On the Twist Recovery Methodologies After Failure

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Abstract In this paper, methodologies for investigating the effect of failures on the performance of manipulators are presented, and the correctional input for recovering the lost motion provided by the remaining joints, for minimum Euclidean norm of the correctional and the overall joint velocity vectors, are presented. The procedure is simulated to examine the norm of the overall input before and after a failure, as well as the norm of the correctional input.

Key words: Parallel manipulator, failure, twist recovery

1 Introduction

In parallel manipulators, the mobile platform is connected to the base by a number of legs, e.g., refer to Figure 1. In general, each leg is a kinematic chain of links connected by active and passive joints. For non-redundant actuation, using the one degree of freedom joints such as revolute or prismatic joints, the number of active joints is equal to the degree of freedom (DOF) of the manipulator, e.g., Figure 1(a), with active prismatic joints. To form a kinematically redundant leg, one or more redundant active joints could be added to the leg, e.g., Figure 1(b).

Failure of a link and/or a joint could result in the loss of DOF, actuation, motion constraint, and information in parallel manipulators [6]. If any of these failures affect the performance of manipulator such that the task cannot be completed as desired, then the manipulator is considered failed. From the kinematics point of view, the failure of a joint occurs if the joint is broken, or jammed (its displacement remains constant), or if the displacement/velocity/acceleration of joint is not at the desired level. Redundancy in joint displacement sensing was investigated in [3] to facilitate the joint sensor fault detection, isolation and recovery. The relative manipulativity index was used in [7] to investigate the Jacobian matrices of manipulators fault tolerant to joint failures. In [1], the task space was partitioned to complete the major

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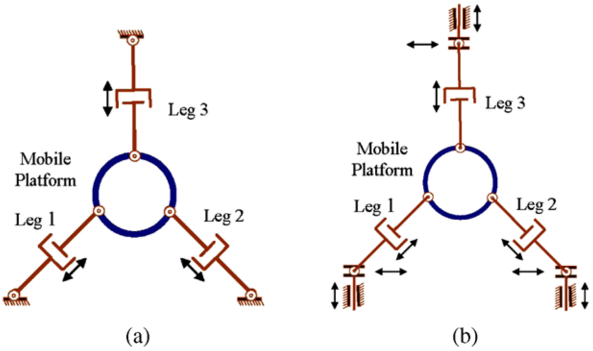

Fig. 1 Planar parallel manipulators: (a) non-redundant; and (b) redundant.

task and optimize a secondary task such as actuator fault tolerance. The effects of joint failures on the force/moment capabilities and motion performance of parallel manipulators were respectively investigated in [4, 5]. In this article, the methodology for recovering the lost motion due to the failure of joints/actuators, presented in [5], is briefly discussed. The properties of the recovered motion are examined in Section 2 by reformulating the correctional and overall velocity vectors for minimum Euclidean norm. The simulation results are reported in Section 3. The article concludes with Section 4.

2 Recovering Lost Velocity

For parallel manipulators, the relation between the $n \times 1$ active joint velocity vector, $\dot{\mathbf{q}}$, and the $m \times 1$ mobile platform twist (velocity vector), $\mathbf{V}$, is given as $\dot{\mathbf{q}} = \mathbf{J}\mathbf{V}$, where $m \leq 6$ depending on the dimension of task space, e.g., $m = 3$ for planar motion. The Jacobian matrix of manipulator, $\mathbf{J}$, is an $n \times m$ matrix; with $n = m$ for non-redundant manipulators and $n > m$ for redundant manipulators. For a given $\dot{\mathbf{q}}$, using the generalized inverse (GI) of $\mathbf{J}$, $\mathbf{J}^{\#}$, the platform twist is

$$\mathbf{V} = \mathbf{J}^{\#}\dot{\mathbf{q}} \tag{1}$$

Hence, to provide the required platform twist, $\mathbf{V}$ should belong to the range space of $\mathbf{J}^{\#}$. In addition, each leg of manipulator should allow the platform twist $\mathbf{V}$.

Considering leg i, its $l \times 1$ joint velocity vector, ${}^i\dot{\mathbf{q}} = {}^i\,[\dot{q}_1\ \dot{q}_2\ \ldots\ \dot{q}_{l-1}\ \dot{q}_l]^T$, and the twist, $\mathbf{V}$, are related by the $m \times l$ Jacobian matrix of the leg, ${}^i\mathbf{J}$, as

$$\mathbf{V} = {}^i\,\mathbf{J}\,{}^i\dot{\mathbf{q}} = \left[{}^i\mathbf{J}_1\ {}^i\mathbf{J}_2\ \ldots {}^i\,\mathbf{J}_h\ \ldots {}^i\,\mathbf{J}_{l-1}\ {}^i\mathbf{J}_l\right]{}^i\dot{\mathbf{q}} = \sum_{k=1}^{l} {}^i\mathbf{J}_k{}^i\dot{q}_k \tag{2}$$

where each column of ${}^i\mathbf{J}$, ${}^i\mathbf{J}_k$, is a screw representing the axis of the corresponding joint of leg i; and $l \geq m$. To provide the platform velocity, $\mathbf{V}$ should be in the range space of all ${}^i\mathbf{J}$, for $i = 1, \ldots, n_l$, where n_l is the number of legs.

After failure, the twist $\mathbf{V}$ should be provided by the active joints of the manipulator, as well as by the joints of each leg. When joint h (active or passive) on leg i is failed its velocity ${}^i\dot{q}_{ch}$ will be different than the desired value ${}^i\dot{q}_h$, and ${}^i\dot{\mathbf{q}}_f ={}^i [\dot{q}_1 \ \ldots \ \dot{q}_{ch} \ \ldots \ \dot{q}_l]^T$. When ${}^i\dot{q}_{ch} \neq {}^i\dot{q}_h$ the velocity equation for leg i is

$$\mathbf{V}_f ={}^i \mathbf{J}\, {}^i\dot{\mathbf{q}}_f = \sum_{k=1}^{l} {}^i\mathbf{J}_k {}^i\dot{q}_k - {}^i\mathbf{J}_h({}^i\dot{q}_h - {}^i\dot{q}_{ch}) \tag{3}$$

Then, the manipulator would be considered as failed unless the lost motion of platform is in the range space of the Jacobian matrix corresponding to the remaining (healthy) joints of that leg [5]. For full recovery of the lost twist ${}^i\mathbf{J}_h({}^i\dot{q}_h - {}^i\dot{q}_{ch})$, in general the leg with a failed joint should have a redundant joint.

2.1 Correctional Input from Healthy Joints

When joint h has a different velocity the correctional velocity to be provided by the remaining joints of leg i, ${}^i\dot{\mathbf{q}}_{corr} = {}^i[\dot{q}_{corr1} \ \dot{q}_{corr2} \ \ldots \ 0 \ \ldots \ \dot{q}_{corr\ l-1} \ \dot{q}_{corr\ l}]^T$, will compensate for the lost twist partially or completely, where in ${}^i\dot{\mathbf{q}}_{corr}$ entry h is replaced by a zero. Then, the recovered velocity of the platform will be

$$\mathbf{V}_r = {}^i\mathbf{J}\, {}^i\dot{\mathbf{q}}_f +{}^i \mathbf{J}\, {}^i\dot{\mathbf{q}}_{corr} = {}^i\mathbf{J}\, {}^i\dot{\mathbf{q}}_f +{}^i \mathbf{J}_f\, {}^i\dot{\mathbf{q}}_{corr} \tag{4}$$

where in ${}^i\mathbf{J}_f = [{}^i\mathbf{J}_1 \ {}^i\mathbf{J}_2 \ \ldots \ \mathbf{0} \ \ldots \ {}^i\mathbf{J}_{l-1} \ {}^i\mathbf{J}_l]$, column h is replaced by zeros.

To fully recover the lost twist, after applying the correctional velocity the change in the twist should be zero, i.e., $\mathbf{V} - \mathbf{V}_r = {}^i\mathbf{J}({}^i\dot{\mathbf{q}} -{}^i \dot{\mathbf{q}}_f) - {}^i\mathbf{J}_f\, {}^i\dot{\mathbf{q}}_{corr} = \mathbf{0}$. Then, the correctional velocity of the healthy joints will be

$${}^i\dot{\mathbf{q}}_{corr} = {}^i\mathbf{J}_f^{\#}\, {}^i\mathbf{J}_h({}^i\dot{q}_h -{}^i \dot{q}_{ch}) = {}^i\mathbf{J}_f^{\#}\, {}^i\mathbf{J}({}^i\dot{\mathbf{q}} - {}^i\dot{\mathbf{q}}_f) \tag{5}$$

where ${}^i\dot{\mathbf{q}} -{}^i \dot{\mathbf{q}}_f = {}^i[0 \ 0 \ \ldots \ (\dot{q}_h - \dot{q}_{ch}) \ \ldots \ 0 \ 0]^T$ is the lost motion due to failure of joint h. Then, the overall joint velocities will be

$${}^i\dot{\mathbf{q}}_{tot} = {}^i\dot{\mathbf{q}}_f + {}^i\dot{\mathbf{q}}_{corr} = {}^i\mathbf{J}_f^{\#}\, {}^i\mathbf{J}\, {}^i\dot{\mathbf{q}} + (\mathbf{I} - {}^i\mathbf{J}_f^{\#}\, {}^i\mathbf{J})^i\, \dot{\mathbf{q}}_f = {}^i\mathbf{J}_f^{\#}\, {}^i\mathbf{J}\, {}^i\dot{\mathbf{q}} \tag{6}$$

If the velocities of g joints of leg i are different than the required values the corresponding g columns of ${}^i\mathbf{J}_f$ will be zero. The lost platform twist will be $\sum {}^i\mathbf{J}_h({}^i\dot{q}_h - {}^i\dot{q}_{ch}) = {}^i\mathbf{J}({}^i\dot{\mathbf{q}} - {}^i\dot{\mathbf{q}}_f)$ and the correctional velocity will be

$${}^i\dot{\mathbf{q}}_{corr} = {}^i\mathbf{J}_f^{\#} \sum {}^i\mathbf{J}_h({}^i\dot{q}_h - {}^i\dot{q}_{ch}) = {}^i\mathbf{J}_f^{\#}\, {}^i\mathbf{J}({}^i\dot{\mathbf{q}} -{}^i \dot{\mathbf{q}}_f) \tag{7}$$

where the summation is taken over the failed joints.

When ${}^i\mathbf{J}_f$ has full row-rank, i.e., $\mathbf{V}$ belongs to the range space of ${}^i\mathbf{J}_f$, $\mathbf{V} \in \Re({}^i\mathbf{J}_f)$, if ${}^i\dot{\mathbf{q}}$ is not physically consistent, e.g., leg i has a combination of revolute and prismatic joints, the weighted right-generalized inverse [2] of ${}^i\mathbf{J}_f$ is formulated such that ${}^i\dot{\mathbf{q}}^T(\mathbf{W}_{\dot{q}}{}^i\dot{\mathbf{q}})$ is physically consistent. Then

$$ {}^i\mathbf{J}^{\#}_{fw} = \mathbf{W}_{\dot{q}}^{-1}\,{}^i\mathbf{J}_f^T\left({}^i\mathbf{J}_f\,\mathbf{W}_{\dot{q}}^{-1}\,{}^i\mathbf{J}_f^T\right)^{-1}. \tag{8} $$

Otherwise, for physically consistent ${}^i\dot{\mathbf{q}}$, ${}^i\mathbf{J}^{\#}_f = {}^i\mathbf{J}_f^T({}^i\mathbf{J}_f\,{}^i\mathbf{J}_f^T)^{-1}$.

The deviation in the platform twist after applying the correctional velocity will be zero when $\mathbf{V} \in \Re({}^i\mathbf{J}_f)$. Hence, the condition for full recovery is

$$ \mathbf{V}_{\Re^\perp} = \left(\mathbf{I} - {}^i\mathbf{J}_f\,{}^i\mathbf{J}^{\#}_f\right)\mathbf{V} = \mathbf{0} \tag{9} $$

If some entries of $\mathbf{V}_{\Re^\perp} = (\mathbf{I} - {}^i\mathbf{J}_f\,{}^i\mathbf{J}^{\#}_f)\mathbf{V}$ are not zero the corresponding components of the platform twist could not be completely recovered. When some of g failed joints have non-zero velocity $\mathbf{V}^* = \mathbf{V} - \sum \mathbf{J}_h{}^i\dot{\mathbf{q}}_{ch}$ could be used in (9). In case ${}^i\mathbf{J}_f$ does not have full row-rank, in general, the lost motion cannot be fully recovered and the platform twist that best approximates the lost motion in the least-square sense is calculated using the weighted left-GI of ${}^i\mathbf{J}_f$.

When the number of failed joints is equal to the number of redundant actuators, $g = l - m$, as long as the leg is not at a singularity, after removing the columns of the $m \times l$ Jacobian matrix corresponding to failed joints, the reduced Jacobian matrix ${}^i\mathbf{J}_r$ will be an $m \times m$ square matrix with rank m and in general there will be a unique solution for the velocity of healthy joints. When $g < l - m$ while the rank of ${}^i\mathbf{J}_f$ is m there will be infinite solutions for ${}^i\dot{\mathbf{q}}_{corr}$ and ${}^i\dot{\mathbf{q}}_{tot}$. In the following subsections, expressions for ${}^i\dot{\mathbf{q}}_{corr}$ and ${}^i\dot{\mathbf{q}}_{tot}$ are derived considering their norms.

2.2 Minimum Norm for Correctional Velocity Vector

When g out of l actuators/joints of leg i are failed, to minimize the jump in the velocity of joints after failure while providing the platform twist, the objective function will be the square of the Euclidean norm of the weighted correctional velocity vector $(\mathbf{W}_{\dot{q}}^{1/2}\,{}^i\dot{\mathbf{q}}_{corr}) \cdot (\mathbf{W}_{\dot{q}}^{1/2}\,{}^i\dot{\mathbf{q}}_{corr}) = {}^i\dot{\mathbf{q}}_{corr} \cdot (\mathbf{W}_{\dot{q}}\,{}^i\dot{\mathbf{q}}_{corr})$. The linear constraint equation in terms of the overall velocity vector ${}^i\dot{\mathbf{q}}_{tot}$ is $\mathbf{V}\,{}^i\mathbf{J}\,{}^i\dot{\mathbf{q}}_{tot} = \mathbf{0}$. The Lagrange function L is formulated by augmenting the constraint equation with the objective function using the Lagrange multiplier vector $\boldsymbol{\lambda}$

$$ L({}^i\dot{\mathbf{q}}_{corr}, \boldsymbol{\lambda}) = \frac{1}{2}{}^i\dot{\mathbf{q}}_{corr} \cdot (\mathbf{W}_{\dot{q}}\,{}^i\dot{\mathbf{q}}_{corr}) - \boldsymbol{\lambda}^T(\mathbf{V} - {}^i\mathbf{J}\,{}^i\dot{\mathbf{q}}_{tot}) \tag{10} $$

If ${}^i\dot{\mathbf{q}}_{corr} \cdot (\mathbf{W}_{\dot{q}}\,{}^i\dot{\mathbf{q}}_{corr})$ is a minimum for the original constrained problem at the stationary point $({}^i\dot{\mathbf{q}}_{corr}, \boldsymbol{\lambda})$ the gradient of L vanishes, i.e., $\nabla L({}^i\dot{\mathbf{q}}_{corr}, \boldsymbol{\lambda}) = \mathbf{0}$.

When ${}^i\dot{q}_{ch} = 0$ the constraint equation is $\mathbf{V} - {}^i\mathbf{J}_f\,{}^i\dot{\mathbf{q}}_{tot} = \mathbf{0}$ and

$$\frac{\partial L({}^i\dot{\mathbf{q}}_{corr}, \boldsymbol{\lambda})}{\partial {}^i\dot{\mathbf{q}}_{corr}} = \mathbf{W}_{\dot{q}}\, {}^i\dot{\mathbf{q}}_{corr} - {}^i\mathbf{J}_f{}^T \boldsymbol{\lambda} = \mathbf{0} \quad \Rightarrow \quad {}^i\dot{\mathbf{q}}_{corr} = \mathbf{W}_{\dot{q}}^{-1}\, {}^i\mathbf{J}_f{}^T \boldsymbol{\lambda} \tag{11}$$

$$\frac{\partial L({}^i\dot{\mathbf{q}}_{corr}, \boldsymbol{\lambda})}{\partial \boldsymbol{\lambda}} = {}^i\mathbf{J}_f\, {}^i\dot{\mathbf{q}}_{tot} - \mathbf{V} = \mathbf{0} \quad \Rightarrow \quad {}^i\mathbf{J}_f\, {}^i\dot{\mathbf{q}}_f + {}^i\mathbf{J}_f\, {}^i\dot{\mathbf{q}}_{corr} = \mathbf{V} \tag{12}$$

then $\boldsymbol{\lambda} = ({}^i\mathbf{J}_f\, \mathbf{W}_{\dot{q}}^{-1}\, {}^i\mathbf{J}_f{}^T)^{-1}(\mathbf{V} - {}^i\mathbf{J}_f\, {}^i\dot{\mathbf{q}}_f)$, ${}^i\mathbf{J}\, {}^i\dot{\mathbf{q}}_f = {}^i\mathbf{J}_f\, {}^i\dot{\mathbf{q}}_f$ for ${}^i\dot{q}_{ch} = 0$

$${}^i\dot{\mathbf{q}}_{corr} = {}^i\mathbf{J}^{\#}_{fw}\, {}^i\mathbf{J}({}^i\dot{\mathbf{q}} - {}^i\dot{\mathbf{q}}_f) \tag{13}$$

and ${}^i\mathbf{J}^{\#}_{fw} = \mathbf{W}_{\dot{q}}^{-1}\, {}^i\mathbf{J}_f{}^T ({}^i\mathbf{J}_f\, \mathbf{W}_{\dot{q}}^{-1}\, {}^i\mathbf{J}_f{}^T)^{-1}$ is the weighted right-GI of ${}^i\mathbf{J}_f$.

When ${}^i\dot{q}_{ch} \neq 0$ the constraint equation is $\mathbf{V} - {}^i\mathbf{J}\, {}^i\dot{\mathbf{q}}_{tot} = \mathbf{0}$. To ensure zero correctional velocity for the failed joints, ${}^i\mathbf{J}^T\, \boldsymbol{\lambda}$ is replaced with ${}^i\mathbf{J}_f{}^T\, \boldsymbol{\lambda}$ for ${}^i\dot{\mathbf{q}}_{corr}$

$$\frac{\partial L({}^i\dot{\mathbf{q}}_{corr}, \boldsymbol{\lambda})}{\partial {}^i\dot{\mathbf{q}}_{corr}} = \mathbf{W}_{\dot{q}}\, {}^i\dot{\mathbf{q}}_{corr} - {}^i\mathbf{J}^T \boldsymbol{\lambda} = \mathbf{0} \quad \Rightarrow \quad {}^i\dot{\mathbf{q}}_{corr} = \mathbf{W}_{\dot{q}}^{-1}\, {}^i\mathbf{J}_f{}^T \boldsymbol{\lambda} \tag{14}$$

$$\frac{\partial L({}^i\dot{\mathbf{q}}_{corr}, \boldsymbol{\lambda})}{\partial \boldsymbol{\lambda}} = {}^i\mathbf{J}\, ({}^i\dot{\mathbf{q}}_f + {}^i\dot{\mathbf{q}}_{corr}) - \mathbf{V} = \mathbf{0} \quad \Rightarrow \quad {}^i\mathbf{J}\, {}^i\dot{\mathbf{q}}_f + {}^i\mathbf{J}\, {}^i\dot{\mathbf{q}}_{corr} = \mathbf{V}. \tag{15}$$

Then $\boldsymbol{\lambda} = ({}^i\mathbf{J}\,\mathbf{W}_{\dot{q}}^{-1}\, {}^i\mathbf{J}_f{}^T)^{-1}(\mathbf{V} - {}^i\mathbf{J}\, {}^i\dot{\mathbf{q}}_f)$, and as ${}^i\mathbf{J}_f{}^T ({}^i\mathbf{J}\, {}^i\mathbf{J}_f{}^T)^{-1} = {}^i\mathbf{J}_f{}^T ({}^i\mathbf{J}_f\, {}^i\mathbf{J}_f{}^T)^{-1}$

$${}^i\dot{\mathbf{q}}_{corr} = {}^i\mathbf{J}^{\#}_{fw}(\mathbf{V} - {}^i\mathbf{J}\, {}^i\dot{\mathbf{q}}_f) = {}^i\mathbf{J}^{\#}_{fw}\, {}^i\mathbf{J}({}^i\dot{\mathbf{q}} - {}^i\dot{\mathbf{q}}_f) \tag{16}$$

As indicated by equations (13) and (16), the failure recovery methodology of Section 2.1 results in minimum 2-norm solution for the correctional velocity vector. To have physically consistent twist $(\mathbf{W}_V^{1/2}\mathbf{V}) \cdot (\mathbf{W}_V^{1/2}\mathbf{V}) = \mathbf{V} \cdot (\mathbf{W}_V\mathbf{V})$, using the reformulated constraint equation $\mathbf{W}_V^{1/2}\mathbf{V} = \mathbf{W}_V^{1/2}\, {}^i\mathbf{J}\, \mathbf{W}_{\dot{q}}^{1/2}\, \mathbf{W}_{\dot{q}}^{-1/2}\dot{\mathbf{q}}_{tot}$, the weighted GI will be ${}^i\mathbf{J}^{\#}_{fw} = \mathbf{W}_{\dot{q}}^{-1/2}(\mathbf{W}_V^{1/2}\, {}^i\mathbf{J}_f{}^T\, \mathbf{W}_{\dot{q}}^{-1/2})^{\#}\, \mathbf{W}_V^{1/2}$, which will result in ${}^i\dot{\mathbf{q}}_{corr}$ of equations (13) and (16) when ${}^i\mathbf{J}_f$ has full row-rank, and ${}^i\mathbf{J}^{\#}_{fw} = ({}^i\mathbf{J}_f{}^T\, \mathbf{W}_V\, {}^i\mathbf{J}_f)^{-1}\, {}^i\mathbf{J}_f{}^T\, \mathbf{W}_V$ when ${}^i\mathbf{J}_f$ has full column-rank.

2.3 Minimum Norm for Overall Velocity Vector

To minimize the actuation energy after failure while providing the required platform twist, the objective function will be the square of the Euclidean norm of the weighted overall velocity vector, ${}^i\dot{\mathbf{q}}_{tot} \cdot (\mathbf{W}_{\dot{q}}\, {}^i\dot{\mathbf{q}}_{tot})$. The linear constraint equation in terms of ${}^i\dot{\mathbf{q}}_{tot}$ is $\mathbf{V} - {}^i\mathbf{J}\, {}^i\dot{\mathbf{q}}_{tot} = \mathbf{0}$. The Lagrange function L is

$$L({}^i\dot{\mathbf{q}}_{tot}, \boldsymbol{\lambda}) = \frac{1}{2}{}^i\dot{\mathbf{q}}_{tot} \cdot (\mathbf{W}_{\dot{q}}\, {}^i\dot{\mathbf{q}}_{tot}) - \boldsymbol{\lambda}^T(\mathbf{V} - {}^i\mathbf{J}\, {}^i\dot{\mathbf{q}}_{tot}) \tag{17}$$

and when ${}^i\dot{\mathbf{q}}_{tot} \cdot (\mathbf{W}_{\dot{q}}\, {}^i\dot{\mathbf{q}}_{tot})$ is a minimum $\nabla L({}^i\dot{\mathbf{q}}_{tot}, \boldsymbol{\lambda}) = \mathbf{0}$.

When ${}^i\dot{q}_{ch} = 0$, to have zero overall (and correctional) velocity for the failed (jammed) joints, the constraint equation is $\mathbf{V} - {}^i\mathbf{J}_r\ {}^i\dot{\mathbf{q}}_{tot\ r} = \mathbf{0}$, where ${}^i\mathbf{J}_r$ and ${}^i\dot{\mathbf{q}}_{tot\ r}$ are respectively obtained by removing the columns and entries of ${}^i\mathbf{J}$ and ${}^i\dot{\mathbf{q}}_{tot}$ corresponding to the failed joints.

$$\frac{\partial L({}^i\dot{\mathbf{q}}_{tot\ r}, \boldsymbol{\lambda})}{\partial {}^i\dot{\mathbf{q}}_{tot\ r}} = \mathbf{W}_{\dot{q}}\ {}^i\dot{\mathbf{q}}_{tot\ r} - {}^i\mathbf{J}_r{}^T\boldsymbol{\lambda} = \mathbf{0} \quad \Rightarrow \quad {}^i\dot{\mathbf{q}}_{tot\ r} = \mathbf{W}_{\dot{q}}^{-1}\ {}^i\mathbf{J}_r{}^T\boldsymbol{\lambda} \tag{18}$$

$$\frac{\partial L({}^i\dot{\mathbf{q}}_{tot\ r}, \boldsymbol{\lambda})}{\partial \boldsymbol{\lambda}} = {}^i\mathbf{J}_r\ {}^i\dot{\mathbf{q}}_{tot\ r} - \mathbf{V} = \mathbf{0} \quad \Rightarrow \quad {}^i\mathbf{J}_r\ {}^i\dot{q}_{tot\ r} = \mathbf{V} \tag{19}$$

then $\boldsymbol{\lambda} = ({}^i\mathbf{J}_r\ \mathbf{W}_{\dot{q}}^{-1}\ {}^i\mathbf{J}_r{}^T)^{-1}\mathbf{V}$ and

$${}^i\dot{\mathbf{q}}_{tot\ r} = \mathbf{W}_{\dot{q}}^{-1}\ {}^i\mathbf{J}_r{}^T\ ({}^i\mathbf{J}_r\ \mathbf{W}_{\dot{q}}^{-1}\ {}^i\mathbf{J}_r{}^T)^{-1}\mathbf{V} = {}^i\mathbf{J}_{rw}^{\#}\mathbf{V} = {}^i\mathbf{J}_{rw}^{\#}\ {}^i\mathbf{J}\ {}^i\dot{\mathbf{q}} \tag{20}$$

and the correctional velocity from the healthy joints will be ${}^i\dot{\mathbf{q}}_{corr\ r} = {}^i\dot{\mathbf{q}}_{tot\ r} - {}^i\dot{\mathbf{q}}_{f\ r}$, where the reduced joint velocity vector after failure ${}^i\dot{\mathbf{q}}_{f\ r}$ is obtained by removing the zero entries of ${}^i\dot{\mathbf{q}}_f$ corresponding to failed joints.

When ${}^i\dot{q}_{ch} \neq 0$ the constraint equation is $\mathbf{V} - {}^i\mathbf{J}\ {}^i\dot{\mathbf{q}}_{tot} = \mathbf{0}$. To calculate the minimum norm overall joint velocity vector, first the portion of the platform twist provided by the failed joints with ${}^i\dot{q}_{ch} \neq 0$ should be removed from the required platform twist. When gc out of g failed joints have non-zero velocity

$$\mathbf{V}^* = \mathbf{V} - \sum_{gc} \mathbf{J}_k\ {}^i\dot{q}_{ck} \tag{21}$$

The Lagrange function and its partial derivatives in terms of the overall velocity of healthy joints are

$$L({}^i\dot{\mathbf{q}}_{tot\ r}, \boldsymbol{\lambda}) = \frac{1}{2}{}^i\dot{\mathbf{q}}_{tot\ r} \cdot (\mathbf{W}_{\dot{q}}\ {}^i\dot{\mathbf{q}}_{tot\ r}) - \boldsymbol{\lambda}^T(\mathbf{V}^* - {}^i\mathbf{J}_r\ {}^i\dot{\mathbf{q}}_{tot\ r}) \tag{22}$$

$$\frac{\partial L({}^i\dot{\mathbf{q}}_{tot\ r}, \boldsymbol{\lambda})}{\partial {}^i\dot{\mathbf{q}}_{tot\ r}} = \mathbf{W}_{\dot{q}}\ {}^i\dot{\mathbf{q}}_{tot\ r} - {}^i\mathbf{J}_r{}^T\boldsymbol{\lambda} = \mathbf{0} \quad \Rightarrow \quad {}^i\dot{\mathbf{q}}_{tot\ r} = \mathbf{W}_{\dot{q}}^{-1}\ {}^i\mathbf{J}_r{}^T\boldsymbol{\lambda} \tag{23}$$

$$\frac{\partial L({}^i\dot{\mathbf{q}}_{tot\ r}, \boldsymbol{\lambda})}{\partial \boldsymbol{\lambda}} = {}^i\mathbf{J}_r\ {}^i\dot{\mathbf{q}}_{tot\ r} - \mathbf{V}^* = \mathbf{0} \quad \Rightarrow \quad {}^i\mathbf{J}_r\ {}^i\dot{\mathbf{q}}_{tot\ r} = \mathbf{V}^*. \tag{24}$$

Then $\boldsymbol{\lambda} = ({}^i\mathbf{J}_r\ \mathbf{W}_{\dot{q}}^{-1}\ {}^i\mathbf{J}_r{}^T)^{-1}\mathbf{V}^*$ and the overall velocity of healthy joints is

$${}^i\dot{\mathbf{q}}_{tot\ r} = {}^i\mathbf{J}_{rw}^{\#}\mathbf{V}^* = {}^i\mathbf{J}_{rw}^{\#}\left({}^i\mathbf{J}\ {}^i\dot{\mathbf{q}} - \sum_{gc}\mathbf{J}_k\ {}^i\dot{\mathbf{q}}_{ck}\right) \tag{25}$$

${}^i\dot{\mathbf{q}}_{tot}$ is obtained by incorporating the velocities of failed joints (zero and non-zero velocities). As indicated by equations (20) and (25), the recovery methodology of Section 2.1 results in the minimum 2-norm solution for the overall velocity vector.

3 Case Study

Considering the manipulator of Figure 1(a), to form kinematically redundant legs, two active prismatic joints, with axes in the Y and X directions, are added to each leg between the base and the first revolute joint; the $\underset{\rightarrow}{P}\,\underset{\rightarrow}{P}\,R\,\underset{\rightarrow}{P}\,R$ layout of Figure 1(b). The platform twist $\mathbf{V}$ is related to the joint velocities of leg i, ${}^{i}\dot{\mathbf{q}}$, as

$$\begin{bmatrix} v_x \\ v_y \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & l_i s\alpha_i + r_{Bi/P}\, s(\alpha_i+\beta_i) & -c\alpha_i & r_{Bi/P}\, s(\alpha_i+\beta_i) \\ 1 & 0 & -l_i c\alpha_i - r_{Bi/P}\, c(\alpha_i+\beta_i) & -s\alpha_i & -r_{Bi/P}\, c(\alpha_i+\beta_i) \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{d}y_i \\ \dot{d}x_i \\ \dot{\alpha}_i \\ \dot{l}_i \\ \dot{\beta}_i \end{bmatrix}$$
$$= {}^{i}\mathbf{J}\,{}^{i}\dot{\mathbf{q}} \tag{26}$$

where $c\alpha_i = \cos\alpha_i$, $s\alpha_i = \sin\alpha_i$, $c(\alpha_i+\beta_i) = \cos(\alpha_i+\beta_i)$ and so on.

The coordinates of the base attachment points of leg i, $A_i, i = 1, \ldots, 3$, are $(-2, -1.5)$, $(2, -1.5)$ and $(0, 1.5)$, respectively. The position of connection points of leg i, B_i, on the platform is set at a constant radius of $r_{Bi/P} = 0.25$ meters with angular coordinates, θ_i, of $-150°, -30°$ and $90°$. When the platform pose is $\mathbf{p} = [0 \;\; 0]^T$ meter and $\varphi = -30°$ leg 3 is in the Y direction with the joint displacements of ${}^{3}\mathbf{q} = [0 \;\; 0.125 \;\; 90 \;\; 1.283 \;\; -30]^T$. Then, the leg Jacobian matrix is

$${}^{3}\mathbf{J} = \begin{bmatrix} 0 & 1.0 & 1.5 & 0 & 0.217 \\ 1.0 & 0 & -0.125 & -1.0 & -0.250 \\ 0 & 0 & 1.0 & 0 & 1.0 \end{bmatrix} \tag{27}$$

For the twist of $\mathbf{V} = [1 \;\; 0.5 \;\; 0]^T$, the minimum norm vector of joint velocity is

$${}^{3}\dot{\mathbf{q}} = {}^{3}\mathbf{J}^{\#}\mathbf{V} = [0.250 \;\; 0.548 \;\; 0.352 \;\; -0.250 \;\; -0.352]^T \tag{28}$$

with a magnitude of $\|{}^{3}\dot{\mathbf{q}}\|_2 = 0.821$. When the second active joint ($h = 2$) of leg 3 is jammed there remain four joints (two active prismatic and two passive revolute joints) for a 3 DOF task. Then ${}^{3}\dot{\mathbf{q}}_f = [0.250 \;\; 0 \;\; 0.352 \;\; -0.250 \;\; -0.352]^T$ with $\|{}^{3}\dot{\mathbf{q}}_f\|_2 = 0.611$, and the platform twist is $\mathbf{V}_f = {}^{3}\mathbf{J}\,{}^{3}\dot{\mathbf{q}}_f = [0.452 \;\; 0.5 \;\; 0.0]^T$.

The failure of this active joint could be fully recovered as $(\mathbf{I} - {}^{3}\mathbf{J}_f\,{}^{3}\mathbf{J}_f^{\#})\,\mathbf{V} = \mathbf{0}$. Using an identity weighting matrix, the correctional velocity is

$${}^{3}\dot{\mathbf{q}}_{corr} = {}^{3}\mathbf{J}_f^{\#}\,{}^{3}\mathbf{J}_2\,\dot{d}x_3 = [0 \;\; 0 \;\; 0.427 \;\; 0 \;\; -0.427]^T \tag{29}$$

with $\|{}^{3}\dot{\mathbf{q}}_{corr}\|_2 = 0.604$. In this configuration of leg 3, because the first and third prismatic joints axes are collinear (and the first and second prismatic joints axes are always perpendicular), the motion of the failed second prismatic joint is fully recovered by the two passive revolute joints. Then, the overall joint velocities are

$${}^{3}\dot{\mathbf{q}}_{tot} = {}^{3}\dot{\mathbf{q}}_f + {}^{3}\dot{\mathbf{q}}_{corr} = [0.250 \;\; 0 \;\; 0.779 \;\; -0.250 \;\; -0.779]^T \tag{30}$$

which is identical to the overall joint velocity vector calculated with the Lagrange multiplier method ${}^3\dot{\mathbf{q}}_{tot\,r} = {}^3\mathbf{J}_r^{\#}\mathbf{V}$ with $\|{}^3\dot{\mathbf{q}}_{tot}\|_2 = 1.157$.

At this pose and required twist, if the first two joints were jammed the leg would reduce to the one in Figure 1(a) and $\mathbf{V}_f = [0.452\ \ 0.250\ \ 0.0]^T$. The unique solution for recovery would be ${}^3\dot{\mathbf{q}}_{corr} = [0\ \ 0\ \ 0.427\ \ -0.250\ \ -0.427]^T$ with $\|{}^3\dot{\mathbf{q}}_{corr}\|_2 = 0.654$. Then, ${}^3\dot{\mathbf{q}}_{tot} = [0\ \ 0\ \ 0.779\ \ -0.500\ \ -0.779]^T$ with $\|{}^3\dot{\mathbf{q}}_{tot}\|_2 = 1.210$, i.e., the third prismatic joint and the two revolute joints would respectively recover the motion of the failed first and second prismatic joints.

4 Conclusion

In this article, methodologies for recovering the lost motion of manipulators due to joint/actuator failures were presented. When a joint (active or passive) is failed the required platform twist should be provided by adjusting the motion of the remaining active joints of the manipulator, as well as the active and passive joints of the leg with failed joint(s). The method discussed here examined the motion of a leg with failed joint(s) utilizing the Jacobian matrix of the leg. A similar process could be adapted to investigate the motion of active joints using the Jacobian matrix of manipulator. It was shown that the procedure based on the projection of the lost joint motion onto the orthogonal complement of the null space of the reduced Jacobian matrix results in minimum Euclidean norm for the correctional velocity vector and the overall velocity vector.

References

1. Chen, Y., McInroy, J., Yi, Y.: Optimal, fault-tolerant mappings to achieve secondary goals without compromising primary performance. IEEE Trans. Robot. Autom. **19**(4), 680–691 (2003)
2. Doty, K., Melchiorri, C., Bonivento, C.: A theory of generalized inverses applied to robotics. Int. J. Robot. Res. **12**(1), 1–19 (1995)
3. Notash, L.: Joint sensor fault detection for fault tolerant parallel manipulators. J. Robot. Syst. **17**(3), 149–157 (2000)
4. Notash, L.: A methodology for actuator failure recovery in parallel manipulators. Mech. Mach. Theory **46**(4), 454–465 (2011)
5. Notash, L.: Motion recovery after joint failure in parallel manipulators. Trans. Can. Soc. Mech. Eng. **35**(4), 559–571 (2011)
6. Notash, L., Huang, L.: On the design of fault tolerant parallel manipulators. Mech. Mach. Theory **38**(1), 85–101 (2003)
7. Roberts, R., Yu, H., Maciejewski, A.: Fundamental limitations on designing optimally fault tolerant redundant manipulators. IEEE Trans. Robot. **24**(5), 1224–1237 (2008)

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