

# Chapter 2

## Design Signal Detection Project of MIMO Communication Systems Based on Improved Grover Algorithm

Lu Xin-Bo

**Abstract** The signal detection scheme of multiple inputs multiple output (MIMO) communication system is introduced, and the existing problems of Grover algorithm is analyzed and improved. The signal detection scheme of MIMO system based on Grover algorithm and improved Grover algorithm is developed, and Grover algorithm is applied to find the minimum value in order to decide the sending sequence. In order to test the efficiency and reliability of this algorithm, the analysis and comparison between Grover algorithm, improved Grover algorithm and other traditional algorithms are made by MATLAB simulation. Experimental results show that it can reduce the complexity, while achieving the same performance of the traditional optimum detection algorithms.

**Keywords** Grover quantum search algorithm • Quantum parallel computation • MIMO detection algorithm • Quantum register

Multiple-input multiple-output (MIMO) wireless communication systems refer to transmitters and receivers that are equipped with multiple antenna units for data transferring process in the wireless communication system. Compared with single-input single-output (SISO) wireless communication systems, additional degrees of freedom (space resource) can be created in MIMO system without increasing neither the bandwidth nor transmit power, which can be exploited for significant improvement of system capacity and enhancement of transmission reliability. As the key technology of the fourth generation mobile communication system (4G), MIMO signal detection has been extensively studied and put forward many signal detection algorithm, such as zero-forcing (ZF) algorithm and minimum

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mean square error (MMSE) algorithm. However, these algorithms generally are based on flat fading MIMO channel. For frequency selective broadband MIMO communication system, its receiving signals exist serious inter-symbol interference (ISI). MIMO detection algorithm cannot be overcome. We must use that can confront the frequency selective decline of technology. It can not only overcome the frequency channel choice function but also make further improvement on the frequency utilization that OFDM technology is introduced into MIMO. Grover algorithm which is based on the quantum parallel computation, can find the desired value precisely with  $O(\sqrt{N})$  iterations in a large unsorted database which contains  $N$  elements. However, to find the desired value in the database, any classical algorithm would need at least  $O(N)$  steps. So quantum algorithm, for the system signal detection, can slash algorithm complexity [1].

In this chapter a kind of signal detection scheme of communication systems based on Grover algorithm is developed. It can not only effectively reduce the algorithm complexity but also reach basically the same performance of classic best receiving algorithm.

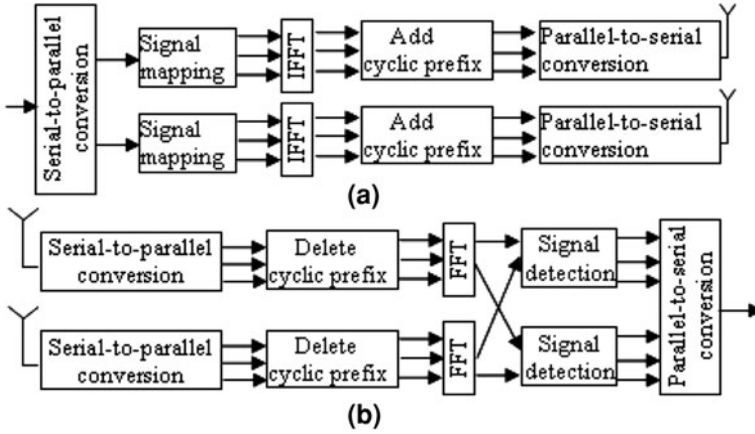
## 2.1 MIMO Communication System

The block diagram of MIMO system signal detection based on spatial multiplexing without coding is shown in Fig. 2.1 [2].

Assume the sender has  $M$  root antenna, the receiver has  $N$  root antenna. The channel, between the  $M$  ( $M = 1, 2 \dots$ ) sender root antenna and the  $N$  ( $N = 1, 2 \dots$ ) receiving root antenna, is the multipath fading channel which obey Rayleigh distribution. OFDM subcarrier number is  $K$ . In the sender, input bit data stream converted into  $M$  parallel data stream after serial-to-parallel conversion, so as to realize the multiple antennas output. For each path flow, we should first signal mapping then IFFT transform. IFFT transform realize the OFDM modulation function here. In other words, the function is that low-speed of multi-channel parallel data flow are modulated to mutually orthogonal  $K$  sub-carrier at the same time. In order to reduce ISI, protect interval that usually use circle prefix form is joined in among symbols after IFFT transformation. At last, the data stream is transmitted after parallel-to-serial conversion.

In the receiver, each antenna received signals that were sent by  $M$  transmitting antennas and MIMO channel linear superposition. The system first desterilizes and removes circulation prefix from every data stream then according to receiving antenna FFT transform, from the time domain transform to frequency domain. At last, parallel data flow is delivered to modem after detector processing and get recovery information bit stream by serializer.

To MIMO system which have  $K$  sub-carriers, we can assume the receiver completely know channel state information and channel characteristic is changeless in a OFDM symbol duration and cyclic prefix is greater than channel delay



**Fig. 2.1** The block diagram of MIMO system signal detection. **a** The sending end. **b** The receiving end

spread and the system does not exist ISI. Therefore, the multipath channel between any pair antennas can be expressed as  $k$  parallel frequency sub channels in a OFDM symbol duration [3].

## 2.2 Grover Algorithm

Grover algorithm intent is increasing probability amplitude of target quantum states by the unitary transformation of initial equal amplitude superposition state while simultaneously reducing probability amplitude of other off-target quantum states. In the end, the more probability amplitude of target quantum states the bigger probability the right target is searched (namely measured).

Grover algorithm described as follows [4]:

(1) Initialization

$$\left( \overbrace{\frac{1}{\sqrt{N}}, \frac{1}{\sqrt{N}}, \dots, \frac{1}{\sqrt{N}}} \right)$$

Hypotheses  $n$  are quantum bits,  $N = 2^n$ . In other words, all basic vectors have initially equal amplitudes. This can be achieved by Walsh–Hadamard transform acts on quantum states  $|0000 \dots 0\rangle_n$ . Total need  $O(\log N)$  steps.

- (2) Grover iteration. Repeat step (a) and (b)  $O(\sqrt{N})$  times. According to the Ref. [5], best iteration number is  $j \approx \lceil \pi/4\sqrt{N/m} \rceil$ , lowercase  $m$  letter is the correct solution number,  $\lceil \rceil$  show integer.
- (a) Selective rotation transformation  $U_f$  (corresponds to solution set marked). Hypotheses  $S$  is a basic vector of input vectors. Vector  $S$  rotate  $180^\circ$  when  $C(S) = 1$ , unchanged when  $C(S) = 0$ .
- (b)  $D$  transforms matrix acts on every input component.  $D$  defined as follows:

$$D = \begin{cases} \frac{2}{N}, & i = j \\ -1 + \frac{2}{N}, & i \neq j \end{cases} \quad D = WRW \quad (2.1)$$

$W$  is W-H transform matrix,  $R$  is conditions transfer matrix.  $R$  matrix reverse probability amplitudes of quantum state  $|S\rangle$  which content with  $C(|S\rangle) = 1$

$$\begin{aligned} R_N &= 0 \quad (i \neq j \text{ and } i \neq 0) \\ R_N &= 1 \quad (i = 0) \\ R_N &= -1 \quad (i = j \text{ and } i \neq 0) \end{aligned} \quad (2.2)$$

- (3) Measure input, observation is  $|\varphi_r\rangle$ , we can get results if  $C(|\varphi_r\rangle) = 1$ , or we should start algorithm again.

Grover algorithm can effectively search the database. But it is invalid under the following circumstances:

- (a)  $m = N/2, \theta = \pi/4$ , no matter how many times iteration occurs, the effect of iteration and no iteration is same and at this time algorithm is invalid.
- (b)  $m > N/4$ , to ensure the algorithm has greater success probability, iteration number does not meet  $O(\sqrt{N/m})$ .

In order to solve the above problems, Grover algorithm was improved in Ref. [6]. In the following, we will study and compare Grover algorithm and improved algorithm effect on MIMO signal detection.

### 2.3 Signal Detection Project Based on Improved Grover Algorithm

Firstly, design two databases. The first database which has  $2^{n_r}$  registers save all possible sending sequence. As necessary judgment information, the correlation matrix  $RS$  and receiving signal stored in the computer. We can get  $2^{n_r}$  decision values by each may be sending sequence and correlation matrix and channel matrix  $H$  are calculated according to  $\|y - Hx\|^2$ . These judgment values put in the second database. Message sending sequences in the first database and judgment values in the second database constitute one-to-one relationships. Message

sequences which correspond to minimum decision values are calculated according to  $\|y - Hx\|^2$ , and are the sending sequences which are detected by the best detection scheme. Therefore, the following work is looking for the smallest judgment value and judging sending sequence.

Grover Algorithm can solve this problem.

The basic thought: Hypothesis in the second database there is one group of judgment value  $(x_1, x_2, x_3, \dots, x_n, N = 2^n)$ , which correspond to  $N$  quantum ground-states  $(\varphi_1, \varphi_2, \varphi_3, \dots, \varphi_N, N_{N-1} = 2^n)$ .  $N$  quantum ground-states placed in a quantum register. So there is  $x_i = \min_{i=0}^{N-1}(x_i)$  in the second database, and we need to make its position in the database. The judgment value and quantum state is corresponding, so we can think, once finding  $x_i$  corresponding quantum state position in quantum registers, also determined  $x_i$  position in the second database and then finding  $x_i$  corresponding sending sequence, and then the sequence is thought of as the message sequence by judgment. Solved classic problems by quantum algorithms become possible.

Specific algorithm described as follows:

- (1) Construct  $N = 2$  bit quantum registers, contains quantum ground-state  $(|\varphi_1\rangle, |\varphi_2\rangle, \dots, |\varphi_N\rangle)$  which correspond with decision value  $(x_1, x_2, x_3, \dots, x_N)$ . They can be shown by  $T(|\varphi_i\rangle) = x_i \quad (i = 1, 2, \dots, N)$ .
- (2) Initialization quantum registers by Walsh-Hadamard transform. At this time the quantum states in quantum register is shown by

$$|\varphi(k_i^{(0)}, l_i^{(0)})\rangle = \sum_{k=1}^N \frac{1}{\sqrt{N}} |\varphi_1\rangle \quad k_i^{(0)}, l_i^{(0)} = \frac{1}{\sqrt{N}} \quad (2.3)$$

- (3) In quantum register random took a quantum ground-state, its corresponding judgment value as a threshold. Here we used judgment value  $x_1$  which corresponds with the first quantum ground-state  $|\varphi_1\rangle$  in quantum register quantum as threshold. Then through Grover algorithm, in quantum register we find quantum ground-state which correspond with judgment value which is less or equal to  $x_i$ . Use rotation operation  $R$  to rotate its probability amplitude, otherwise remain unchanged.  $R$  defines for:

$$1 \leq m \leq \frac{N}{3} \quad R_{pq} = \begin{cases} 0 & p \neq q \\ 1 & p = q, T(|\varphi_p\rangle) > T(|\varphi_1\rangle) \\ -1 & p = q, T(|\varphi_p\rangle) \leq T(|\varphi_1\rangle) \end{cases} \quad (2.4)$$

$$m > \frac{N}{3} \quad R_{pq} = \begin{cases} 0 & p \neq q \\ 1 & p = q, T(|\varphi_p\rangle) > T(|\varphi_1\rangle) \\ i & p = q, T(|\varphi_p\rangle) \leq T(|\varphi_1\rangle) \end{cases} \quad (2.5)$$

- (4) Vector of quantum ground-state probability amplitude unitary transform by matrix  $D$ , amplify the probability amplitude of quantum ground-state which is searching. Matrix  $D$  defines for:

$$1 \leq m \leq \frac{N}{3} \quad D_{pq} = \begin{cases} \frac{2}{N} & p \neq q \\ -1 + \frac{2}{N} & p = q \end{cases} \quad (2.6)$$

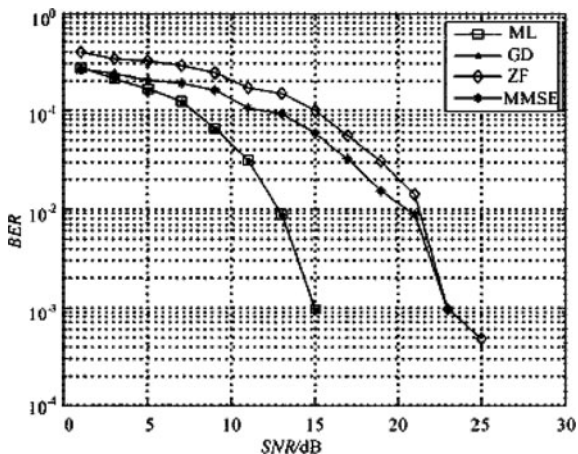
$$m > \frac{N}{3} \quad D_{pq} = \begin{cases} \frac{1}{N} + \frac{i}{N} & p \neq q \\ \frac{1}{N} - i \left( \frac{N-1}{N} \right) & p = q \end{cases} \quad (2.7)$$

- (5) After iteration function of  $R$  operator and  $D$  operator, all the quantum ground-state have been discovered which correspond judgment value less or equal to  $x_1$  and are in quantum register, seeking scope of target quantum states dramatically narrow. We repeat (3) (4) operation for all searched quantum state and search the quantum ground-state which correspond judgment value less than or equal to threshold by Grover algorithm until the only quantum ground-state remains.
- (6) At that time, quantum state is  $|\varphi(k_i^{(0)}, l_i^{(0)})\rangle$ . By measured quantum registers, we can obtain petitions solution.

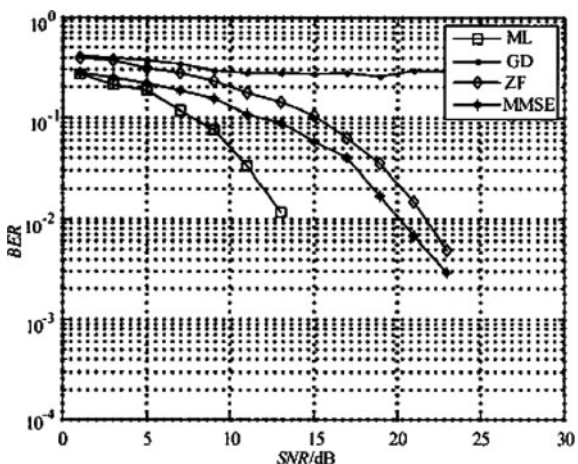
## 2.4 Simulation Results and Analysis

In order to analyze and compare performance of MIMO detection method based on Grover algorithm, this chapter does following hypothesis: (1)  $K = 16$ ,  $K$  is sub-carrier number of OFDM, each carrier send symbols number is 128, (2) Channel matrix  $H$  known and unchanged in 128 symbols cycle, then independently changed, (3) The receiver know the exact channel state information, the sender use no coding QPSK modulation, sending power is 1, (4) Noise is white Gauss noise, (5)  $4 \times 4$  single diameter MIMO-OFDM systems. On the basis of hypothesis, this thesis gives Grover algorithm detection (GD) and improves Grover algorithm detection (IGD) apply to MIMO-OFDM signal detection and compares with the traditional maximum likelihood (ML) algorithm, ZF algorithm and MMSE algorithm detection. If the difference of detected signals in receiver and the original signals in sender is in a certain range, the receiver judges no error and correct answer. This range defines for search errors.

**Fig. 2.2** GD and the traditional algorithm performance comparison when search error is 0.001



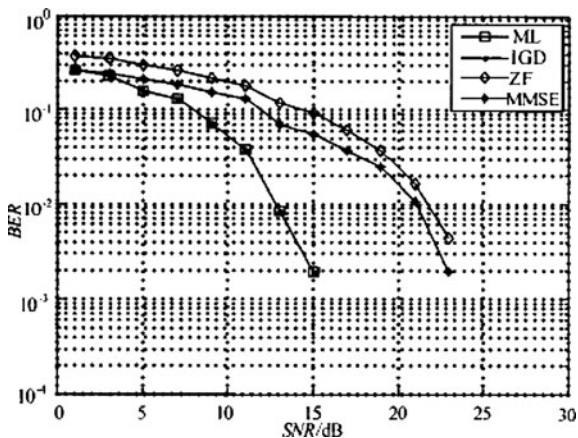
**Fig. 2.3** GD and the traditional algorithm performance comparison when search error is 0.0001



Simulation results analysis: It can be seen from the Fig. 2.2 that GD performance is better in MMSE and ZF detection performance and close to ML detection performance even the same when search error is 0.001. But the original GD search error narrowed to 0.0001 even smaller, its detection effect is very poor and search failure, this can be seen from the Fig. 2.3. However, when search error is 0.0001 IGD performance is better in MMSE and ZF detection performance and GD performance in Fig. 2.2. This can be seen from Fig. 2.4. IGD effect is very good when taking smaller search error, since the selection of threshold is random at the beginning. The different selection of search error size not only relation computational complexity but also affects the algorithm's accuracy.

IGD search results are more accurate and valid than the original GD, when GD algorithm has many solutions its maximum iterating times appear indeterminacy. However IGD can assure solutions with great probability by iteration when the number of solutions is greater than  $N/3$ . All these show the improved Grover

**Fig. 2.4** IGD and the traditional algorithm performance comparison when search error is 0.0001



algorithm can clearly improve performance of traditional MIMO detection algorithm and achieve the performance which is very near to ML detection algorithm during effectively reducing the algorithm complexity.

## 2.5 Conclusions

The complexity of ML detection algorithm is exponential relation with antenna numbers. When antenna number and modulation order number is bigger, this all-over search process is difficult to realize for real-time or cannot come true in actual system for computing complexity, so it applies only to the theoretical analysis. ZF and MMSE receiver greatly reduces the computing complexity, but the two algorithms have to sacrifice property for complexity reduction because their performance was decreased obviously in high signal-to-noise ratio. For Grover algorithm, search steps reduce from classic  $N$  to  $\sqrt{N}$ , its time complexity is  $O(\sqrt{N})$ , thus showing quantum acceleration. Time complexity of IGD is  $O(\sqrt{N}/t)$  that has been proved by Ref. [6] and the algorithm is more effective and fast. The detection scheme that has been presented by this text can effectively reduce the complexity of the algorithm, and its effect is consistent with classic optimum detecting scheme in error code performance.

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