

## Chapter 2

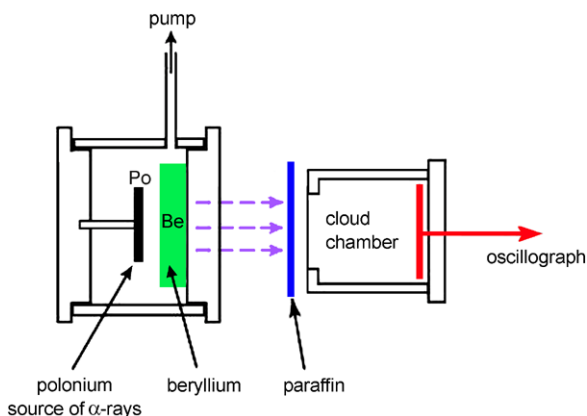
# Particle Interactions with Matter and Detectors

### Problems

- 2.1. **Atom number density.** In the interaction of particles (or nuclei) with matter, the number of collisions depends on the number of scattering centers per unit volume. Often, the scattering centers are atomic nuclei. Consider for example the case of carbon, which has an atomic mass number  $A = 12$  and a density (specific mass)  $\rho \simeq 2.265 \text{ g cm}^{-3}$ . Determine:
- (a) the number of atoms per  $\text{cm}^3$ ;
  - (b) the number of atoms per gram.
- [See solutions]
- 2.2.  **$\alpha$  particle energy loss.** An  $\alpha$  particle with 7.4 MeV kinetic energy crosses a target consisting of a thin copper foil  $5 \cdot 10^{-4} \text{ cm}$  thick. Determine:
- (a) the ionization energy loss in the copper foil;
  - (b) the particle kinetic energy and (c) the Coulomb multiple scattering angle when going out of the foil.
- Hint: see Supplement 2.1.  
[See solutions]
- 2.3. **Muon Energy loss.** A muon of 100 GeV energy crosses without being absorbed a detector whose mass is mainly due to the hadronic calorimeter and to the muon detector. The thickness of the crossed material can be considered as a layer of 3 m of iron. Determine:
- (a) what is the dominant energy loss process;
  - (b) the average energy loss of the muon inside the detector.
- Hint: see Supplement 2.2.  
[See solutions]

- 2.4. **Energy transferred.** Calculate the maximum energy  $\nu_{max}$  transferred in elastic scattering of a charged particle with mass  $M$  and energy  $E = T + Mc^2$  to an electron at rest:  
 (a) in the non relativistic case ( $T \ll Mc^2$ );  
 (b) in the relativistic case with  $M \gg m_e$ ;  
 (c) in the general case.  
 [A: (c)  $\nu_{max} = \frac{2m_e c^2 (E^2 - M^2 c^4)}{M^2 c^4 + m_e^2 c^4 + 2Em_e c^2}$ ]
- 2.5. **Kinematics of the Compton effect.** Using the energy and momentum conservation, describe the kinematics of the Compton effect and derive Eq. (2.19).  
 Calculate the maximum energy of the recoiling electron Eq. (2.21).
- 2.6. **Electromagnetic shower.** Calculate the average number of particles in an electromagnetic shower initiated by a 50 GeV photon, after 10, 13 and 20 cm of crossed iron.  
 [See solutions]
- 2.7. **Muon from pion decay.** Consider a  $\pi^+$  at rest decaying in  $\pi^+ \rightarrow \mu^+ \nu_\mu$ . Calculate the  $\mu^+$  kinetic energy and evaluate approximately the  $\mu^+$  range in liquid hydrogen (specific mass  $\rho = 0.07 \text{ g cm}^{-3}$ ).  
 [See solutions]
- 2.8. **Neutron discovery.** In his Letter to the Editor of Nature of February 27, 1932 (*Possible Existence of a Neutron*), J. Chadwick described the observation of protons emitted from a target containing hydrogen atoms. The hydrogenated target was exposed to an unknown radiation of strong penetrating power emitted by beryllium when bombarded by  $\alpha$ -particles from polonium. See the layout presented in Fig. 2.1. The protons (with mass  $m_p$ ) were emitted with velocities up to a maximum of nearly  $3 \times 10^9 \text{ cm/s}$ . Since the penetrating radiation emitted by the beryllium was observed to be neutral, it could consist either of photons or, according to Chadwick's hypothesis, of neutral particles with a mass similar to that of the proton, i.e., the neutrons. Assuming that the neutral radiation emitted by the Be is composed of photons and that the protons are emitted through the Compton effect induced by these incident photons, calculate the photon energy  $E_\gamma$ . Discuss why this  $E_\gamma$  is inconsistent with the observation. Finally, discuss the reasons that led Chadwick to formulate the hypothesis of the neutron existence.  
 [See solutions]
- 2.9. **Multiple Scattering-1.** Calculate the Coulomb multiple scattering angle in the plane  $\theta_{plane}^0$  for protons  
 (a) of 50 MeV/c momentum in  $0.1 \text{ g cm}^{-2}$  of aluminum;  
 (b) of 200 MeV kinetic energy in 2 mm of copper.  
 [See solutions]

**Fig. 2.1** Layout of Chadwick experimental apparatus that led to the neutron discovery. A beryllium target is exposed to high-energy  $\alpha$  rays from a polonium source. A strong penetrating power radiation is emitted from the Be and hits the protons contained in the paraffin layer. The emitted protons are observed in the cloud chamber on the *right* [2w3]



**2.10. Multiple Scattering-2.** From considerations based on the Coulomb multiple scattering on nuclei, determine when a target is thin or thick.

[See solutions]

**2.11. Neutron moderation.** Neutrons produced in nuclear reactors are emitted with energies of order of a few MeV and must be slowed down to thermal energies through elastic scattering on nuclei of a *moderator*. Determine the neutron speed variation in each collision assuming that the moderator is (a) hydrogen; (b) carbon; (c) iron. Show that a non-relativistic calculation is sufficient.

[See solutions]

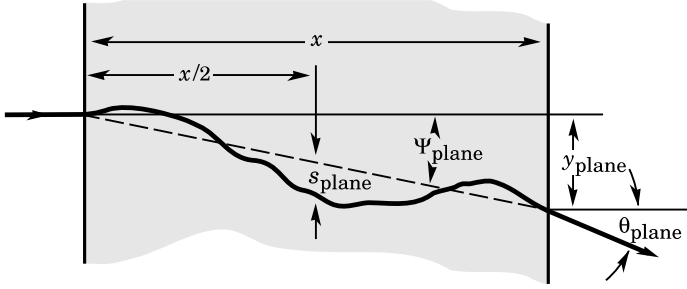
## Supplement 2.1: Multiple Scattering at Small Angles

A charged particle traversing a medium is deflected by many small-angle scatters. This deflection is due to the superposition of many Coulomb scattering from individual nuclei, and hence the effect is called *multiple Coulomb scattering*. When the particle is a hadron, the strong interaction also contributes. The cumulative effect (for thick targets) is a deflection as that shown in Fig. 2.2.

For small deflection angles, the Coulomb scattering distribution is well represented by a Gaussian distribution. At larger angles (i.e., larger than the angle  $\theta_0$  defined below), the distribution shows larger tails and the behavior is more similar to that of the Rutherford scattering. In many applications, scattering at large angles is negligible and the Gaussian approximation for small angles describes well enough the projected angle distribution, with a width [P10]:

$$\theta_0 = \theta_{plane}^{rms} = \frac{13.6 \text{ MeV}}{\beta c p} z \sqrt{\frac{x}{X_0}} \left[ 1 + 0.038 \ln(x/X_0) \right] \quad (2.1)$$

where  $p$ ,  $\beta c$ , and  $z$  are respectively the momentum, velocity, and charge number of the incident particle;  $x/X_0$  is the thickness of the scattering medium in units of



**Fig. 2.2** Quantities used to describe the multiple Coulomb scattering. The particle is incident from the *left* in the plane of the figure. From [P10]

radiation length (Eq. (2.15)). The distribution of the angle in the space has width  $\theta_{space}^{rms} = \sqrt{2}\theta_{plane}^{rms}$ .

## Supplement 2.2: Muon Energy Loss at High Energies

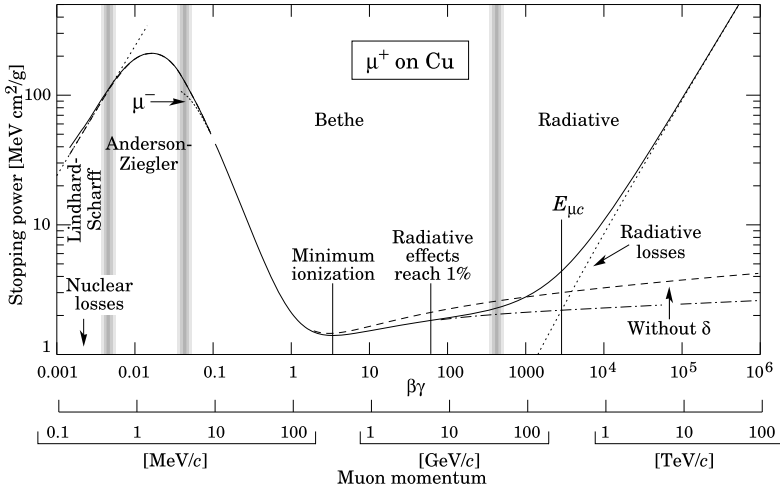
As for electrons (see Sect. 2.2.3), at sufficiently high energies, radiative processes become more important than ionization for all charged particles. In particular for muons, the critical energy occurs at several hundred GeV [2G01]. Radiative effects dominate the energy loss of energetic muons found in cosmic rays or produced at high energy accelerators. Radiative effects are characterized by small cross-sections, hard spectra, large energy fluctuations, and generation of electromagnetic or (in the case of photonuclear interactions) hadronic showers. Above the critical energy, the treatment of energy loss as a uniform and continuous process is inadequate. It is convenient to write the average muon energy loss rate as:

$$-dE/dx = a(E) + b(E)E \quad (2.2)$$

where  $a(E)$  is the ionization energy loss, and  $b(E)$  is the sum of energy losses due to  $e^+e^-$  pair production, bremsstrahlung, and photonuclear processes. In most approximations, the quantities  $a, b$  can be considered constant and independent of the muon energy  $E$ . In this case, the mean range  $x_0$  of a muon with initial energy  $E$  is obtained by integrating Eq. (2.2):

$$\int_0^{x_0} dx = \int_E^0 dE/(a + bE) \rightarrow x_0 \simeq (1/b) \ln(1 + E/E_c^\mu) \quad (2.3)$$

where  $E_c^\mu = a/b$ .  $b(E)$  can be computed for different materials; it changes only very slowly with energy. In water,  $b$  ranges between  $(2 \div 4) \cdot 10^{-6} \text{ g}^{-1} \text{ cm}^2$  for muon energies between  $10^2 \div 10^7 \text{ GeV}$ . In standard rock,  $b$  is 20%÷30% higher than in water. Since  $a(E) \sim 2 \text{ MeV g}^{-1} \text{ cm}^2$ , the critical energy  $E_c^\mu = a/b \sim 500 \text{ GeV}$  and the radiative losses dominate above several hundred GeV. The rates of energy loss for positive muons in copper as a function of  $\beta\gamma = p/Mc$  over nine orders of magnitude in momentum is reported in Fig. 2.3.



**Fig. 2.3** Total energy loss  $-(dE/dx)$  (solid curves) for positively charged muons in copper as a function of the muon momentum [P10]

## Solutions

**Problem 2.1** If the scattering centers are the material atoms (or atomic nuclei), one has:

$$\left( N_c = \frac{\text{scattering centers}}{\text{cm}^3} \right) = \left( N_a = \frac{\text{atoms}}{\text{cm}^3} \right) = \left[ \frac{\text{n. of gram-molecule}}{\text{cm}^3} \cdot N_A \cdot H \right]$$

where  $H$  is the number of atoms per molecule,  $N_A$  is *Avogadro's number*, i.e., the number of molecules in a gram-molecule. The number of atoms per  $\text{cm}^3$  is  $N_a = \frac{m}{M} \frac{1}{v} N_A H = \frac{\rho}{M} N_A H$ , where  $m$  is the mass in gram;  $M$  is the molecular weight in gram,  $v$  is the volume in  $\text{cm}^3$  and  $\rho = m/v$  is the specific mass. In the case of a monatomic element, for which  $H = 1$  and  $M = A$  ( $A$  = atomic mass), one has  $N_a = \rho N_A / A$ . In the case of carbon for example, one has  $A = 12$  and  $\rho \simeq 2.265 \text{ g cm}^{-3}$ , and the number of atoms per  $\text{cm}^3$  is:

$$N_a = \frac{\rho N_A}{A} \simeq \frac{2.265 \cdot 6.03 \cdot 10^{23}}{12} \frac{\text{g}}{\text{cm}^3} \frac{\text{molecule}}{\text{g moles}} = 1.137 \cdot 10^{23} \frac{\text{carbon atoms}}{\text{cm}^3}.$$

The number of atoms per gram is:

$$\frac{N_a}{\rho} \simeq \frac{1.137 \cdot 10^{23}}{2.265} \frac{\text{atoms}}{\text{g}} = 5.02 \cdot 10^{22} \frac{\text{carbon atoms}}{\text{g}}.$$

**Problem 2.2** Consider the energy loss given in Eq. (2.9):

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln \frac{\gamma^2 m_e v^3}{ze^2 \bar{v}}. \quad (2.4)$$

Taking into account that  $N_e = N_A Z \rho / A$  and that  $r_e = e^2 / m_e c^2$ , Eq. (2.4) can be rewritten as:

$$-\frac{dE}{dx} = 4\pi r_e^2 m_e c^2 \frac{N_A Z \rho}{A} \frac{z^2}{\beta^2} \ln \frac{m_e c^2 \beta^2 \gamma^2}{I} = C \rho \frac{Z}{A} \frac{z^2}{\beta^2} \ln \frac{m_e c^2 \beta^2 \gamma^2}{I} \quad (2.5)$$

where  $C = 4\pi r_e^2 m_e c^2 N_A$  is a constant numerically equal to  $C = 0.30 \text{ MeV/g cm}^{-2}$  and  $I$  is the average ionization potential that may be parameterized as  $I = 13.6 \cdot Z \text{ eV}$  (13.6 eV is the hydrogen ionization potential). In Eq. (2.5), the energy loss is factorized in three terms: the constant  $C$ , the term  $(\rho \frac{Z}{A})$  which depends on the crossed material, the term  $(\frac{z^2}{\beta^2})$  which depends on the particle charge and  $\beta$  times a logarithmic term which slightly depends on the particle  $\beta\gamma$ .

For copper, one has  $\rho_{Cu} = 8.9 \text{ g cm}^{-3}$ ,  $Z = 29$ ,  $A = 64$ . For the considered  $\alpha$  particle, one must determine the  $\gamma$  and  $\beta$  values from its known kinetic energy  $T = E - m_\alpha$ . For  $c = 1$ , one has (in natural units):

$$\beta\gamma = \frac{p}{m_\alpha} = \frac{\sqrt{E^2 - m_\alpha^2}}{m_\alpha} \simeq \sqrt{\frac{2T}{m_\alpha}} = \sqrt{\frac{2 \cdot 7.4}{3700}} = 0.064.$$

It is straightforward to verify that  $\gamma = E/m_\alpha \simeq 1$ . Placing these values in Eq. (2.5), one obtains:

$$\begin{aligned} -\frac{dE}{dx} &= C \rho \frac{Z}{A} \frac{z^2}{\beta^2} \ln \frac{m_e c^2 \beta^2 \gamma^2}{I} = 0.30 \cdot 8.9 \frac{29}{64} \frac{2^2}{0.064^2} \ln \frac{0.511 \cdot 10^6 \cdot 0.064^2 \cdot 1^2}{13.6 \cdot 29} \\ &= \frac{4.84 \text{ MeV/cm}}{0.064^2} \ln(5.14) = 1997 \text{ MeV/cm}. \end{aligned} \quad (2.6)$$

When passing through a thickness of  $5 \cdot 10^{-4} \text{ cm}$ , the total energy loss is:

$$\Delta E = 1997 \text{ MeV/cm} \times 5 \cdot 10^{-4} = 1.0 \text{ MeV}.$$

(b)  $T' = T - 1.0 = 6.4 \text{ MeV}$

(c) Let us use Eq. (2.1) and take into account that the radiation length of copper (Table 2.1) corresponds to a path of 1.43 cm. The particle momentum is:

$$p = \sqrt{(T + m_\alpha)^2 - m_\alpha^2} \simeq \sqrt{2Tm_\alpha} = 234 \text{ MeV/c}.$$

Therefore (in the plane perpendicular to the motion), one has:

$$\theta_0 = \frac{13.6 \text{ MeV}}{0.064 \cdot 234 \text{ MeV}} \cdot 2 \cdot \sqrt{\frac{5 \cdot 10^{-4}}{1.43}} (1 - 0.30) = 23 \text{ mrad}$$

$$\text{and } \theta_0^{\text{space}} = \sqrt{2} \theta_0 = 32 \text{ mrad}.$$

### Problem 2.3

(a) As shown in Fig. 2.3, at the momentum of 100 GeV/c (remember that in the relativistic range  $E = pc$ ) the dominant energy loss process is still that of excitation-ionization, with  $dE/dx \sim 3 \text{ MeV cm}^2/\text{g}$  in the case of copper ( $Z = 29$ ,  $A = 64$ ). Since the energy loss depends only on the ratio  $Z/A$  of the crossed medium, it does not change for iron ( $Z = 26$ ,  $A = 56$ ).

- (b) The density of iron is  $7.87 \text{ g/cm}^3$ , and then the muon energy loss in 3 m of iron is on average

$$\Delta E = 0.003 [\text{GeV cm}^2/\text{g}] \cdot 7.87 [\text{g/cm}^3] \cdot 300 [\text{cm}] = 7.1 \text{ GeV}.$$

**Problem 2.6** The processes considered here are the  $e^+e^-$  pair creation from photons, and the bremsstrahlung of electrons and positrons (i.e., the radiation of a high energy photon and the consequent energy decreases of the electron or positron). In both cases (see Fig. 4.7), the pair production and bremsstrahlung processes can be approximated as a process corresponding, on average, to the production of two particles sharing half the energy of the *parent particle*. The process stops when the particle energy drops below the *critical energy*. From that point on, the particles do not lose energy by pair production or bremsstrahlung (with increasing number of particles), but through the excitation and ionization processes.

The residual energy of a particle which has crossed a section of material of thickness  $x$  is given in Eq. (2.14). The radiation length and the path length of particles in iron (given in Table 2.1) are respectively  $13.84 \text{ g cm}^{-2}$  and  $1.76 \text{ cm}$ . After 10 cm of iron, the average energy of each particle is:

$$E_{10} = E_0 e^{-x/L_{rad}} = (5 \times 10^4) \cdot e^{-10/1.76} = 170.4 \text{ MeV}$$

higher than the critical energy  $E_c = 27.4 \text{ MeV}$  in iron (see again Table 2.1). Since (on average) all particles have the same energy, the number of particles in the shower is:

$$n_{10} = E_0/E = \frac{5 \times 10^4}{170.4} = 293.$$

Applying the same calculation after 13 cm of iron, one finds  $E_{13} = 31.0 \text{ MeV}$ . This value is slightly larger than the critical energy, and the corresponding number of particles is  $n_{13} = 1613$ . For an iron thickness larger than 13.2 cm, the average energy of the particles becomes smaller than  $E_c$ . The multiplicative process becomes less important with respect to the continuous energy loss mechanism. At a distance of 20 cm, the number of “surviving” particles is less than  $n_{13}$ .

**Problem 2.7** The four-vector of the particles involved in the  $\pi^+ \rightarrow \mu^+ \nu_\mu$  decay at rest are:

$$(m_\pi, 0) \rightarrow (E_\mu, \mathbf{p}_\mu) + (p_\nu, \mathbf{p}_\nu).$$

The neutrino mass is null (or completely negligible at this energy scale), and  $E_\nu = p_\nu$ . The condition for the momenta of the final state particles is simply:

$$|p_\nu| = |p_\mu|$$

while for the energy, one has:

$$m_\pi = E_\mu + E_\nu = E_\mu + |p_\nu| = E_\mu + |p_\mu| \longrightarrow |p_\mu| = m_\pi - E_\mu.$$

Finally,  $E_\mu$  can be calculated using the mass-energy-momentum relation:

$$m_\mu^2 = E_\mu^2 - p_\mu^2 = E_\mu^2 - (m_\pi - E_\mu)^2$$

from which, one finds:

$$E_\mu = \frac{m_\mu^2 + m_\pi^2}{2m_\pi} = \frac{(105)^2 + (138)^2}{2 \times 138} = 109 \text{ MeV}.$$

The momentum of the emitted muon is:

$$p_\mu = m_\pi - E_\mu = 138 - 109 = 29 \text{ MeV}/c.$$

Figure 2.3 allows to determine the range of particles with known momentum. In fact, in this case, one has:

$$\beta\gamma = p_\mu/m_\mu = 29/105 \simeq 0.3$$

which corresponds to a value of  $R/M = R/m_\mu = 1 \text{ g cm}^{-2} \text{ GeV}^{-1}$  (from inspection of the figure). Taking into account the specific mass of liquid hydrogen ( $\rho = 0.07 \text{ g cm}^{-3}$ ) and the muon mass ( $m_\mu = 0.105 \text{ GeV}$ ), one has:

$$\text{range} = \frac{R/m_\mu}{\rho} \cdot m_\mu = \frac{1 \times 0.105}{0.07} \simeq 1.5 \text{ cm}.$$

**Problem 2.8** Protons emitted by the hydrogenated target have maximum velocity  $\beta = v/c = 0.1$  and maximum momentum (in natural units,  $c = 1$ )  $p_p = m_p\beta = 938 \times 0.1 \simeq 94 \text{ MeV}$ . The maximum kinetic energy  $T_p$  of the proton is:

$$E = T_p + m_p = \sqrt{m_p^2 + p_p^2} = \sqrt{938^2 + 94^2} = 942.7 \text{ MeV} \rightarrow T_p = 4.7 \text{ MeV}.$$

Let us assume that the protons in the hydrogenated target are extracted through Compton elastic scattering from photons coming from the beryllium. The photon energy  $h\nu$  can be derived from the kinematics of the Compton effect given in Eq. (2.21). The maximum energy of the scattered particle is:

$$T_p = h\nu \frac{2\Gamma}{1 + 2\Gamma} \quad \text{with } \Gamma = \frac{h\nu}{m_p}.$$

Therefore, one can write:

$$T_p = \frac{2(h\nu)^2}{m_p + 2h\nu}.$$

Denoting  $h\nu \equiv x$ , this corresponds to a second degree equation:

$$2x^2 - 2xT_p - m_pT_p = 0 \rightarrow x = \frac{2T_p \pm \sqrt{4T_p^2 + 4 \cdot 2 \cdot m_pT_p}}{4}.$$

The solution with the negative sign must be excluded because it gives negative  $x$ . Thus, one has:

$$x \equiv h\nu = \frac{2T + \sqrt{4T^2 + 4 \cdot 2 \cdot m_pT_p}}{4} \simeq \frac{9.4 + \sqrt{8 \cdot 938 \cdot 4.7}}{4} = 51 \text{ MeV}.$$



Using energy and momentum conservation laws, Compton concluded that the radiation emitted by the beryllium is incompatible with the hypothesis of photons. The  $\gamma$  radiation emitted by excited nuclei is indeed below 10 MeV.

A more likely solution is that the neutral radiation incoming to the hydrogenated target is made of neutral particles with a mass similar to that of the proton. The elastic scattering between two particles with equal mass (one moving and one at rest) allows the transfer of the whole kinetic energy of the moving particle to the particle at rest (see Problem 2.11). Therefore, a *neutron* (the neutral counterpart of the proton) emitted from the Be target with a kinetic energy of  $\sim 4.7$  MeV is able to transfer such an energy to a proton at rest.

### Problem 2.9

- (a) The deflection of charged particles due to the multiple scattering is discussed in Supplement 2.1. According to Table 2.1, the radiation length of Aluminum is  $X_0^{Al} = 24.0 \text{ g cm}^{-2}$ . Here,  $x = 0.1 \text{ g cm}^{-2}$ . The width of the projected angle distribution is given in Eq. (2.1). To evaluate  $\beta pc$ , remember that  $p = m\beta c\gamma$ ;  $E = mc^2\gamma$  and thus  $pc/E = \beta$ . The energy for a  $p = 50 \text{ GeV}/c$  proton is

$$E = \sqrt{p^2 + m_p^2 c^4} \simeq m_p c^2 = 938 \text{ MeV} \longrightarrow \beta = \frac{pc}{E} = \frac{50}{938} = 0.053.$$

This corresponds to  $\beta pc = 0.053 \times 50 \text{ MeV} = 2.66 \text{ MeV}$ .

The width of the projected angle is equal to:

$$\theta_0^a = \frac{13.6}{2.66} \sqrt{\frac{0.1}{24.0}} (1 - 0.208) = 5.1 \times 0.064 \times 0.79 = 0.26 \text{ rad}.$$

- (b) According to Table 2.1, the radiation length of Copper is  $X_0^{Cu} = 12.9 \text{ g cm}^{-2}$ . The Copper density is  $\rho_{Cu} = 8.96 \text{ g cm}^{-3}$ . Thus, 2 mm of Copper corresponds to  $x = 0.2 \times 8.96 = 1.79 \text{ g cm}^{-2}$ . The energy of the protons with kinetic energy  $T = 200 \text{ MeV}$  is  $E = T + m_p c^2 = 1138 \text{ MeV}$ . The corresponding momentum is  $pc = \sqrt{E^2 - m_p^2 c^4} = 644 \text{ MeV}$ . The relativistic factor  $\beta = pc/E = 644/1138 = 0.56$  and  $\beta pc = 644 \times 0.56 = 365 \text{ MeV}$ .

The width of the projected angle distribution is:

$$\theta_0^b = \frac{13.6}{365} \sqrt{\frac{1.79}{12.9}} \left( 1 + 0.038 \ln \frac{1.79}{12.9} \right) = 0.037 \times 0.37 \times 0.925 = 0.013 \text{ rad}.$$

**Problem 2.10** Equation (2.1) depends on three factors: (i) a kinematic factor  $\sim (\beta pc)^{-1}$  which does not depend from the characteristic of the target; (ii) the factor  $z$  which depends on the particle electric charge; and (iii) the factor  $\sqrt{x/X_0} (1 + 0.038 \ln x/X_0)$ . This is the only term which depends on the material. The condition of a *thin* target corresponds to  $x \ll X_0$ . Remember that (Table 2.1)  $X_0 = 36; 24.0; 12.9; 13.8 \text{ g cm}^{-2}$  in air, aluminum, copper and iron, respectively. Considering their respective densities, the values correspond to a path length of 300 m in air, 8.9 cm in Al, 1.43 cm in Cu, 1.76 cm in Fe.

**Problem 2.11** Let us consider the collision in the system in which the nucleus is at rest. It can be considered that the nucleus mass is  $M \simeq m_n A$ . The incoming neutron has a mass  $m_n$  and its velocity is  $v, v'$ , before and after the collision, respectively. After the collision, the nucleus has a velocity  $V$ . By imposing the non-relativistic energy and momentum conservation laws, one has:

$$\frac{1}{2}m_n v^2 = \frac{1}{2}m_n v'^2 + \frac{1}{2}(m_n A)V^2 \quad (2.7)$$

$$m_n v = m_n v' + (m_n A)V \quad (2.8)$$

The non-relativistic formulae are valid because the neutron kinetic energy  $T$  is of the order of a few MeV and  $\beta = p/M \simeq \sqrt{\frac{2T}{m_n}} \simeq O(0.1)$ .

Solving the system by eliminating  $V$ , one obtains a second order equation which admits two solutions. One of the two ( $v' = v$ ) should be eliminated, because it predicts a behavior independent of  $A$  corresponding to a non-physical solution. The other solution is:

$$v' = v \frac{1 - A}{1 + A} \quad (2.9)$$

For the different nuclei considered here, one has:

- (a) Hydrogen ( $A = 1$ ),  $v' = 0$ . All the energy of the neutron is transferred to the proton (the small difference in mass between  $n$  and  $p$  is neglected).
- (b) Carbon ( $A = 12$ ),  $v' = -0.85v$ . The percentage variation in speed (in absolute value) is:  $\Delta v/v = |v - v'|/v = 15\%$ .
- (c) Iron ( $A = 56$ ),  $v' = -0.965v$ . The percentage variation in speed is:  $\Delta v/v = 3.5\%$ .

The best moderators are therefore the elements with an atomic number  $A$  as small as possible.

## References

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Particles and Fundamental Interactions: Supplements,  
Problems and Solutions

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