Chapter 2
Basic Logarithm for a Better Understanding of Statistical Methods

Non-linear relationships in clinical research are often linear after logarithmic transformations. Also, logarithmic transformation normalizes skewed frequency distributions and is used for the analysis of likelihood ratios. Basic knowledge of logarithms is, therefore, convenient for a better understanding of many statistical methods. Almost always natural logarithm (ln), otherwise called naperian logarithm, is used, i.e., logarithm to the base e. Log is logarithm to the base 10, ln is logarithm to the base e (2.718281828).

Theory and Basic Steps

log 10 = 10 log 10 = 1
log 100 = 10 log 100 = 2
log 1 = 10 log 1 = 0
antilog 1 = 10
antilog 2 = 100
antilog 0 = 1

Casio fx-825 scientific, Scientific Calculator, Texas TI-30XA, Sigma, Commodore
Press: 100….log….2
Press: 2….2ndf….log…100

Electronic Calculator, Kenko KK-82MS-5
Press: 100…. = ….log…. = ….2
Press: 2…. = ….shift….log….100

ln e = e log e = 1
ln e^2 = e log e^2 = 2
ln 1 = e log 1 = 0

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antiln 1 = 2.718…
antiln 2 = 7.389…
antiln 0 = 1

Casio fx-825 scientific, Scientific Calculator, Texas TI-30XA, Sigma
Press: 7.389….ln….2
Press: 2….2ndf….ln….7389

Electronic Calculator, Kenko KK-82MS-5
Press: 7.389…. = ….ln…. = ….2
Press: 2…. = ….shift….ln….7.389

Example, Markov Model

In patients with diabetes mellitus (* = sign of multiplication):

After	1 year 10% has beta-cell failure, and	90% has not.
	2	90 * 90 = 81% has not.
	3	90 * 90 * 90 = 73% has not.

When will 50% have beta-cell failure?

$0.9^x = 0.5$
$x \log 0.9 = \log 0.5$
$x = \log 0.5/\log 0.9 = 6.5788$ years.

Example, Odds Ratios

	Events	No events	
	Numbers of patients		
Group 1	15(a)	20(b)	35(a + b)
Group 2	15(c)	5(d)	20(c + d)
	30(a + c)	25(b + d)	55(a + b + c + d)

The odds of an event = the number of patients in a group with an event divided by the number without. In group 1 the odds of an event equals = a/b.

The odds ratio (OR) of group 1 compared to group 2

$$\begin{aligned} &= (a/b)/(c/d) \\ &= (15/20)/(15/5) \\ &= 0.25 \\ \ln\mathrm{OR} &= \ln 0.25 = -1.386 (\ln = \text{natural logarithm}) \end{aligned}$$

The standard error (SE) of the above term

$$
\begin{aligned}
&= \sqrt{(1/\ a\ + 1/\ b\ + 1/\ c\ + 1/\ d)} \\
&= \sqrt{(1/15 + 1/20 + 1/15 + 1/5)} \\
&= \sqrt{0.38333} \\
&= 0.619
\end{aligned}
$$

The odds ratio can be tested using the z-test.

$$
\begin{aligned}
\text{The test} - \text{statistic} &= \text{z} - \text{value} \\
&= (\ln \text{odds ratio}) / (\text{SE} \ln \text{odds ratio}) \\
&= -1.386/0.619 \\
&= -2.239
\end{aligned}
$$

Z table

	Z-value	P-value
	1.645	0.1
	1.960	0.05
	2.5576	0.01
	3.090	0.002

The z table shows that if this value is smaller than −1.96 or larger than +1.96, then the odds ratio is significantly different from 1 with p-value <.05. There is, thus, a significant difference in numbers of events between the two groups.

Conclusion

We conclude that basic knowledge of logarithms is convenient for a better understanding of many statistical methods. Odds ratio tests, log likelihood ratio tests, Markov modeling and many regression models use logarithmic transformations.

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