

Chapter 2

Modeling

Abstract Nowadays, non rigid airships with a cigar shaped profile are the most common type. These airships do not have any rigid internal framework. The objective of this chapter is to present kinematics and dynamics models of a lighter than air robot, taking into account wind effect. Newton-Euler and Hamilton-Lagrange approaches are used for this discussion then the translational model is presented. Here, motion is referenced to a system of orthogonal body axes fixed in the airship, with the origin assumed to coincide with the bow. Finally, some aerology characteristics are briefly discussed.

2.1 Introduction

The airship is essentially a low speed vehicle and is fundamentally a balloon with a means of propulsion but in order to produce an effective vehicle, the balloon element has to be considerably modified. The primary requirement of the envelope is that it should be of streamline form to reduce air resistance and improve controllability and that it should be able to sustain the additional loads imposed by aerodynamic forces and propulsive components. A representative mathematical model of a lighter than air vehicle differs from the usual aircraft model because the lighter than air vehicle displaces a very large volume and its virtual mass and inertia properties become significant and thus the aerodynamics and control characteristics are described quite differently.

What makes a vehicle lighter than air is the fact that it uses a lifting gas (i.e. Helium as Hydrogen has been banned) in order to be lighter than the surrounding air. While Hydrogen gas is approximately 7 percent more buoyant, Helium has the advantage of being non-inflammable in addition to being fire retardant.

The principle of Archimedes applies in the air as well as under water. The difference between airships and balloons is that: balloons simply follow the direction of the winds. In contrast, airships are powered and have some means of controlling their direction. There are basically three different kinds of airships: rigid, semi-rigid and non-rigid airships (see Fig. 2.1).

Non rigid airships or pressure airships, also known as blimps, are the most common form nowadays. They are basically large gas balloons. Their shape is maintained by their internal overpressure. The only solid parts are the gondola, the set of

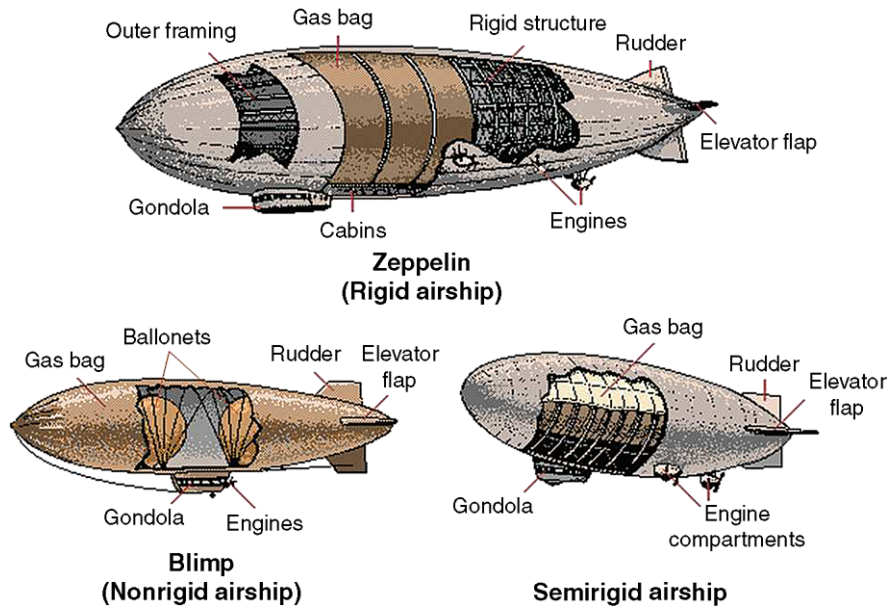


Fig. 2.1 Different kind of *airships*, taken from <http://library.thinkquest.org/18033/airship.html>

propellers and the tail fins. The envelope holds the Helium that makes the airship lighter than air. In addition to the lift provided by Helium, airships derive aerodynamic lift from the shape of the envelope as it moves through the air. An envelope as the gas containment membrane encloses the lifting gas and the ballonets and provides protection from the environment. Ballonets are filled with air by blowers to maintain a fixed pressure inside as the temperature of the lifting gas or the airship altitude changes. Ballonets permit the envelope pressure to be controlled and relative fullness of fore and aft ballonets is associated with pitch control. Adjustment of air volume in ballonets and gas volume in the envelope produces the change of buoyancy. The vertical portion of the gondola loads is supported by an internal suspension system which is contained in the envelope and runs from the top of the envelope to the gondola. The principal function of the external suspension system attached to the bottom part of the gondola loads into the envelope. The envelope fabric consists of laminated composite and is designed to withstand environment and flight loads. Non rigid airships have simple structures and are easy to design, build and maintain.

The most common form of a dirigible is like an ellipsoid. It is a highly aerodynamically profile with a good resistance to aerostatics pressures. Aerodynamics forces occur when a body moves relative to the air in which it is immersed. These forces are classified as steady when they are invariant with time or transient when they are not. The steady forces arise when a body is in uniform motion in a still atmosphere. Transient forces occur during maneuvers or owing to turbulence in the atmosphere. Aerodynamic forces and moments arise from the local surface pres-

tures, which when integrated over the whole body, give the overall forces and moments. It is therefore necessary to determine the local pressures. At non-neutral lift, it becomes necessary to trim the airship to generate aerodynamic lift to balance the difference between the weight and lift forces. This in turn creates additional drag with the consequent penalty in performance.

Several research studies have been carried out on dynamic modeling and simulation of airships. Dynamic models are built to study guidance and control techniques of airships, trajectory, stability and maneuverability. This chapter consists of 5 sections. Section 2.2 presents kinematics using two different approaches: Euler angles and Euler parameters. In Sect. 2.3, six degrees of freedom dynamics are presented, using respectively Newton-Euler and Lagrange-Hamilton methods, followed by three degrees of freedom translational dynamics and aerology characteristics, in Sect. 2.4. Finally some concluding remarks are given in Sect. 2.5.

2.2 Kinematics

The model used was written originally for a buoyant underwater vehicle [73]. It was modified later to take into account the specificity of the airship [10, 15, 23, 24, 89, 108]. The position of all points belonging to the airship rigid body with respect to the inertial fixed frame $\mathcal{R}_f$ can be completely defined by knowing the orientation of a body fixed frame $\mathcal{R}_m$ to the airship body and the position of its origin with respect to the reference frame $\mathcal{R}_f$. The choice of the body fixed frame origin can be made at the center of gravity, center of volume (assumed to coincide with the gross center of lift) or also at the bow of the hull. The Aerodynamic frame $\mathcal{R}_a$ is used extensively in flight mechanics with flight equations of motion and through an aircraft's air data probe and other sensors. The airship's air speed, V_a is referenced to the aerodynamic frame. These three frames are classically considered in the derivation of the kinematical and dynamical equations of the motion of the aerial vehicles. There are many ways to describe finite rotations between frames. Direction cosines, quaternions, Euler angles, can serve as examples. Some of these groups of variables are very close to each other in their nature.

Remark 2.1 In classical vector mechanics, it is customary to represent the acceleration of a rigid body by means of the linear acceleration of a specified point in the body and an angular acceleration vector which applies to the whole body.

Remark 2.2 The derivation relationship in two different frames can be written as

$$\mathbf{V} = \dot{\mathbf{r}} + \boldsymbol{\Omega} \times \mathbf{r} \quad (2.1)$$

$$\dot{\mathbf{V}} = \ddot{\mathbf{r}} + \dot{\boldsymbol{\Omega}} \times \mathbf{r} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}) \quad (2.2)$$

where $\boldsymbol{\Omega}$ is the angular velocity.

Remark 2.3 The finite rotation of a rigid body does not obey to the laws of vector addition (in particular commutativity) and as a result the angular velocity of the body cannot be integrated to give the attitude of a body [13].

These remarks are extensively used in the following demonstrations.

2.2.1 Euler Angles

The usual minimal representation of orientation is given by a set of three Euler angles. Assembled with the three position coordinates, they allow the description of the situation of a rigid body. A 3×3 direction cosine matrix (of Euler parameters) is used to describe the orientation of the body (achieved by 3 successive rotations) with respect to some fixed frame reference. This parametrization is a three parameters set corresponding to the yaw ψ , pitch θ and roll ϕ , which can be directly measured by the attitude sensors. Adopting this formulation, the rotation matrix R can be written as a function of $\eta_2 = (\phi, \theta, \psi)^T$ given by:

$$R(\eta_2) = R_z(\psi)R_y(\theta)R_x(\phi) \quad (2.3)$$

with

$$R_x(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{pmatrix}$$

$$R_y(\theta) = \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix}$$

$$R_z(\psi) = \begin{pmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

This transformation, also called the Direction Cosine Matrix (DCM), can be explicitated as:

$$R(\eta_2) = \begin{pmatrix} \cos \psi \cos \theta & -\sin \psi \cos \phi + \cos \psi \sin \theta \sin \phi & \sin \psi \sin \phi + \cos \psi \sin \theta \cos \phi \\ \sin \psi \cos \theta & \cos \psi \cos \phi + \sin \psi \sin \theta \sin \phi & -\cos \psi \sin \phi + \sin \psi \sin \theta \cos \phi \\ -\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi \end{pmatrix} \quad (2.4)$$

The rotation matrix R is an orthogonal matrix. The set of such matrices constitutes the special orthogonal matrix group $SO(3)$ defined as:

$$SO(3) = \left\{ R \in \mathbb{R}^{3 \times 3}, R^T R = I_{3 \times 3}, \det(R) = 1 \right\} \quad (2.5)$$

The relationship between the body fixed angular velocity vector $v_2 = (p, q, r)^T$ and the rate of change of the Euler angles $\dot{\eta}_2 = (\dot{\phi}, \dot{\theta}, \dot{\psi})^T$ can be determined by resolving the Euler rates into the body fixed coordinate frame:

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} \dot{\phi} \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} 0 \\ \dot{\theta} \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \dot{\psi} \end{pmatrix} \quad (2.6)$$

or equivalently

$$\begin{aligned} p &= \dot{\phi} - \dot{\psi} \sin \theta \\ q &= \dot{\theta} \cos \phi + \dot{\psi} \sin \phi \cos \theta \\ r &= -\dot{\theta} \sin \phi + \dot{\psi} \cos \phi \cos \theta \end{aligned} \quad (2.7)$$

The configuration of the airship is completely defined by associating the orientation matrix and the $\mathcal{R}_m$ frame origin position vector, $\eta_1 = (x, y, z)^T$, with respect to $\mathcal{R}_f$ using homogeneous matrix formulation as:

$$A_M = \begin{pmatrix} R(\eta_2) & \eta_1 \\ 0_{3 \times 3} & 1 \end{pmatrix} \quad (2.8)$$

The set of all such matrices is called the special Euclidean group of rigid-body transformations in three dimensions noted $SE(3)$ defined by:

$$SE(3) = \left\{ A_M \mid A_M = \begin{pmatrix} R(\eta_2) & \eta_1 \\ 0_{3 \times 3} & 1 \end{pmatrix}, R \in SO(3), \eta_1 \in \mathbb{R}^3 \right\} \quad (2.9)$$

The set $SE(3)$ is a Lie group. Given a curve $C(t) : [-a, a] \rightarrow SE(3)$, an element $S(t)$ of the Lie algebra $\mathfrak{se}(3)$ can be associated to the tangent vector $\dot{C}(t)$ at an arbitrary configuration $A(t)$ by:

$$S(t) = A_M^{-1}(t) \dot{A}_M(t) = \begin{pmatrix} Sk(v_2) & R^T \dot{\eta}_1 \\ 0 & 0 \end{pmatrix} \quad (2.10)$$

where

$$Sk(v_2) = R^T(t) \dot{R}(t) \quad (2.11)$$

is a 3×3 skew symmetric operator on a vector defined by:

$$Sk(v_2) = Sk((p, q, r)^T) = \begin{pmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{pmatrix} \quad (2.12)$$

such that

$$\forall x, y \in \mathbb{R}^3: \quad Sk(y)x = y \times x \quad (2.13)$$

A curve on $SE(3)$ physically represents a motion of the rigid body. If $(\omega(t), v(t))$ is the pair corresponding to $S(t)$, then $v_2 = \Omega = (p, q, r)^T$ physically corresponds to the angular velocity of the rigid body, while $v_1 = V = (u, v, w)^T$ is the linear velocity of the origin O_m of the frame $\mathcal{R}_m$ with respect to the inertial frame $\mathcal{R}_f$, both the linear and angular velocities are expressed in the body fixed frame $\mathcal{R}_m$.

The kinematic relationship between the different velocities are given by:

$$\begin{pmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{pmatrix} = \mathbf{R}\mathbf{V} = \begin{pmatrix} R(\eta_2) & 0_{3 \times 3} \\ 0_{3 \times 3} & J(\eta_2) \end{pmatrix} \begin{pmatrix} V \\ \Omega \end{pmatrix} \quad (2.14)$$

where $\mathbf{R}$ is the rotation matrix defined by Eq. (2.4) and $\mathbf{J}(\eta_2)$ is defined by:

$$J(\eta_2) = \begin{pmatrix} 1 & 0 & -\sin \theta \\ 0 & \cos \phi & \sin \phi \cos \theta \\ 0 & -\sin \phi & \cos \phi \cos \theta \end{pmatrix}^{-1} = \begin{pmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \frac{\sin \phi}{\cos \theta} & \frac{\cos \phi}{\cos \theta} \end{pmatrix} \quad (2.15)$$

This matrix presents a singularity for $\theta = \pm \frac{\pi}{2}$. During practical operations of the airship, the pitch angle of $\theta = \pm \frac{\pi}{2}$ is not likely to be encountered.

2.2.2 Euler Parameters

To avoid the singularity inherent to this representation, Euler parameters can be presented. They are unit quaternions and are represented by a normalized vector of four real numbers [7, 9, 15, 24]. Taking the vector part of a unit quaternion and normalizing it gives the rotational axis and the scalar part allows to obtain the angle of rotation.

2.2.2.1 Introduction

A general spatial displacement of a rigid body consists of a finite rotation about a spatial axis and a finite translation along some vector. The rotational and translational axes in general are not related to each other. It is often easier to describe a spatial displacement as a combination of a rotation and a translation motions, where the two axes are not related. However, the combined effect of the two partial transformations can be expressed as an equivalent unique screw displacement. The concept of a screw thus represents an ideal mathematical tool to analyze spatial transformation [177].

The Euler representation, a four parameter representation, is based on the idea that a transformation from one co-ordinate frame to another may be effected by a single rotation about a vector μ defined with respect to the reference frame. Let $\mathbf{q}$ denote the Euler parameters which are expressed by the rotation axis n and the

rotation angle μ about the axis as follows:

$$\mathbf{q} = \begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} \cos(\frac{\mu}{2}) \\ \sin(\frac{\mu}{2})n_x \\ \sin(\frac{\mu}{2})n_y \\ \sin(\frac{\mu}{2})n_z \end{pmatrix}, \quad 0 \leq \mu \leq 2\pi \quad (2.16)$$

The quaternion can also be written as:

$$\mathbf{q} = q_0 + q_1\hat{i} + q_2\hat{j} + q_3\hat{k} \quad (2.17)$$

q_0 is real, q_1, q_2, q_3 are imaginary. The following properties apply:

$$\begin{aligned} \hat{i}.\hat{i} &= -1 & \hat{j}.\hat{j} &= -1 & \hat{k}.\hat{k} &= -1 \\ \hat{i}.\hat{j} &= -\hat{j}.\hat{i} = \hat{k} & \hat{k}.\hat{i} &= -\hat{i}.\hat{k} = \hat{j} & \hat{k}.\hat{j} &= -\hat{j}.\hat{k} = -\hat{i} \end{aligned} \quad (2.18)$$

The quaternion product may be expressed in matrix form as:

$$\mathbf{q}.\mathbf{p} = \begin{pmatrix} q_0 & -q_1 & -q_2 & -q_3 \\ q_1 & q_0 & -q_3 & q_2 \\ q_2 & q_3 & q_0 & -q_1 \\ q_3 & -q_2 & q_1 & q_0 \end{pmatrix} \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix} \quad (2.19)$$

2.2.2.2 Propagation of Quaternion with Time

The configuration space $R^3 \times SO_3$ may be replaced by $R^3 \times S^3$ where

$$S^3 = \left\{ x \in R^4 \text{ such that } \|x\|_2 = 1 \right\}$$

is the unit sphere in R^4 for the Euclidean norm $\|\cdot\|_2$. The associated kinematics differential equations of Euler parameters define a vector field on S^3 (the 3 sphere on R^4) and are given by:

$$\begin{aligned} \dot{q} &= \frac{1}{2} (q_0 I_{3 \times 3} + Sk(q)) \Omega = \frac{1}{2} (q_0 \Omega - \Omega \times q) \\ \dot{q}_0 &= -\frac{1}{2} q^T \Omega \end{aligned} \quad (2.20)$$

where q is the vectorial part of the quaternion. Thus, the time derivative of Euler parameters can be determined at any configuration of the body if the angular velocity is given.

Remark 2.4 Equation of this form may be solved in a navigation system to keep track of the quaternion parameters which define body orientation. The quaternion parameters may then be used to compute an equivalent direction cosine matrix, or used directly to transform the measured specific force vector into the chosen reference frame.

2.2.2.3 Kinematic Model

Let us introduce $\eta = (\eta_1, \eta_2)^T$ where $\eta_1 = (x, y, z)^T$ is the position vector of the airship (expressed in the Earth fixed frame), $\eta_2 = (q_0, q_1, q_2, q_3)^T$ is the quaternion representing the orientation, V as the linear velocity of the origin and Ω as the angular velocity (expressed in the airship fixed frame):

$$V = \begin{pmatrix} u \\ v \\ w \end{pmatrix} \quad \Omega = \begin{pmatrix} p \\ q \\ r \end{pmatrix} \quad (2.21)$$

The kinematics of the lighter than air robot can be expressed in the following way:

$$\dot{\eta} = \begin{pmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{pmatrix} = \begin{pmatrix} R(\eta_2) & 0 \\ 0 & J(\eta_2) \end{pmatrix} \mathbf{V} = \mathbf{R}\mathbf{V} \quad (2.22)$$

where $\mathbf{V} = (V, \Omega)^T$.

The orientation matrices R and J are formulated as follows [167, 218]:

$$R(\eta_2) = \begin{pmatrix} 1 - 2(q_2^2 + q_3^2) & 2(q_1q_2 + q_3q_0) & 2(q_1q_3 - q_2q_0) \\ 2(q_1q_2 - q_3q_0) & 1 - 2(q_1^2 + q_3^2) & 2(q_2q_3 + q_1q_0) \\ 2(q_1q_3 + q_2q_0) & 2(q_2q_3 - q_1q_0) & 1 - 2(q_1^2 + q_2^2) \end{pmatrix} \quad (2.23)$$

or equivalently

$$R = (q_0^2 - q^T q) I_{3 \times 3} + 2q q^T + 2q_0 Sk(q) \quad (2.24)$$

and

$$J(\eta_2) = \frac{1}{2} \begin{pmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{pmatrix} \quad (2.25)$$

with $J(\eta_2)^T J(\eta_2) = \frac{1}{4} I_{3 \times 3}$. If the following diagonal matrix is defined:

$$\Delta_{6 \times 6} = \begin{pmatrix} \mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \frac{1}{4} \mathbf{I}_{3 \times 3} \end{pmatrix} \quad (2.26)$$

Then:

$$\mathbf{R}^T \mathbf{R} = \Delta_{6 \times 6} \quad (2.27)$$

giving way to the formulation for the left pseudo-inverse of $\mathbf{R}$ as

$$\mathbf{R}^+ = \Delta_{6 \times 6}^{-1} \mathbf{R}^T \quad (2.28)$$

This formulation is only achievable due to the fact that the quaternions are used instead of the Euler angles for attitude representation.

Remark 2.5 Euler parameters cannot be used in a Lagrangian approach of dynamic modeling since this 7 parameters configuration is not a minimal one.

2.3 Dynamics

A mathematical description of the flight dynamics must include the aerodynamic, structural and other internal dynamics effects that influence the response of the lighter than air robot to inputs control and external atmospheric disturbances [17, 23, 82, 89]. In this section, analytic expressions for the forces and moments on the lighter than air robot are derived. The forces and moments are referred to a system of body-fixed axes, centered at the lighter than air bow. There are in general two approaches in deriving equations of motion. One is to apply Newton-Euler's law and the other one is the Lagrange-Hamilton framework.

Remark 2.6 For preliminary analysis, an airship can be treated as a loaded beam subjected to bending forces from unequal distribution of weight and buoyancy and shear forces are calculated by dividing the airship body into many longitudinal segments. The computation of the static bending moments can be separated into three parts: vertical forces of distributed weight and buoyancy, longitudinal components of suspension rope tension and longitudinal forces due to increase of gas superpressure [133]. The contribution of the gondola weight is included in the first two parts. The aerodynamic bending moment can be computed from the effect of the empenage lift and transverse forces on the envelope (using Munk's momentum theory [154]).

The lighter than air platform considered in this book is a generic airship. It is a non rigid airship equipped with two vectorable engines on the sides of the gondola, one tail rotor and 4 control surfaces at the stern. This airship is controlled with elevator and rudder surfaces for controllability of the pitch and heading angles, respectively, at significant airspeeds. These surfaces are mounted at the rear of the airship. Bow and/or stern thrusters are also used for landing and docking operations.

Remark 2.7 Some simplifying assumptions are made in this derivation: the Earth fixed reference frame is inertial, the gravitational field is constant. The hull is considered as a solid. Elasticity is not considered in this derivation [10]. The motion of the Helium inside the hull is neglected. The volume of the lighter than air robot is considered constant. As the speed of a lighter than air robot is generally low, the coupling between dynamics and thermal phenomena are neglected and the density of the air is not locally modified by the system's motion.

2.3.1 Mass Characteristics

A significant difference of an airship from a typical aircraft is that its mass characteristics strongly depends on the change of altitude [105]. The pressure difference between the surrounding atmosphere and the inner gas (Helium) should be kept as constant as possible at each altitude level. This permanent pressure difference is required for maintaining the aerodynamic shape of the envelope under most operational conditions. As the atmospheric pressure changes with the altitude, it should be compensated by the internal pressure. For this purpose, the envelope is equipped with two air filled ballonets. The volume occupied by the inner gas and the ballonets represents the inner volume of the hull envelope and is nearly constant. By filling the ballonets with the air, they displace the volume of the inner gas, increasing the total pressure of the gas in the envelope. In general, the total mass of the airship can be expressed by the following equation:

$$m = m_{gas} + m_{Bal} + m_{ic} + m_F + m_P \quad (2.29)$$

where m_{gas} is the mass of the inner gas (Helium), m_{Bal} is the total mass of air ballonets, m_{ic} represents the mass of all internal components (skin, structures...), $m_F(t)$ the time varying fuel mass and finally m_P the payload mass. The mass of the Helium can be considered as constant if leakage through the hull's skin is insignificant. The mass of all internal components can be derived by accounting all elements of the airship as a consolidation of point and distributed masses [215, 217]. Each ballonet can be modeled as a fixed point with variable mass [42]. The volume of ballonets depends on change of the atmospheric pressure gradient. At sea level, where the atmospheric level is high, the ballonet volume has its maximum level and reduces with the increased altitude.

2.3.2 6 DOF Dynamics: Newton-Euler Approach

In this section, the dynamics model is defined as the set of equations relying the situation of the vehicle in its position, velocity and acceleration to the control vector. The forces and moments are referred to a system of body-fixed axes, centered at the bow N of the lighter than air robot.

Let the motion of the lighter than air robot be described by its velocity $\mathbf{V}$ a 6D vector including the linear $\mathbf{V}$ and angular $\mathbf{\Omega}$ velocities. Let the surrounding air be described by an wind velocity V_a . The wind coordinates vector is defined as: $\eta_w = (x_w \ y_w \ z_w \ \phi_w \ \theta_w \ \psi_w)^T$ whose time derivative is relative to the wind velocity

$$\dot{\eta}_w = \mathbf{R}\mathbf{V}_w \quad (2.30)$$

The robot has thus a relative air velocity $V_a = V - V_w$. As the vector from point N , being the bow to point G , being the center of gravity, is represented by $\overline{OG} =$

$\overline{ON} + \overline{NG}$ where

$$\overline{NG} = (x_g \quad 0 \quad z_g)^T$$

Both velocities are related via:

$$V_G = V - Sk(\overline{NG}) \Omega \quad (2.31)$$

where $Sk(\overline{NG})$ is the skew matrix related to $\overline{NG}$.

2.3.2.1 Newton-Euler Equations

Newton's laws of motion are used to relate the applied forces and moments to the resulting translational and rotational accelerations. Translational and rotational motions are considered in the sequel.

Translational Motion Concerning forces, the Newton-Euler equations of motion may be given by the following relations:

$$\sum F_{ext} = \left. \frac{dP_d}{dt} \right|_{\mathcal{R}_f} = \left. \frac{dP_d}{dt} \right|_{\mathcal{R}_m} + \Omega \times P_d \quad (2.32)$$

The generalized force vector is given by F_{ext} while the linear momentum is given by:

$$P_d = mV_G = m(V - Sk(\overline{NG}) \Omega) \quad (2.33)$$

Rotational Motion Concerning moments, the Newton-Euler equations of motion may be given by the following relations:

$$\sum M_{ext} = \left. \frac{d\Pi_N}{dt} \right|_{\mathcal{R}_f} + V \times P_d = \left. \frac{d\Pi_N}{dt} \right|_{\mathcal{R}_m} + \Omega \times \Pi_N + V \times P_d \quad (2.34)$$

with

$$\Pi_N = \Pi_G + \overline{NG} \times mV_G = I_G \Omega + \overline{NG} \times mV_G = I_N \Omega + \overline{NG} \times mV \quad (2.35)$$

where Π_N and Π_G are the angular momentum computed respectively at the points N and G. From Koenig theorem, the inertia operator in N is calculated as:

$$I_N = I_G + m\overline{NG} \times (\Omega \times (\overline{NG})) \quad (2.36)$$

Thus, these forces and torques equations may be written as:

$$\begin{aligned} \sum F_{ext} &= m\dot{V}_{/\mathcal{R}_m} - mSk(\overline{NG}) \dot{\Omega}_{/\mathcal{R}_m} + \dot{m}V_{/\mathcal{R}_m} - \dot{m}Sk(\overline{NG}) \Omega_{/\mathcal{R}_m} \\ &\quad + m\Omega \times V_{/\mathcal{R}_m} - m\Omega \times Sk(\overline{NG}) \Omega_{/\mathcal{R}_m} \end{aligned} \quad (2.37)$$

and

$$\begin{aligned} \sum M_{ext} = I_N \dot{\Omega}_{/\mathcal{R}_m} + mSk(\overline{NG}) \dot{V}_{/\mathcal{R}_m} + \Omega \times I_N \Omega \\ + \Omega \times mSk(\overline{NG}) V + M_{var} \end{aligned} \quad (2.38)$$

where

$$M_{var} = \dot{I}_N \Omega_{/\mathcal{R}_m} + \dot{m}Sk(\overline{NG}) V_{/\mathcal{R}_m} + V \times mV - V \times mSk(\overline{NG}) \Omega \quad (2.39)$$

The total kinetic energy W of the mechanical system is thus defined as:

$$W = \frac{1}{2} \begin{pmatrix} V_G \\ \Omega \end{pmatrix}^T \begin{pmatrix} P_d \\ \Pi \end{pmatrix} = \frac{1}{2} \begin{pmatrix} V \\ \Omega \end{pmatrix}^T \mathbf{M}_{RB} \begin{pmatrix} V \\ \Omega \end{pmatrix} \quad (2.40)$$

where:

- $\mathbf{M}_{RB} = \begin{pmatrix} mI_{3 \times 3} & -mSk(NG) \\ mSk(NG) & I_N \end{pmatrix}$ is the lighter than air robot rigid body inertia matrix, m being the lighter than air robot total mass.
- $I_N = \begin{pmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{zx} & 0 & I_{zz} \end{pmatrix}$ is the moment of inertia, assuming the symmetry about the XZ plane.

Remark 2.8 The terms $\Omega \times mV - \Omega \times mSk(NG)\Omega$ are considered as Coriolis and centrifugal forces while the Coriolis and centrifugal moments terms are $\Omega \times mSk(NG)V + V \times mV - V \times mSk(NG)\Omega + \Omega \times I_N \Omega$.

Taking into account these assumptions, the analytic expressions of forces and moments acting on the lighter than air robot expressed in body-fixed frame $\mathcal{R}_m$, can be given by:

$$\begin{aligned} \mathbf{M}_{RB} \dot{\mathbf{V}} &= \mathbf{C}(\mathbf{V})\mathbf{V} + \tau_s + \tau_a + \tau_p + \mathbf{M}_d \mathbf{V} \\ \dot{\eta} &= \mathbf{R}\mathbf{V} \\ \mathbf{V} &= \begin{pmatrix} V \\ \Omega \end{pmatrix} \end{aligned} \quad (2.41)$$

where:

- The Coriolis and centrifugal tensor $T_c = \mathbf{C}(\mathbf{V})\mathbf{V}$ is given by

$$T_c = \begin{pmatrix} \Omega \times mV - \Omega \times mSk(\overline{NG})\Omega \\ \Omega \times I_N \Omega + V \times mV + \Omega \times mSk(\overline{NG})V - V \times mSk(\overline{NG})\Omega \end{pmatrix}$$

where

$$\mathbf{C}(\mathbf{V}) = \begin{pmatrix} 0_{3 \times 3} & -mSk(V + \Omega \times \overline{NG}) \\ -mSk(V + \Omega \times \overline{NG}) & -Sk(I_N \Omega) - mSk(NG)V \end{pmatrix}$$

is the Coriolis-centrifugal matrix associated to the lighter than air robot body mass, depending on its local velocities.

- τ_s is the static tensor due to the weight and lift forces.
- τ_p is the propulsion system tensor.
- τ_a is the aerodynamic tensor due to the atmosphere-lighter than air robot interaction and depends on the relative velocity.
- $\mathbf{M}_d = \begin{pmatrix} \dot{m}I_{3 \times 3} & -\dot{m}Sk(\overline{NG}) \\ \dot{m}Sk(\overline{NG}) & \dot{I}_N \end{pmatrix}$

In presence of wind with velocity V_W , the aerodynamic forces and moment depend on the airship relative linear velocity v_{1a} with respect to wind velocity, $V_a = V - V_W$, and hence $V_A = (V_a, \Omega)^T$.

2.3.2.2 Static Tensor

The gravity and lift forces must be considered in the following expressions:

$$F_{gravity} = R^T \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix} \quad (2.42)$$

and

$$F_{lift} = -R^T \begin{pmatrix} 0 \\ 0 \\ B = \rho \cdot Vol \cdot g \end{pmatrix} \quad (2.43)$$

where B is the buoyancy force, ρ is the volumic mass of the inner gas, Vol is the lighter than air robot volume and g is the gravity acceleration.

The static force vector is given by the difference between the lighter than air robot weight (acting at the center of gravity G) and the lift force (acting upwards on the center of lift C). The static moment is given by the following relation

$$M_s = \overline{NG} \times F_{gravity} + \overline{NC} \times F_{lift} \quad (2.44)$$

where C represents the center of lift in the body fixed frame. Thus the static tensor can be given by:

$$\tau_s = \begin{pmatrix} F_{gravity} + F_{lift} \\ \overline{NG} \times F_{gravity} + \overline{NC} \times F_{lift} \end{pmatrix} \quad (2.45)$$

Remark 2.9 In hover mode, the lighter than air robot stability is mainly affected by its center of lift in relation to the center of gravity which can be adjusted to obtain either stable, neutral or unstable conditions. Putting all weight on the top would create a highly unstable lighter than air robot with a tendency to roll over in a stable position.

2.3.2.3 Propulsive Tensor

Actuators provide the means for maneuvering the lighter than air robot along its course, propelled by thrust. The required thrust and power change with flight conditions according to:

$$\begin{aligned} T_{req} &= \frac{1}{2} \rho(h) V^2 U_H^{2/3} C_{D0} \\ P_{req} &= \frac{T_{req}}{\nu} \end{aligned} \quad (2.46)$$

where ν is the efficiency of the propellers, and $U_H^{2/3}$ is the standard aerodynamic reference area for airships.

Propellers are designed to exert thrust to drive the vehicle forward. The most popular propulsion system layout for pressurized lighter than air robot is twin ducted propellers mounted either side of the envelope bottom. Another one may exist in the stern for torque correction and attitude control. Hence, the lighter than air robot is an under-actuated system with two types of control: forces generated by thrusters and angular input μ_m controlling the direction of the thrusters. The lighter than air robot actuators list may be split into the following two sets:

1. Force inputs are available from the vectored two main propellers on each side of the gondola, providing a complementary lift to oppose the weighting mass, as well as a forward thrust controlling the longitudinal speed. When a differential input is added between two propellers, they also provide torque to control the rolling motion near hover. Finally a stern lateral thruster may be present to provide yaw control at low airspeed.
2. Surface deflections of the tail, when a minimum airspeed is present, provide torque inputs mostly for the control of the pitching and yawing motions.

When the air is perfectly still, the hover control is reduced to the use of the first set only.

Therefore for the first set, the propulsion torques can be written as:

$$\tau_p = \begin{pmatrix} F_M \cos \mu \\ F_T \\ -F_M \sin \mu \\ 0 \\ F_M O_z \cos \mu + F_M O_x \sin \mu \\ F_T O_y \end{pmatrix} \quad (2.47)$$

where F_M , F_T represent respectively the main and tail torques, μ is the tilt angle of the main engine, the vector $\begin{pmatrix} O_x \\ 0 \\ O_z \end{pmatrix}$ is the position of the main engines while the vector $\begin{pmatrix} O_y \\ 0 \\ 0 \end{pmatrix}$ represent the position of the stern rotor. The surface deflections are considered in the aerodynamic tensor presented in the following paragraph.

Remark 2.10 Maximum in-flight incremental acceleration load factors owing to worst gusts or maneuvering, in general, may be: 0.75 g vertical, 0.5 g side and 0.5 g longitudinal.

2.3.2.4 Aerodynamic Tensor

The total aerodynamic forces and moments are related to

- Non stationary terms related to translational and rotational accelerations
- Terms involving translation-rotation products equivalent to Coriolis forces
- Terms involving rotation-rotation products equivalent to centrifugal forces
- Terms involving translation-translation products equivalent to stationary phenomena

Some elements of the theory of slender bodies are given in the following references [11, 12, 36, 64, 89, 108, 123, 207, 231].

Stationary Phenomena A pure translation depending aerodynamic tensor is considered. This phenomena arises from the forces and moment coming from the distribution of the pressure around the body and also the friction forces due to the viscosity of the air [123, 154, 207]. This tensor can be expressed as:

$$\tau_{sta} = \frac{1}{2} \rho v_a^2 S_{ref} \begin{pmatrix} C_T \\ C_L \\ C_N \\ C_l \\ C_n \\ C_m \end{pmatrix} \quad (2.48)$$

where $C_L, C_N, C_T, C_l, C_m, C_n$ are aerodynamic coefficients depending on the geometrical shape of the airship and the positions of the control surfaces: rudders and elevators. These coefficients may be calculated in two different ways. The first one, is a pure experimental procedure which consists on collecting data in a wind tunnel, the second one, is an analytical procedure based on geometric quantities procedure [89, 113, 120, 131, 135, 137, 139, 149].

Remark 2.11 The bare hull of classic streamline form is directionally unstable, tending to turn broadside on the direction of motion. However, the ellipsoidal form remains a good compromise between structural and aerodynamic requirements for in-flight motion.

All aerodynamic lift and drag forces result from the combination of shear and pressure forces. Drag forces that are function of lift are known as induced drag.

Drag forces not strongly related to lift are usually known as parasite drag or zero lift drag [63].

If the theoretical pressure forces in a perfect fluid are integrated over a streamlined body without flow separation, the pressure over a body that yields a drag force in the flight direction should be exactly matched by the pressure around the body which yield a forward force. Thus, if skin friction is ignored, the net drag would be zero. This is known to be false, because even if the lift of a body of revolution is less than the lift due to a wing, it is different from zero.

The boundary layer which is produced by viscosity causes the flow to separate somewhere on the back half of the lighter than air robot. This prevents the full attainment of the forward acting force leaving a net drag force due to viscous separation. This drag force depends upon the location of the separation point of the body. This separation point is function of the curvature of the body and the amount of energy in the flow. Thus, Kirchoff's theory must be completed by the theory of slender bodies, initiated by Munk (1924) [154]. This theory is expressed for bodies whose thickness ratio $\ll 1$ (ratio between the maximum diameter of the hull and its length). Typically airship hulls experience skin friction drag, form (or pressure) drag and induced drag.

Owing to the large surface area of the airship, skin friction is the largest fraction of the total drag. For this reason, the reference area S_{ref} (the wetted area) is in general approximated as the surface area. However, another approach is to consider the surface area as $(Buoyant \cdot Volum)^{2/3}$.

In the stationary part, two additive terms appear [89, 208, 217]:

- The first one for low values of the angle of attack α

$$\begin{aligned} C_{L1} &= C_1 \cos \alpha \sin 2\beta \\ C_{N1} &= C_1 \sin 2\alpha \cos^2 \beta \\ C_{n1} &= C_0 \cos \alpha \sin 2\beta \\ C_{m1} &= C_2 \sin 2\alpha \cos^2 \beta \end{aligned} \quad (2.49)$$

β being the side-slip angle

- The second one in turbulent situation, augmenting rapidly with the angle of attack

$$\begin{aligned} C_{L2} &= C_3 \sin \beta \sqrt{\sin^2 \alpha \cos^2 \beta + \sin^2 \beta} \\ C_{N2} &= C_3 \sin \alpha \cos \beta \sqrt{\sin^2 \alpha \cos^2 \beta + \sin^2 \beta} \\ C_{l2} &= (C_4 \sin \alpha + C_{10} \cos \alpha) \sin 2\beta \\ C_{m2} &= C_5 \sin \alpha \cos \beta \sqrt{\sin^2 \alpha \cos^2 \beta + \sin^2 \beta} \\ C_{n2} &= C_9 \sin \beta \sqrt{\cos^2 \alpha \cos^2 \beta + \sin^2 \beta} \end{aligned} \quad (2.50)$$

Finally, the following general relations are obtained:

$$\left(\begin{array}{c} C_L = C_{L1} + C_{L2} \\ C_N = C_{N1} + C_{N2} \\ C_T = \left[C_6 \left(\text{sign}(C_L) \sqrt{C_N^2 + C_L^2} + C_7 \right)^2 + C_8 \right] \cos^2 \alpha \cos^2 \beta \\ C_l = C_{l2} \\ C_n = C_{n1} + C_{n2} \\ C_m = C_{m1} + C_{m2} \end{array} \right) \quad (2.51)$$

where the C_i , $i = 0..10$ are constants depending on the reference area, the separation point location, the base surface at this point and the diameter function of the lighter than air robot.

Non Stationary Aerodynamic Phenomena When the lighter than air robot moves, the air close to its body is moved. Contrary to the other aerial vehicles, the mass of displaced air is close to those of the lighter than air robot and consequently cannot be neglected. The displaced air mass is known as *added mass* or *virtual mass*. Added mass and inertia coefficients received their name because they can be linearly combined with the true lighter than air robot mass in the equations of motion to form one coefficient [11, 36, 42, 64, 80, 137, 207].

Each component of the airship is represented by an ellipsoid with three independently sized principal axes, this allows the added masses to be calculated analytically. Ellipsoid geometry, orientation and relative location are chosen so that both added masses and added moments of inertia are optimally modeled. Interference effects between the main hull component and an appendage are approximately accounted for by using the flow field around a replacement ellipsoid for the hull to modify the flow at the appendage, interference effects between appendages are neglected. The analysis uses incompressible potential flow theory. It does not account for any circulation in the flow [214].

The added mass matrix is, in general, an extra-diagonal matrix:

$$\mathbf{M}_A = - \begin{pmatrix} X_{\ddot{u}} & X_{\ddot{v}} & X_{\ddot{w}} & X_{\ddot{p}} & X_{\ddot{q}} & X_{\ddot{r}} \\ Y_{\ddot{u}} & Y_{\ddot{v}} & Y_{\ddot{w}} & Y_{\ddot{p}} & Y_{\ddot{q}} & Y_{\ddot{r}} \\ Z_{\ddot{u}} & Z_{\ddot{v}} & Z_{\ddot{w}} & Z_{\ddot{p}} & Z_{\ddot{q}} & Z_{\ddot{r}} \\ K_{\ddot{u}} & K_{\ddot{v}} & K_{\ddot{w}} & K_{\ddot{p}} & K_{\ddot{q}} & K_{\ddot{r}} \\ M_{\ddot{u}} & M_{\ddot{v}} & M_{\ddot{w}} & M_{\ddot{p}} & M_{\ddot{q}} & M_{\ddot{r}} \\ N_{\ddot{u}} & N_{\ddot{v}} & N_{\ddot{w}} & N_{\ddot{p}} & N_{\ddot{q}} & N_{\ddot{r}} \end{pmatrix} \quad (2.52)$$

The inertial effects of this added mass constitute the first component of the aerodynamic tensor. Another part of the aerodynamic forces is coming from the translation-rotation and rotation-rotation coupling motions and can be assimilated to Coriolis-centrifugal effects associated to the added mass and can also be represented as a damping effect representation. Due to the importance of the added mass of the

lighter than air robot, this tensor must be included. In addition, a pure translation depending aerodynamic tensor is considered. The total added masses of the vehicle are taken to be the sum of the added masses of each of the component ellipsoids, except that interference effects between the hull and fins are accounted for first. The interference effects can be modeled approximately by superimposing a uniform velocity over a relatively small ellipsoid which finds itself in the presence of a larger one; this interference velocity is the velocity field around the large ellipsoid evaluated at the location of the centroid of the smaller one, without the smaller ellipsoid being present.

The added mass matrix of a rigid body lighter than air robot must include the contributions of both the hull and the fins as:

$$M_{A_{rigid}} = M_{AH} + M_{AF} \quad (2.53)$$

It is in general acknowledged that added mass coefficients are not strongly dependent on viscous effects (such as circulation and boundary layer growth). Indeed, many of the experimental techniques currently in use for determining the coefficients make this assumption since they ignore the dependence of added mass on both the orientation of the vehicle to the oncoming flow and time in an unsteady flow. A potential flow analysis, neglecting circulation, is an appropriate method for estimating the added mass coefficient.

Fortunately, many of the added mass derivatives contained in the general expressions for added mass are either zero or mutually related when the body has various symmetries.

Added Mass Derivatives for a Prolate Ellipsoid An ellipsoid considered with the origin at the center of the ellipsoid, can be described as:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad (2.54)$$

Here a, b, c are the semi-axes. A prolate spheroid is obtained by letting $b = c$ and $a > b$. Cross coupling terms will be zero due to body symmetry about three planes:

$$\begin{aligned} X_{\dot{u}} &= -m \frac{\alpha_0}{2 - \alpha_0} \\ Y_{\dot{v}} &= Z_{\dot{w}} = -m \frac{\beta_0}{2 - \beta_0} \\ K_{\dot{p}} &= 0 \\ N_{\dot{r}} &= M_{\dot{q}} = -\frac{1}{5} \frac{(b^2 - a^2)^2 (\alpha_0 - \beta_0)}{2(b^2 - a^2) - (b^2 + a^2)(\alpha_0 - \beta_0)} \end{aligned} \quad (2.55)$$

where the mass of the prolate spheroid is

$$m = \frac{4}{3} \pi \rho a b^2 \quad (2.56)$$

The eccentricity e is introduced

$$e^2 = 1 - \frac{b^2}{a^2} \quad (2.57)$$

Hence, the constants α_0 and β_0 can be calculated as

$$\begin{aligned} \alpha_0 &= \frac{2(1-e^2)}{e^3} \left(\frac{1}{2} \ln \left(\frac{1+e}{1-e} \right) - e \right) \\ \beta_0 &= \frac{1}{e^2} - \left(\frac{1-e^2}{2e^3} \ln \left(\frac{1+e}{1-e} \right) \right) \end{aligned} \quad (2.58)$$

An alternative representation of these mass derivatives is presented by Lamb [123] who defines Lamb's k -factors as

$$\begin{aligned} k_1 &= \frac{\alpha_0}{2 - \alpha_0} \\ k_2 &= \frac{\beta_0}{2 - \beta_0} \\ k' &= \frac{e^4(\beta_0 - \alpha_0)}{(2 - e^2)(2e^2 - (2 - e^2)(\beta_0 - \alpha_0))} \end{aligned} \quad (2.59)$$

Hence the definition of the added mass derivatives simplifies to

$$\begin{aligned} X_{\dot{u}} &= -k_1 m \\ Y_{\dot{v}} &= Z_{\dot{w}} = -k_2 m \\ N_{\dot{r}} &= M_{\dot{q}} = -k' I_y \end{aligned} \quad (2.60)$$

where the moment of inertia of the prolate spheroid is

$$I_y = I_z = \frac{4}{15} \pi \rho a b^2 (a^2 + b^2) \quad (2.61)$$

The lighter than air robot hull is generally responsible for the greatest contribution to the added mass coefficient. In practice, a simple approach to obtain the added mass and moments of inertia of the hull is to approximate the hull as a set of ellipsoids of revolution. For each ellipsoid, all the off-diagonal terms in the added mass matrix are considered to be zero.

Fins added masses are calculated semi-empirically using flat plate models. The added mass and moment of inertia of the fins can be computed by integrating the 2D added mass of the cross section over the fin region. The contribution of the fins to these 2D added mass terms can be written as:

$$\begin{aligned} \bar{m}_{F,22} &= \bar{m}_{F,33} = \rho \pi \left(b - \frac{R_s^2}{b} \right)^2 \\ \bar{m}_{F,44} &= \frac{2}{\pi} k_{44} \rho b^4 \end{aligned} \quad (2.62)$$

where R_s is the hull cross sectional radius and b is the fin semi span. The factor k_{44} is a function of R_s/b . The non-zero elements in the added mass matrix of the fins are obtained from the following integrals:

$$m_{F,22} = m_{F,33} = \eta_f \int_{x_{FS}}^{x_{FE}} (\bar{m}_{F,22} dx) \quad (2.63)$$

$$m_{F,35} = -m_{F,26} = m_{F,33} = \eta_f \int_{x_{FS}}^{x_{FE}} (\bar{m}_{F,22} x dx) \quad (2.64)$$

$$m_{F,44} = \eta_f \int_{x_{FS}}^{x_{FE}} (\bar{m}_{F,44} dx) \quad (2.65)$$

$$m_{F,55} = m_{F,66} = \eta_f \int_{x_{FS}}^{x_{FE}} (\bar{m}_{F,22} x^2 dx) \quad (2.66)$$

where x_{FS}, x_{FE} are respectively the x coordinates of the start and end positions of the fins. An efficiency factor η_f is included to account for 3D effects.

Complete Lighter than Air Robot Model To make an operational lighter than air robot, it is necessary to provide control surfaces and propulsion units (in the gondola). Each of these items has its own drag modified when it is fitted to the hull as it is then operating in the flow field of the hull and vice-versa. The ailerons modify all the forces and moments coefficients. However, correctly dimensioned and situated, they stabilize the streamlined body. The gondola introduces an asymmetry in the geometry of the vehicle. It influences all the forces sometimes only modifying the first and second term introduced above through the coefficients C_i and sometimes adding a third term independent of the angle of attack α and the skid angle β .

Hence, the aerodynamical tensor, defined by the contribution of all aerodynamic phenomena, can be synthesized as: Coriolis-centrifugal effects associated to the added mass and can also be represented as an damping effect representation, i.e. $D(\mathcal{Q})V_A$. Moreover, the pure translation motion action tensor, $\tau_1 = (0_{1 \times 3}, (V \times M_{A_{rigid}} V)^T)^T$, is usually ignored in the other aerial applications.

Using the Complementarity between the coefficients of Kirchoff and Bryson theories, the following damping coefficient $\mathbf{D} = [D_V \ D_{\mathcal{Q}}]^T$, is obtained:

$$D_V = \begin{pmatrix} 0 & a_{22}r & -a_{33}q \\ pm_{13} + r(x_{m11} - a_{11}) & 0 & a_{33}p \\ (a_{11} - x_{m22})q & -a_{22}p & 0 \\ pm_{33} + r(a_{15} + x_{m13}) & -(a_{62} + a_{35})q & (a_{62} + a_{35})r + a_{24}p \\ (a_{35} + x_{m22})q & -a_{42}r + a_{62}p & -a_{15}q \\ -(a_{51} + a_{24} - x_{m13})p - (a_{26} - x_{2m11})r & (a_{15} + a_{42})q & -a_{53}p \end{pmatrix}$$

$$D_{\Omega} = \begin{pmatrix} a_{24}r & -a_{35}q & a_{26}r \\ a_{35}q & -a_{15}q & 0 \\ -a_{24}p - a_{26}r & a_{15}q & 0 \\ -a_{64}q & (a_{55} - a_{66})r & 0 \\ a_{64}p + (a_{66} - a_{44})r & 0 & -a_{64}r \\ (a_{44} - a_{55})q & a_{46}r & 0 \end{pmatrix} \quad (2.67)$$

The aerodynamic torques are thus given by the following relation:

$$\tau_a = \mathbf{M}_{RB} \dot{\mathbf{V}}_A + \mathbf{D}(\Omega) \mathbf{V}_A + \tau_{sta} \quad (2.68)$$

2.3.2.5 Complete Model with Euler Angles

The complete mechanical-aerodynamical lighter than air robot model can be given by:

$$\mathbf{M} \dot{\mathbf{V}} = \mathbf{M}_{RB} \dot{\mathbf{V}}_w + \mathbf{C}(\mathbf{V}) \mathbf{V} + \tau_s + \mathbf{D}(\Omega) \mathbf{V}_A + \tau_{sta} + \tau_p + \mathbf{M}_d \mathbf{V} \quad (2.69)$$

$$\dot{\eta} = \mathbf{R} \mathbf{V} \quad (2.70)$$

where $\mathbf{M} = \mathbf{M}_{A_{rigid}} + \mathbf{M}_{RB}$ is the total mass matrix.

Remark 2.12 For a system with added masses, the term $\mathbf{V} \times \mathbf{M} \mathbf{V}$ is different from zero. The terms $\mathbf{V} \times \mathbf{M} \mathbf{V}$, $\Omega \times \mathbf{M} \mathbf{V}$, $\mathbf{V} \times \mathbf{M} \Omega$ and $\Omega \times \mathbf{M} \Omega$ show the centrifugal and Coriolis components.

Another modeling approach is considered in the next section.

2.3.3 6 DOF Dynamics: Lagrange Approach

One advantage with the Lagrangian approach is to deal with two scalar energy functions $\mathbf{W}$ (kinetic energy) and E_p (potential energy). The Newtonian approach is vector oriented since everything is derived from Newton's second law. The Lagrangian description uses generalized coordinates (one for each degree of freedom), all of which must be independent. The equations of motion for lighter than air vehicles are derived in a Lagrangian framework. The Lagrangian approach has several distinctive features to the Newtonian approach. The derivation of the aerodynamic added inertia and the vehicle's rigid body equations of motion can be done in a common framework [123, 143, 150, 231]. The added inertia is given a physical interpretation when the vehicle-ambient air system is considered from an energy point of view instead of a force moment approach. Recall that:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\eta}} \right) - \frac{\partial L}{\partial \eta} + \frac{\partial P_d}{\partial \dot{\eta}} = \tau_{\eta} = \sum_{i=1}^m F_i(\eta) u_i \quad (2.71)$$

where L is the Lagrangian F defines the input directions and the additional term describes the dissipative forces P_d as a power function.

$$\frac{\partial P_d}{\partial \dot{\eta}} = D_{\eta}(\eta, v) \dot{\eta} \quad (2.72)$$

Forces dissipating energy are called dissipative forces: friction, air drag... The generalized coordinates are taken to be $\eta = (x, y, z, \phi, \theta, \psi)^T$ so that $\mathbf{V} = (u, v, w, p, q, r)^T$ are dependent variables related to the generalized coordinates [11, 12, 41, 42, 64, 105, 207].

A rigid body moving in an unbounded fluid is holonomic due to, the unboundness and the infinite extent of the fluid. Hence a rigid body moving without constraints in 6 degrees of freedom is an ordinary Lagrangian system. However, the infinite degrees of freedom system by the ambient air particles cannot be said to be holonomic.

The Lagrangian for the vehicle-ambient air system is defined as:

$$L = W_{RB} + W_A - E_p \quad (2.73)$$

where W_{RB} is the rigid body kinetic energy, W_A is the fluid kinetic energy and E_p is the potential energy defined implicitly by:

$$\frac{\partial E_p}{\partial \eta} = \tau_s(\eta) \quad (2.74)$$

Hence the total kinetic energy can be expressed as:

$$W_{RB} + W_A = \frac{1}{2} \dot{\eta}^T \mathbf{R}^{-T} (M_{RB} + M_{A_{rigid}}) \mathbf{R}^{-1} \dot{\eta} \quad (2.75)$$

where the following notation is used: $\mathbf{R}^{-T} = (\mathbf{R}^{-1})^T$.

The equations of motion for lighter than air vehicles can be written as follows:

$$\begin{aligned} \mathbf{M}\dot{\mathbf{V}} + \mathbf{C}(\mathbf{V})\mathbf{V} + \mathbf{D}(\mathbf{V})\mathbf{V} + \tau_s(\eta) &= \varpi(\phi) + \tau_a + \tau_p \\ \dot{\eta} &= \mathbf{R}\mathbf{V} \end{aligned} \quad (2.76)$$

$\mathbf{M}$ is the inertia matrix including the aerodynamic virtual inertia (added mass), $\mathbf{C}(\mathbf{V})\mathbf{V}$ contains the nonlinear forces and moments due to centrifugal and Coriolis forces and $\mathbf{D}(\mathbf{V})\mathbf{V}$ is the vehicle damping matrix where the potential damping and the viscous effects are lumped together. $\tau_s(\eta)$ is a vector containing the restoring terms formed by the vehicle's buoyancy and gravitational terms, $\varpi(\phi)$ is the atmospheric disturbance vector and $\tau_a + \tau_p$ is a tensor containing the aerodynamic and propulsion tensors. $\mathbf{R}$ is the velocity transformation matrix which transforms velocities from the body fixed frame $\mathcal{R}_m$ to the Earth fixed reference frame $\mathcal{R}_f$.

2.3.3.1 Kinetic Energy

The kinetic energy of a solid in motion can be written as:

$$W_{RB}(\eta, \dot{\eta}) = \frac{1}{2} \int \int \int \left[\rho (V + (\Omega \times r))^T (V + (\Omega \times r)) \right] dv \quad (2.77)$$

or equivalently:

$$W_{RB}(\eta, \dot{\eta}) = \frac{1}{2} \dot{\eta}^T \mathbf{M}_{RB} \dot{\eta} \quad (2.78)$$

with the symmetric positive definite matrix:

$$\mathbf{M}_{RB} = \begin{pmatrix} m & 0 & 0 & 0 & mz_g & -my_g \\ 0 & m & 0 & -mz_g & 0 & mx_g \\ 0 & 0 & m & my_g & -mx_g & 0 \\ 0 & -mz_g & my_g & I_x & -I_{xy} & -I_{xz} \\ mz_g & 0 & -mx_g & -I_{xy} & I_y & -I_{yz} \\ -my_g & mx_g & 0 & -I_{xz} & -I_{yz} & I_z \end{pmatrix} \quad (2.79)$$

The radiation induced forces and moments can be identified as the sum of three components:

- Added mass due to the inertia of the surrounding fluid
- Radiation induced potential damping due to the energy carried away by the wind
- Weight and buoyancy (Archimedes) forces

Added mass should be understood as pressure-induced forces and moments due to a forced harmonic motion of the body which are proportional to the acceleration of the body. In order to allow the vehicle to pass through the air, the fluid must move aside and then close behind the vehicle. As a consequence, the fluid passage possesses kinetic that it would lack if the vehicle was not in motion. The kinetic energy of the air particles generated by a vehicle in motion can be written as:

$$W_A(\eta, \dot{\eta}) = \frac{1}{2} \dot{\eta}^T \mathbf{M}_{A_{rigid}} \dot{\eta} \quad (2.80)$$

with $\mathbf{M}_{A_{rigid}}$ the added mass and inertia matrix, symmetric positive definite. Mathematically, added mass is the proportionality constant relating the kinetic energy in the air surrounding the vehicle to the square of the vehicle speed, in the same way that vehicle mass relates vehicle kinetic energy to speed squared. During vehicle accelerations, both the kinetic energy of the vehicle and the air are changed, their ratio remaining constant in an ideal fluid. Therefore, an accelerating vehicle must overcome the effective inertia of both the vehicle and added mass. The added mass for a lighter than air robot can be equivalent to the actual vehicle mass, and so must be accounted for properly.

Remark 2.13 The air is assumed to be an incompressible and homogeneous fluid of infinite extent.

2.3.3.2 Lighter than Air Robot Model

Let the motion of the lighter than air robot be described by its inertial velocity $\mathbf{V}$ a 6D vector including the inertial linear V and angular Ω velocities. Let the surrounding air be described by an inertial wind velocity $\mathbf{V}_W = (V_W, \Omega_W)^T$. The lighter than air robot has thus a relative air velocity $\mathbf{V}_a$:

$$\mathbf{V}_a = \mathbf{V} - \mathbf{V}_W \quad (2.81)$$

The total kinetic energy is defined as a sum [9, 80, 150, 206, 214]:

$$\mathbf{W} = \mathbf{W}_G + \mathbf{W}_B + \mathbf{W}_V \quad (2.82)$$

Accounting for:

- $\mathbf{W}_G$: The kinetic energy of the vehicle expressed in the center of gravity G located at a distance $\overline{NG}$ from the body fixed frame center N:
- $\mathbf{W}_B$: The kinetic energy added to the buoyancy air (displaced by the lighter than air robot volume) expressed in the center of lift C and where $\mathbf{M}_B$ is the generalized mass matrix of the buoyancy air. In the case of the buoyancy air, the kinetic energy is the difference between the extra kinetic energy and the contribution of the air mass motion.
- $\mathbf{W}_V$: The energy due to an extra virtual mass also expressed in C and where $\mathbf{M}_v$ is the generalized virtual mass matrix of the mass of air around the lighter than air robot and displaced with its relative motion in the air.

where

$$\mathbf{W}_G = \frac{1}{2} \mathbf{V}_G^T \mathbf{M}_G \mathbf{V}_G \quad (2.83)$$

with

$$\mathbf{W}_B = \frac{1}{2} \mathbf{V}_a^T \mathbf{M}_B \mathbf{V}_a - \frac{1}{2} \mathbf{V}^T \mathbf{M}_B \mathbf{V} \quad (2.84)$$

and

$$\mathbf{W}_v = \frac{1}{2} \mathbf{V}_a^T \mathbf{M}_v \mathbf{V}_a \quad (2.85)$$

where the generalized mass matrices being given by

$$\mathbf{M}_G = \begin{pmatrix} m I_{3 \times 3} & 0 \\ 0 & I_G \end{pmatrix} \quad \mathbf{M}_B = \begin{pmatrix} m_B I_{3 \times 3} & 0 \\ 0 & J_B \end{pmatrix} \quad \mathbf{M}_v = \begin{pmatrix} M_v & 0 \\ 0 & I_v \end{pmatrix} \quad (2.86)$$

Therefore, the total kinetic energy may be written as:

$$\mathbf{W} = \frac{1}{2} \mathbf{V}_G^T \mathbf{M}_G \mathbf{V}_G + \frac{1}{2} \mathbf{V}_a^T \mathbf{M}_v \mathbf{V}_a - \frac{1}{2} \mathbf{V}^T \mathbf{M}_B \mathbf{V} + \frac{1}{2} \mathbf{V}_a^T \mathbf{M}_B \mathbf{V}_a \quad (2.87)$$

All terms of the kinetic energy in the body-fixed frame consider that the linear speed of the center of gravity G is related to the linear speed of the body fixed frame center: the bow N , through the angular speed:

$$\mathbf{V}_G = \begin{pmatrix} I_{3 \times 3} & -Sk(\overline{NG}) \\ 0_{3 \times 3} & I_{3 \times 3} \end{pmatrix} \mathbf{V} \quad (2.88)$$

Substituting in the previous Eq. (2.87) gives:

$$\mathbf{W} = \frac{1}{2} \mathbf{V}^T \mathbf{M}_N \mathbf{V} - \frac{1}{2} \mathbf{V}^T \mathbf{M}_B \mathbf{V} + \frac{1}{2} \mathbf{V}_a^T \mathbf{M}_B \mathbf{V}_a + \frac{1}{2} \mathbf{V}_a^T \mathbf{M}_v \mathbf{V}_a \quad (2.89)$$

with

$$\mathbf{M}_N = \begin{pmatrix} m I_{3 \times 3} & -m Sk(\overline{NG}) \\ m Sk(\overline{NG}) & I_G - m (Sk(\overline{NG}))^2 \end{pmatrix} \quad (2.90)$$

Therefore,

$$\mathbf{W} = \frac{1}{2} (\mathbf{V} - \mathbf{V}_W)^T (\mathbf{M}_v + \mathbf{M}_B) (\mathbf{V} - \mathbf{V}_W) + \frac{1}{2} \mathbf{V}^T (\mathbf{M}_N - \mathbf{M}_B) \mathbf{V} \quad (2.91)$$

or

$$\mathbf{W} = \frac{1}{2} \mathbf{V}^T (\mathbf{M}_N + \mathbf{M}_v) \mathbf{V} + \frac{1}{2} \mathbf{V}_w^T (\mathbf{M}_B + \mathbf{M}_v) \mathbf{V}_w - \mathbf{V}^T (\mathbf{M}_B + \mathbf{M}_v) \mathbf{V}_w \quad (2.92)$$

Finally, the kinetic energy can be expressed as a function of the lighter than air robot and wind inertial velocities

$$\mathbf{W} = \mathbf{W}_1 + \mathbf{W}_2 + \mathbf{W}_3 \quad (2.93)$$

or equivalently:

$$\mathbf{W}_1 = \frac{1}{2} \mathbf{V}^T \mathbf{M}_{vn} \mathbf{V} \quad (2.94)$$

with

$$\mathbf{W}_2 = \frac{1}{2} \mathbf{V}_W^T \mathbf{M}_{Bv} \mathbf{V}_W \quad (2.95)$$

and

$$\mathbf{W}_3 = -\mathbf{V}^T \mathbf{M}_{Bv} \mathbf{V}_W \quad (2.96)$$

where

$$\mathbf{M}_{vn} = \mathbf{M}_N + \mathbf{M}_v = \begin{pmatrix} m I_{3 \times 3} + M_v & -m Sk(\overline{NG}) \\ m Sk(\overline{NG}) & I_G - m (Sk(\overline{NG}))^2 + I_v \end{pmatrix}$$

$$\mathbf{M}_{Bv} = \mathbf{M}_v + \mathbf{M}_B = \begin{pmatrix} m_B I_{3 \times 3} + M_v & 0 \\ 0 & J_B + I_v \end{pmatrix}$$

Therefore, the Lagrange-Hamilton equations of motion are applied to each of the three terms of the kinetic energy in the sequel.

2.3.3.3 First Term of the Kinetic Energy

The first term of the kinetic energy corresponding to the no wind case is then:

$$\mathbf{W}_1 = \frac{1}{2} \mathbf{V}^T \mathbf{M}_{vn} \mathbf{V} = \frac{1}{2} \dot{\eta}^T \mathbf{R}^{-T} \mathbf{M}_{vn} \mathbf{R}^{-1} \dot{\eta} \quad (2.97)$$

The partial derivative of this kinetic energy $\mathbf{W}_1$ relative to $\dot{\eta}$ is

$$\frac{\partial \mathbf{W}_1}{\partial \dot{\eta}} = \mathbf{R}^{-T} \mathbf{M}_{vn} \mathbf{R}^{-1} \dot{\eta} \quad (2.98)$$

since

$$\mathbf{M}_{vn}^T = \mathbf{M}_{vn} \quad (2.99)$$

Now taking the time derivative gives:

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial \mathbf{W}_1}{\partial \dot{\eta}} \right) &= \dot{\mathbf{R}}^{-T} \mathbf{M}_{vn} \mathbf{R}^{-1} \dot{\eta} + \mathbf{R}^{-T} \dot{\mathbf{M}}_{vn} \mathbf{R}^{-1} \dot{\eta} \\ &\quad + \mathbf{R}^{-T} \mathbf{M}_{vn} \dot{\mathbf{R}}^{-1} \dot{\eta} + \mathbf{R}^{-T} \mathbf{M}_{vn} \mathbf{R}^{-1} \ddot{\eta} \end{aligned} \quad (2.100)$$

Or equivalently:

$$\frac{d}{dt} \left(\frac{\partial \mathbf{W}_1}{\partial \dot{\eta}} \right) = \dot{\mathbf{R}}^{-T} \mathbf{M}_{vn} \mathbf{V} + \mathbf{R}^{-T} \dot{\mathbf{M}}_{vn} \mathbf{V} + \mathbf{R}^{-T} \mathbf{M}_{vn} \dot{\mathbf{V}} \quad (2.101)$$

Where the following relations are used:

$$\begin{aligned} \dot{\mathbf{R}}^{-1} \mathbf{R} + \mathbf{R}^{-1} \dot{\mathbf{R}} &= \mathbf{0}_{6 \times 6} \\ \dot{\mathbf{V}} &= \mathbf{R}^{-1} \ddot{\eta} - \mathbf{R}^{-1} \dot{\mathbf{R}} \mathbf{R}^{-1} \dot{\eta} \end{aligned} \quad (2.102)$$

The partial derivative of the kinetic energy $\mathbf{W}_1$ relative to η is thus:

$$\frac{\partial \mathbf{W}_1}{\partial \eta} = \frac{1}{2} \frac{\partial}{\partial \eta} \left(\dot{\eta}^T \mathbf{R}^{-T} \mathbf{M}_{vn} \mathbf{R}^{-1} \dot{\eta} \right) = \mathbf{K} \mathbf{M}_{vn} \mathbf{V} \quad (2.103)$$

with

$$\mathbf{K} = \frac{\partial}{\partial \eta} \left(\mathbf{R}^{-1} \dot{\eta} \right) \quad (2.104)$$

or

$$\left(\frac{\partial}{\partial \eta_1} (\mathbf{R}^{-1} \dot{\eta}) \right) = \begin{pmatrix} 0_{3 \times 3} & 0_{3 \times 3} \\ K_1 & K_2 \end{pmatrix} \quad (2.105)$$

The generalized force relative to the kinetic energy with no wind is then obtained:

$$\mathbf{F}_1(\eta, \dot{\eta}) = \mathbf{R}^{-T} \mathbf{M}_{vn} \dot{\mathbf{V}} + \left(\mathbf{R}^{-T} \dot{\mathbf{M}}_{vn} + \dot{\mathbf{R}}^{-T} \mathbf{M}_{vn} - \mathbf{K} \mathbf{M}_{vn} \right) \mathbf{V} \quad (2.106)$$

2.3.3.4 Second Term of the Kinetic Energy

The second term of the kinetic energy is computed, defining the wind coordinates vector as $\eta_W = (x_W, y_W, z_W, \phi_W, \theta_W, \psi_W)^T$ whose time derivative is related to the wind velocity:

$$\dot{\eta}_W = \mathbf{R} \mathbf{V}_W \quad (2.107)$$

The second term of the kinetic energy corresponds to:

$$\mathbf{W}_2 = \frac{1}{2} \mathbf{V}_W^T \mathbf{M}_{Bv} \mathbf{V}_W = \frac{1}{2} \dot{\eta}_W^T \mathbf{R}^{-T} \mathbf{M}_{Bv} \mathbf{R}^{-1} \dot{\eta}_W \quad (2.108)$$

with

$$\frac{d}{dt} \left(\frac{\partial \mathbf{W}_2}{\partial \dot{\eta}} \right) = \frac{d}{dt} \left(\frac{\partial}{\partial \dot{\eta}} \left(\frac{1}{2} \dot{\eta}_W^T \mathbf{R}^{-T} \mathbf{M}_{Bv} \mathbf{R}^{-1} \dot{\eta}_W \right) \right) = 0 \quad (2.109)$$

The partial derivative of the kinetic energy $\mathbf{W}_2$ relative to the generalized coordinate η is

$$\frac{\partial \mathbf{W}_2}{\partial \eta} = \frac{\partial}{\partial \eta} \left(\frac{1}{2} \dot{\eta}_W^T \mathbf{R}^{-T} \mathbf{M}_{Bv} \mathbf{R}^{-1} \dot{\eta}_W \right) \quad (2.110)$$

or

$$\frac{\partial \mathbf{W}_2}{\partial \eta} = \mathbf{K}_W \mathbf{M}_{Bv} \mathbf{V}_W \quad (2.111)$$

where

$$\mathbf{K}_W = \begin{pmatrix} 0_{3 \times 3} & 0_{3 \times 3} \\ \mathbf{K}_{W1} & \mathbf{K}_{W2} \end{pmatrix} \quad (2.112)$$

The generalized force relative to is then obtained:

$$\mathbf{F}_2 = -\mathbf{K}_W \mathbf{M}_{Bv} \mathbf{V}_W \quad (2.113)$$

2.3.3.5 Third Term of the Kinetic Energy

Applying the same procedure to the third term, $W_3 = -\dot{\eta}^T \mathbf{R}^{-T} \mathbf{M}_{Bv} \mathbf{V}_W$ we obtain:

$$\frac{d}{dt} \left(\frac{\partial \mathbf{W}_3}{\partial \dot{\eta}} \right) = -\dot{\mathbf{R}}^{-T} \mathbf{M}_{Bv} \mathbf{V}_W - \mathbf{R}^{-T} \mathbf{M}_{Bv} \dot{\mathbf{V}}_W \quad (2.114)$$

leading to:

$$\mathbf{F}_3(\eta, \dot{\eta}) = -\mathbf{R}^{-T} \mathbf{M}_{Bv} \dot{\mathbf{V}}_W - \dot{\mathbf{R}}^{-T} \mathbf{M}_{Bv} \mathbf{V}_W + \mathbf{K} \mathbf{M}_{Bv} \mathbf{V}_W + \mathbf{K}_W \mathbf{M}_{Bv} \mathbf{V} \quad (2.115)$$

2.3.3.6 Forces

Summing up all the generalized forces, we have:

$$\begin{aligned} \mathbf{F}(\eta, \dot{\eta}) = & \mathbf{R}^{-T} \mathbf{M}_{vn} \dot{\mathbf{V}} + \left(\mathbf{R}^{-T} \dot{\mathbf{M}}_{vn} + \dot{\mathbf{R}}^{-T} \mathbf{M}_{vn} - \mathbf{K} \mathbf{M}_{vn} \right) \mathbf{V} - \mathbf{K}_W \mathbf{M}_{Bv} \mathbf{V}_W \\ & - \mathbf{R}^{-T} \mathbf{M}_{Bv} \dot{\mathbf{V}}_W - \dot{\mathbf{R}}^{-T} \mathbf{M}_{Bv} \mathbf{V}_W + \mathbf{K} \mathbf{M}_{Bv} \mathbf{V}_W + \mathbf{K}_W \mathbf{M}_{Bv} \mathbf{V} \end{aligned} \quad (2.116)$$

Let's introduce

$$\mathbf{F}(\mathbf{V}) = \mathbf{R}^T \mathbf{F}(\eta, \dot{\eta}) \quad (2.117)$$

Thus

$$\begin{aligned} \mathbf{F}(\mathbf{V}) = & \mathbf{M}_{vn} \dot{\mathbf{V}} + (\Omega_6 + \mathbf{V}_6) \mathbf{M}_{vn} \mathbf{V} + \dot{\mathbf{M}}_{vn} \mathbf{V} - \mathbf{M}_{Bv} \dot{\mathbf{V}}_W \\ & - (\Omega_6 + \mathbf{V}_6) \mathbf{M}_{Bv} \mathbf{V}_W - \mathbf{V}_{W6} \mathbf{M}_{Bv} \mathbf{V} \end{aligned} \quad (2.118)$$

where

$$\mathbf{V}_6 = \begin{pmatrix} 0_{3 \times 3} & 0_{3 \times 3} \\ Sk(V) & 0_{3 \times 3} \end{pmatrix} \quad (2.119)$$

and

$$\Omega_6 = \begin{pmatrix} Sk(\Omega) & 0_{3 \times 3} \\ 0_{3 \times 3} & Sk(\Omega) \end{pmatrix} \quad (2.120)$$

and

$$\mathbf{V}_{w6} = \begin{pmatrix} 0_{3 \times 3} & 0_{3 \times 3} \\ Sk(V_W) & 0_{3 \times 3} \end{pmatrix} \quad (2.121)$$

where $Sk(V)$, $Sk(\Omega)$, $Sk(V_W)$ denote the antisymmetric cross product matrices corresponding to the vectors V , Ω , V_W and using the following equations

$$\begin{aligned} Sk(V) &= -J^T \mathbf{K}_1 \\ Sk(\Omega) &= J^T ((J)^{-T}) - \mathbf{K}_2 \\ Sk(V_W) &= -J^T \mathbf{K}_{w1} \end{aligned} \quad (2.122)$$

2.3.3.7 Potential Energy

The potential energy stored in the vehicle is the sum of the contributions relative to each subsystem [55, 74, 105]. The contribution due only to the gravitational force is expressed by:

$$U_g = - \int g_0^T p_i^* \rho dv \quad (2.123)$$

The gravitational force vector is given by the difference between the lighter than air robot weight (acting at the center of gravity) and the lift force (acting upwards on the center of lift):

$$\mathbf{F}_g = \frac{\partial L}{\partial \mathbf{q}}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{pmatrix} (mg - B) \sin \theta \\ -(mg - B) \cos \theta \sin \phi \\ -(mg - B) \cos \theta \cos \phi \\ Bz_b \cos \theta \sin \phi - By_b \cos \theta \cos \phi \\ Bx_b \cos \theta \cos \phi + Bz_b \sin \theta \\ -By_b \sin \theta - Bx_b \cos \theta \sin \phi \end{pmatrix} \quad (2.124)$$

where B is the buoyancy force, $\bar{C}N = (x_b, y_b, z_b)^T$ represents the position of the center of lift with respect to the body fixed frame. This leaves the final kinematics/wind torque equations as

$$\mathbf{T} = \mathbf{F}(\mathbf{V}) - \mathbf{F}_g \quad (2.125)$$

Both 6 degrees of freedom models are necessary for control purpose closed loop. However, for guidance purpose, the following 3 degrees of freedom gives enough information about the lighter than air robot motion, while keeping some simplicity.

2.3.4 Translational Dynamics

For the purpose of flight path generation, it is usually sufficient to treat only the translational motion. Using a point mass model for the lighter than air robot implies that the yaw with respect to the relative wind frame is always zero. This is also demonstrated by the absence of side slip in the aerodynamic model [126].

The translational equations of an aerial vehicle through the atmosphere are directly derived from Newton's law. In this derivation, the assumptions are that the Earth is non rotating and flat, and that the vehicle weight is constant. The external forces consist of the thrust, the gravity and buoyancy forces and the aerodynamic force. The gravity and buoyancy forces are treated in the same way.

The position of the lighter than air robot is described in the local coordinate system ENU with unit vectors $\mathbf{e}$, $\mathbf{n}$, $\mathbf{u}$ pointing East, North and Up. Derivation of kinematic equations involves three velocity concepts: inertial velocity, local velocity and wind-relative velocity. The flight path coordinate system is defined, that relates the velocity vector of the vehicle with respect to Earth to the geographic system. Two angles relate the velocity coordinates to the geographic system. The heading angle χ is measured from North to the projection of $\mathbf{V}$ (the aircraft velocity relative to the wind) in the local tangent plane and the flight path angle γ takes vertically up to $\mathbf{V}$. The wind relative velocity vector is defined by the airspeed V , the flight path angle γ and the heading χ . The variables x , y , z are the aircraft inertial coordinates. The x , y directions are chosen such that the xy plane is horizontal, the x -direction is aligned with the principal axis of symmetry and the z -direction is ascending vertically. The

lighter than air equations of motion are expressed in a velocity coordinate frame attached to the lighter than air robot, considering the velocity of the wind $W = \begin{pmatrix} W_x \\ W_y \\ W_z \end{pmatrix}$ (components of the wind velocity in the inertial frame). The frame matrix being given by:

$$R = \begin{pmatrix} \cos \gamma & 0 & \sin \gamma \\ 0 & 1 & 0 \\ -\sin \gamma & 0 & \cos \gamma \end{pmatrix} \begin{pmatrix} \cos(\frac{\pi}{2} - \chi) & \sin(\frac{\pi}{2} - \chi) & 0 \\ -\sin(\frac{\pi}{2} - \chi) & \cos(\frac{\pi}{2} - \chi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos \gamma \sin \chi & \cos \gamma \cos \chi & \sin \gamma \\ \cos \chi & -\sin \chi & 0 \\ \sin \gamma \sin \chi & \sin \gamma \cos \chi & \cos \gamma \end{pmatrix}$$

The velocity with respect to the local **ENU** frame is determined as:

$$V_{local} = V + W = R^T \begin{pmatrix} V \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} W_x \\ W_y \\ W_z \end{pmatrix}$$

It thus may be deduced that:

$$V_{local} = (V \sin \gamma + W_z) \mathbf{u} + (V \cos \gamma \sin \chi + W_x) \mathbf{e} + (V \cos \gamma \cos \chi + W_y) \mathbf{n}$$

The translational kinematics of an aerial vehicle taking into account the wind effect can thus be expressed by the following equations:

$$\begin{aligned} \dot{x} &= V \sin \chi \cos \gamma + W_x \\ \dot{y} &= V \cos \chi \cos \gamma + W_y \\ \dot{z} &= V \sin \gamma + W_z \end{aligned} \tag{2.126}$$

Three degrees of freedom dynamical models used in trajectory optimization for aerial vehicles typically include wing body aerodynamics force effects but ignore the aerodynamic force effects produced by the control surfaces. The implementation uses the translational equations of motion formulated from Newton's law and expressed in flight path coordinates. The state variables and their derivatives are the speed of the vehicle center of mass with respect to Earth V , $\dot{V}$, the heading angle and rate χ , $\dot{\chi}$ and the flight path angle and rate γ , $\dot{\gamma}$. Newton's law is applied with aerodynamic, propulsive and gravity forces as externally applied forces.

The acceleration expressed in the local frame is [201, 231]

$$\begin{pmatrix} \dot{V} \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} V \\ 0 \\ 0 \end{pmatrix} \times \left[\begin{pmatrix} \cos \gamma & 0 & -\sin \gamma \\ 0 & 1 & 0 \\ \sin \gamma & 0 & \cos \gamma \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \dot{\chi} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot{\gamma} \end{pmatrix} \right] = \begin{pmatrix} \dot{V} \\ \dot{\chi} V \cos \gamma \\ -\dot{\gamma} V \end{pmatrix}$$

The approach followed in [126, 147, 228, 231] is proposed in this section. All the external forces must be expressed in the wind axes coordinates. The contribution of

the added mass phenomenon is described as:

$$\begin{aligned}
 F_{AM} &= - \left(C_w^b \right)^T M_a C_w^b \begin{pmatrix} \dot{V} \\ \dot{\chi} V \cos \gamma \\ -\dot{\chi} V \end{pmatrix} \\
 &= - \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{pmatrix} \begin{pmatrix} \dot{V} \\ \dot{\chi} V \cos \gamma \\ -\dot{\chi} V \end{pmatrix} \quad (2.127)
 \end{aligned}$$

where the added mass matrix is given by $M_a = \begin{pmatrix} X_{\dot{u}} & X_{\dot{v}} & X_{\dot{w}} \\ Y_{\dot{u}} & Y_{\dot{v}} & Y_{\dot{w}} \\ Z_{\dot{u}} & Z_{\dot{v}} & Z_{\dot{w}} \end{pmatrix}$ and the matrix C_w^b can be written as:

$$C_w^b = \begin{pmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{pmatrix} \quad (2.128)$$

with this diagonal dominant matrix $(C_w^b)^T M_a C_w^b$ approximated by a diagonal matrix whose inverse can be calculated:

$$\begin{aligned}
 m_{11} &= X_{\dot{u}} \cos^2 \alpha - 2X_{\dot{w}} \cos \alpha \sin \alpha + Z_{\dot{w}} \sin^2 \alpha \\
 m_{22} &= Y_{\dot{v}} \\
 m_{33} &= X_{\dot{u}} \sin^2 \alpha + 2X_{\dot{w}} \sin \alpha \cos \alpha + Z_{\dot{w}} \cos^2 \alpha
 \end{aligned} \quad (2.129)$$

The gravity force is expressed by:

$$f_g = \begin{pmatrix} \cos \gamma \sin \chi & \cos \gamma \cos \chi & \sin \gamma \\ \cos \chi & -\sin \chi & 0 \\ \sin \gamma \sin \chi & \sin \gamma \cos \chi & \cos \gamma \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ B - mg \end{pmatrix}$$

The aerodynamic f_a and thrust f_p forces are given in body coordinates:

$$\begin{aligned}
 f_{ap} &= f_a + f_p \\
 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \sigma & \sin \sigma \\ 0 & -\sin \sigma & \cos \sigma \end{pmatrix} \left(\begin{pmatrix} \cos \alpha & 0 & -\sin \alpha \\ 0 & 1 & 0 \\ \sin \alpha & 0 & \cos \alpha \end{pmatrix} \begin{pmatrix} T \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -D \\ 0 \\ L \end{pmatrix} \right) \\
 &= \begin{pmatrix} T \cos \alpha - D \\ (T \sin \alpha + L) \sin \sigma \\ (T \sin \alpha + L) \cos \sigma \end{pmatrix}
 \end{aligned}$$

The angle σ is the bank angle.

The translational dynamics are given by the following relations:

$$\begin{aligned}
 \dot{V} &= \frac{1}{m + m_{11}} (T \cos \alpha - D + (B - mg) \sin \gamma) \\
 &\quad - \frac{m}{m + m_{11}} (\dot{W}_x \cos \gamma \sin \chi + \dot{W}_y \cos \gamma \cos \chi + \dot{W}_z \sin \gamma) \quad (2.130)
 \end{aligned}$$

$$\dot{\chi} = \frac{1}{(m + m_{22})V \cos \gamma} (L + T \sin \alpha) \sin \sigma - \frac{m}{m + m_{22}} \left(\frac{\dot{W}_x \cos \chi - \dot{W}_y \sin \chi}{V \cos \gamma} \right) \quad (2.131)$$

$$\dot{\gamma} = \frac{-1}{(m + m_{33})V} (L \cos \sigma + T \cos \sigma \sin \alpha + (B - mg) \cos \gamma) + \frac{m}{(m + m_{33})V} (\dot{W}_x \sin \gamma \sin \chi + \dot{W}_y \sin \gamma \cos \chi + \dot{W}_z \cos \gamma) \quad (2.132)$$

The parameter A_r is the reference area or characteristic area, ρ is the atmospheric density, m is the mass of the lighter than air robot, M is the Mach number, $B = \rho \cdot Vol \cdot g$ is the buoyancy force, $C_L(M, \alpha)$ and $C_D(M, \alpha)$ are respectively the lift and drag parameters, while the angle α is the angle of attack. The lift L and drag D are given by:

$$D = \frac{1}{2} C_D(M, \alpha) V^2 A_r \rho \quad (2.133)$$

$$L = \frac{1}{2} C_L(M, \alpha) V^2 A_r \rho$$

Atmospheric density is computed using the standard atmosphere. Generally the lift coefficient is a linear function of the angle of attack and the drag coefficient is a quadratic function of $C_L(M, \alpha)$:

$$C_L(M, \alpha) = C_{L_0}(M) + k_{L_\alpha}(M) \alpha \quad (2.134)$$

$$C_D(M, \alpha) = C_{D_0} + K C_L^2 = k_{D_0}(M) + k_{D_1}(M) \alpha + k_{D_2}(M) \alpha^2$$

where C_{L_0} , C_{L_1} and k_{D_0} , k_{D_1} , k_{D_2} are resulting coefficients with respect to α . The induced drag factor can be determined from the aerodynamic efficiency E_{max} and the zero-lift drag coefficient C_{D_0} as

$$K = \frac{1}{4 C_{D_0} E_{max}^2} \quad (2.135)$$

These equations have an important place in aerospace vehicle study because they can be assembled from trimmed aerodynamic data and simple autopilot designs. Nevertheless, they give a realistic picture of the translational and rotational dynamics unless large angles and cross coupling effects dominate the simulations. Trajectory studies, performance investigations, navigation and guidance evaluations can be successfully executed with simulations of these equations.

2.4 Aerology Characteristics

The medium of flight is the relatively thin layer of air that envelopes the Earth (about 1 percent of its diameter). This atmosphere is driven into motion by Earth rotation

and solar heating and is host to a variety of complex electromagnetic, chemical and thermodynamical processes.

The physical atmospheric boundary layer flows over the Earth surface, which influences the thickness and spatial growth of the layer, in such a way that a typical Atmospheric Boundary Layer extends roughly 300 m above ground level. Because the Atmospheric Boundary Layer is turbulent, a time averaged description does not account for all of the velocity fluctuations and turbulent dispersions. To model the 3D unsteady motion, a direct numerical simulation of the incompressible Navier-Stokes equation is employed for $Reu_\tau = 270$ where Re is the number of Reynolds based on the boundary-layer thickness and u_τ is the wall friction velocity based on the shear stress.

The motion of a lighter than air robot in turbulence is subjected to buffeting by random external forces and as a result the attitude angles and trajectory experiences random variations with time [1, 8, 108, 228, 229]. The time scale and intensity of these responses are governed by the scale and intensity of the turbulence, as well as the speed and characteristics of the vehicle. The total velocity field of the atmosphere is variable in both space and time, composed of a mean value and variations from it. The mean wind is a parameter primarily for navigation and guidance.

At landing and takeoff, lighter than air robots fly close to the ground. The presence of the ground modifies the flow past the lighter than air robot significantly so that large changes may take place in the trim and stability. The presence of the ground imposes a boundary condition which inhibits the downward flow of air normally associated with the lifting action.

Excessive surface winds can cause an unsafe situation due to three types of wind vectors: headwind, tailwind and crosswind. In addition to steady state surface winds, essentially all high winds are accompanied by gusts. Wind gusts are a quick change in the wind speed and/or direction [2].

- Windshift is a sustained change in the average wind direction of 45 degrees or more which takes place in less than 15 mn or a lesser shift if the wind speed during this period is 6 knots or greater. A wind shift could possibly turn a strong headwind into an unacceptable crosswind that suspends operations. Windshifts are typically associated with frontal passages.
- Windshear is an abrupt change in wind speed or direction, it can be vertical or horizontal. Windshear is usually associated with frontal passages, gusty winds and convective activity. Windshear becomes a hazard when an airship is close to the ground and at slow speeds, typically during the take-off and landing phases of flight.
- En route wind is not a hazard by itself. Strong headwinds will cause longer flight times which then require more fuel to be carried out and burned.
- En route turbulence can affect the maximum altitude capability of the airship. A typical flight cruising altitude is based on smooth air. Turbulence cannot be seen at any distance ahead and is only known when it is encountered. Weather radar are an excellent indication of precipitation and moisture content which when severe, can be an indication of turbulence.

- Convective induced turbulence is caused by the instability and resulting up and down drafts due to convective activity typically associated with thunderstorms. Significant Convective induced turbulence is typically associated with moderate or heavy precipitation. Airborne radar can indicate the precipitation regions where Convective induced turbulence is likely to exist. However, it exists also outside the boundaries of precipitation and both the existence and magnitude of Convective induced turbulence is impossible to predict with current technology.
- Clear air turbulence is typically caused by the windshear that exists at the boundaries between large air masses. It exists when there is no visible sign of moisture or weather event.
- Low altitude windshear is usually associated with a severe meteorological phenomenon, called the downburst. It involves a descending column of air, which then spreads horizontally in the neighborhood of the ground. This condition is hazardous because a low and slow airship might encounter a headwind coupled with a downdraft followed by a tailwind coupled with a downdraft. Ideally, what the downburst model should generate is the wind vector as a function of position in 3D field. Such a model would yield not only the wind at the origin of the body fixed frame, but also the gradients of the vertical, horizontal and lateral wind along the airship. These gradients may provide important inputs to the aerodynamic moments and hence to the attitude dynamics, during a critical portion of the landing or take-off. The domain of principal interest is a region close to the ground. The velocities involved are low and the ground boundary layer associated with the downburst can be expected to be relatively thin.
- Another dangerous form of windshear is the microburst. Microburst is an isolated wind event associated with the mature stage of a thunderstorm cell. A microburst is formed when a column of air at high altitudes quickly cools because of evaporation of ice, snow or air and becoming denser than the surrounding air, falls rapidly to the ground. On hitting the ground, this mass of air spreads radially outward in all directions. A core of such a downburst can be about 2–3 km in width and the winds generated can be as high as 150–200 km/h, the life of a microburst is typically 5–15 mn. The microburst wind may be accompanied by rain or may be dry. The wind originated at the base of a thunderstorm starts as a downburst flowing shaft of air less than 4.5 Km in diameter. As the air falls to the ground, it accelerates to speeds that can exceed 240 Km/h at its core. As the wind shift gets close to the ground, the wind slows and gets deflected radially outward from the shaft center. The outflow wind eventually dissipates. This phenomenon can be extremely dangerous especially if the airship is low and slow [115].

2.4.1 Wind Profile

The winds change considerably with altitude. The general trend is that wind speed gradually increases as the vehicle ascends through the troposphere, reaching a peak in the jet stream altitude range of 10–15 km. It then gradually decreases again to

reach a minimum speed in the lower portion of the stratosphere, generally between the 18–25 km altitude. The current wind profile changes with geographic location and the time of year. Typical wind behavior can be adequately characterized based upon meteorological data, which provides a mechanism for design phase planning. The Earth's surface creates a boundary-layer effect so that winds generally increase with altitude [147, 227].

Random turbulence is a chaotic motion of the air that is described by its statistical properties. The main statistical features that need to be considered are: stationarity, homogeneity, isotropy, time and distance scalars, probability distributions, correlations and spectra.

Moreover, unlike scalar random processes in which there is only one dependent and one independent variable, turbulence is a vector process in which the velocity vector is a random function of the position vector and of time.

Recognizing that the scale of atmospheric turbulence is usually larger than the flight vehicle, in a small region, the velocity of the wind can be regarded as a linear function of distance. It then follows that in this region, the motion is a superposition of three simple basic fields: translation, rotation and strain:

$$W(r) = W_0 + \omega_A r + E_1 r + E_2 r$$

where $W(r)$ is the wind velocity at r , W_0 is its value at the origin and ω_A , E_1 , E_2 are matrices containing the nine velocity gradients as elements, ω_A represents solid-body rotation of the air and E_1 , E_2 are axial and shear rates of strain.

- In the case of high altitude when the scale of atmospheric turbulence is large, the assumption of uniform velocity can be made with little loss in accuracy. This is because the components of ω_A , E_1 , E_2 become small as the scale becomes large.
- In small scale turbulence, close to the ground, ω_A , E_2 are important. The wind conditions that exist close to the ground (altitude < 600 m) within the Earth are very different from those at higher altitude and are governed by quite different parameters. At these low altitudes, flight path control is crucial. The principal factors that govern the wind structure (profile of mean wind, turbulence' intensities and turbulence scales) are the roughness and uniformity of the underlying terrain and the thermal stability of the atmosphere. Thermal stability is a powerful factor, in fixing turbulence characteristics at low wind speeds, but for high winds, a neutrally stable atmosphere is a good approximation

Two wind profiles are often used to model the variation in wind close to the surface:

$$\begin{aligned} W &= \frac{V^*}{K} \left(\ln \left(\frac{h}{h_0} \right) - \psi \frac{h}{L^*} \right) \\ W &= V_r \left(\frac{h}{h_r} \right)^A \end{aligned} \tag{2.136}$$

where W is the wind speed at altitude h , h_0 is a roughness length, K is von Karman constant, $\psi \frac{h}{L^*}$ is a correction for non neutral stratification defined as a function of

altitude and the Monin-Obukhov length L^* , the subscript r refers to some reference velocity and height and A is a power law index that varies as a function of surface friction (e.g. ≈ 0.10 for smooth surfaces such as a lake or ocean and ≈ 0.40 for an urban area with tall buildings). The reference height h_r typically varies between 10 and 100 m, depending on the location of the measurement equipment. In addition, the power law index has been found to vary as a function of the surface wind speed. Above a particular height h^* , the wind speed can be treated as constant ($h^* \geq h_r$).

Although the mean wind direction usually veers with height, the basic model assumes that the wind vector does not change direction in the boundary layer. The following cases may be studied:

- Uniform gust approximation (simplest case)
- Linear field approximation
- Correlations and spectra of gust gradients
- Unsteady aerodynamics

Discrete gusts are isolated encounters with steep gradients (horizontal or vertical) in the horizontal and vertical speed of the air. These gradients may occur at the edges of thermals and down drafts, in the wakes of structures, mountains, hills or at temperature inversions. They may also appear as rare extremes of turbulence in clouds, storms and the jet stream, possibly associated with organized structures embedded in the otherwise chaotic background.

2.4.2 Down Burst

A down burst is one mass of cold air that descends to the ground in a column. It is a particular case of down drafts. As the wind approaches the ground the wind changes its direction and radiates outward. A down burst usually picks up dust and may be visually detected. Doppler radar is able to look through a down burst and measure its air movement. A down-draft flow is classically modeled by a pair of 3D vortex rings that are placed symmetrically with respect to the ground. The stream function of a 3D irrotational and incompressible ring vortex is expressed as complete elliptic integrals. In this paragraph, the analytic formulation presented in [228] will be followed where a wind velocity components for a primary vortex ring located at $(0,0, H)$ are modeled by:

$$\begin{aligned} W_x &= \frac{x}{r} W_m H_m \\ W_y &= \frac{y}{r} W_m H_m \\ W_z &= \frac{4}{3} W_m H_z \end{aligned} \tag{2.137}$$

with

$$H_m = \frac{h_p/r_{2p} - h_p/r_{1p}}{\sqrt{r^2 + h_p^2 + R_v^2}} - \frac{h_m/r_{2m} - h_m/r_{1m}}{\sqrt{r^2 + h_m^2 + R_v^2}} \quad (2.138)$$

$$W_m = \frac{1.182}{2\pi} R \Gamma (1 - e^{r_{1p}/r_{1m}}) \quad (2.139)$$

$$H_z = \frac{(\frac{r_1}{r_{1p}})^{0.75} - (\frac{r_2}{r_{2p}})^{0.75}}{(0.25r^2 + h_p^2 + R_v^2)^{0.75}} - \frac{(\frac{r_1}{r_{1m}})^{0.75} + (\frac{r_2}{r_{2m}})^{0.75}}{(0.25r^2 + h_m^2 + R_v^2)^{0.75}} \quad (2.140)$$

and

$$\begin{aligned} r &= \sqrt{x^2 + y^2} & h_p &= h + H & h_m &= h - H \\ r_1 &= r - R & r_2 &= r + R & r_{1p} &= r_1^2 + h_p^2 \\ r_{1m} &= r_1^2 + h_m^2 & r_{2p} &= r_2^2 + h_p^2 & r_{2m} &= r_2^2 + h_m^2 \end{aligned} \quad (2.141)$$

R_v is the ring vortex radius and Γ is its intensity. In this paragraph, the downburst wind field is assumed stationary, for example for W_x :

$$\dot{W}_x = \frac{\partial W_x}{\partial x} \dot{x} + \frac{\partial W_x}{\partial y} \dot{y} + \frac{\partial W_x}{\partial z} \dot{z} \quad (2.142)$$

The same stands for W_y , W_z .

In a surveillance flight, an aerial vehicle would circle around a given area. The onboard warning/detection systems may be: radar, lidar (light detection and ranging) or infrared type devices. Radar devices measure the Doppler velocity of water droplets moving with the winds. When there is no rain, the Lidar devices perform well because the laser signal reflects from aerosol particles carried in the atmosphere at low altitudes. The infrared devices rely on the temperature changed in the farfield (1–3 km for the lighter than air robot) caused by the windshear/microburst activity. Once a down-draft is discovered, a lighter than air vehicle may repeatedly fly through it during its limited lifespan. A downburst may last about 5–10 min. It can create vertical winds up to 20 ms/s and as it spreads out near the ground, horizontal winds up to 40 m/s. If the potential peak airspeed exceeds the performance capability of a lighter than air vehicle, it should try to avoid the down-draft. Some collision avoidance methods are presented in the next chapter.

2.5 Conclusions

Lighter than air robots are a highly interesting study object due to their stability properties. Here, motion is referenced to a system of orthogonal body axes fixed in the lighter than air robot, with the origin assumed to coincide with its bow. In

the first part of this chapter, six degrees of freedom dynamic modeling of LTAR were discussed with two different approaches: Newton-Euler method and Lagrange method. The second section of this chapter addresses the problem of translational dynamics. Atmospheric dynamics, presented in the last section of this chapter, display hardly predictable behavior making the system output differ from the nominal conditions.

After establishing the equations of motion of lighter than air robots, some questions arise:

- How can be planned missions?
- How to avoid obstacles?
- How to characterize trim trajectories
- What are the handling qualities of this lighter than air robot?
- What are their controllability and stabilizability properties? How can closed loop control systems be solved?...

Answers to these questions are the topics of the following chapters.

Lighter than Air Robots
Guidance and Control of Autonomous Airships
Bestaoui Sebbane, Y.
2012, XVIII, 254 p., Hardcover
ISBN: 978-94-007-2662-8