to quotations are nearly all redrawn to allow a better comprehension. Symbols of formulas are always those of the authors, except in easily identifiable cases. Translations of text from French, Latin, German and Italian are as much as possible close to the original.

Rome, September 2011

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History of Virtual Work Laws

A History of Mechanics Prospective

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2012, XII, 492 p., Hardcover

ISBN: 978-88-470-2055-9