Chapter 2
Predicate Logic with Anaphora

Dynamic semantics has often been preoccupied with pronouns. Pronouns are essential in interpretation and they succeed in oiling the wheels of efficient linguistic information exchange. They are naturally involved in all basic features of ordinary language, like reference, coreference, indexicality, modality, belief, and presupposition. Pronouns are ubiquitous. (In English, and closely related languages, that is.) But they almost go unnoticed when they are, as they usually are, conveniently accommodated in a context. Taken out of their natural habitat, however, they speak up. Dynamic semantics provides the means to get a grip on their role in the flow of information.

In this first substantial chapter of this monograph I lay out the basic framework, the system of Predicate Logic with Anaphora (*PLA*). It is a system of interpretation which extends ordinary first order predicate logic just to capture the results on anaphoric relationships achieved within *Discourse Representation Theory* (*DRT* , Kamp 1981; Kamp and Reyle 1993), *File Change Semantics* (*FCS*, Heim 1982), *Situation Semantics* (*SitS*, Barwise and Perry 1983), *Game Theoretical Semantics* (*GTS*, Hintikka 1983), proof-theoretic semantics (*PTS*, Sundholm 1984), and *Dynamic Predicate Logic* (*DPL*, Groenendijk and Stokhof 1991). I will take time and space to explain and motivate this system even though it covers worn-out results. (The first presentation of *PLA* dates from 1994 (Dekker 1994), and related formal literature dates back to the early eighties of that century.)

The mentioned interpretational architectures, *DRT* , *FCS*, *SitS*, *GTS*, and *DPL*, all have their own sound, formal and intuitive, motivation for deviating from what may have been the paradigm of truth-conditional semantics, the prime formal semantic paradigm of the latter half of the twentieth century, conceived by Alfred Tarski, conceptualized by Donald Davidson, rigourously formulated by Richard M. Montague and implemented by others, e.g., Theo M.V. Janssen. The reasons for departing from this paradigm have been manifold, ranging across the intuitive insights that meanings are computed (*DRT*), that interpretation is essentially tied to the update of information (*FCS*, *DPL*), and that its use is essentially tied to situated agents (*SitS*, *GTS*).

P. J. E. Dekker, *Dynamic Semantics*, Studies in Linguistics and Philosophy 91,
DOI: 10.1007/978-94-007-4869-9_2,

There is no reason to complain about any of these enterprises. Still, it seems that all five of them come with the implication, suggestion, or even slogan, that meaning is something dynamic. Again, I do not at all object to such a supposition. On the contrary I do hold the same supposition, but I do believe that it is not motivated by any of the linguistic data these theories aim to account for.

Meaning is not dynamic in any intuitive sense of the word. In ordinary language, if we talk about the meaning of an expression, simple or compound, it is simply just that: the meaning, if any, of the expression. Meanings do not do things. If meanings would be able to do things, change my commitments or beliefs, I would not want to use them, and advise you not to do so either. Interpretation, on the other hand, is, in a certain specific sense, dynamic. If interpretation is conceived of as the assignment of meanings, it is, by this very description, a dynamic thing. It involves an agent, the interpreter, and a process, the assignment of meanings.

Logically speaking, and also from a theoretical linguistic perspective, there does not seem to be independent reason to be interested in this dynamic process. It is the concern of the psychologist, or sociolinguist, or ethnographer, to see how agents actually interpret utterances. This is not, or does not seem to be, the objective of the theoretical linguist. Surely, any serious linguist wants to say something about the meanings of expressions, and how they relate to how these expressions are interpreted, or could be interpreted, or should be interpreted. In this sense, indeed, the so-called 'interpretation' is a matter of concern for the theoretical linguist, but it comes to be something of a fossilized notion. The idea of 'the interpretation of an expression' is something put forward, be it in a normative or descriptive way, but once conceived thus it is no longer anything dynamic. Besides, that is, from one fundamental insight. The fact that meanings really have to do with actual interpretations, intended interpretations, or conventionalized interpretations means that there are or can be systematic aspects of use that a semantic theory should account for, and such a theory, then, may appropriately be called 'dynamic'.

Peter F. Strawson rightly claimed that names do not refer. How could they, linguistic types or tokens, do that? It is normally humans or other agents that refer, and significantly they do this by using names—or variables. If, on an account of proper names like that of John Stuart Mill, or Saul Kripke, we say that a certain type of name refers, rigidly, if you want, and statically, to a certain type of individual, you can, if you want, equate this with saying that it is used to refer, generally, rigidly, but dynamically, to that particular individual. Not much seems to be gained by stating what seems to be the same fact in a static or dynamic way. It is, or seems to be, quite a different fact that certain expressions, in different contexts, can be used to refer to different individuals in each of these contexts. As indicated above, indexical expressions can be typically used so, and pronouns, in general do so all over the place. More intriguingly, the kinds of context which seem to define, or at least determine, the specific interpretations of these terms, can be entirely linguistic as well.

This point, doubtlessly, must have contributed to the fascination of linguists for the typical use of certain terms to 'introduce' discourse referents, and for the use of other terms, pronouns, to typically pick them up. Any elementary presentation of *DRT* , *FCS*, or *DPL*, focuses on especially this phenomenon, originally observed by Lauri Karttunen. In certain contexts certain terms, like indefinite descriptions, introduce discourse referents, and subsequent expressions refer back to them. It is interesting to see that this may happen without there being a decisive answer to the question which individuals are actually referred to. Often it is just "the one previously mentioned", also referred to as "the such and such who did so and so" without there being any implication that there was one and only such and such who did so and so. No matter how these discourse referents, their introduction, revitalization, and death are actually conceived of, they have shown to be a pertinently useful device in the treatment of a vast number of phenomena involving quantifiers, tense, aspect, and models, as shown in the works of Hans Kamp, Nicholas Asher, Reinhard Muskens, Maria Bittner, and Adrian Brasoveanu.

While the idea of introducing and picking up discourse referents has been very appealing and successful in the presentation of the dynamic systems of interpretation mentioned above, Henk Zeevat (1989) has shown at a quite early stage that it does not presuppose a dynamic notion of meaning in the ordinary sense. The results of *DRT* , like those of *FCS*, and *DPL*, can be fully captured by means of standard algebraic operations on suitably structured meanings, statically conceived. More recently, Max Cresswell has defended the point, from both a philosophical and linguistic point of view. In response to certain phenomena the analysis of which has been argued to require a dynamic treatment he sets out: "The purpose of this article is (…) to question the need for any change in the basic aims of semantics." (Cresswell 2002). And even though Zeevat's algebraic operations are not dynamic, intuitively speaking, a proper assessment of the system presented in this chapter will show that there really are dynamic ways of composing meanings. Arguably, Cresswell's observations point in the same direction. This intuition then will be key to the system presented in this chapter, as well as a more technical insight from Kees Vermeulen. In one or another way, the three explicitly dynamic systems of *DRT* , *FCS*, and *DPL*, deal with updates of meanings which are sets of variable assignments. For certain technical reasons Vermeulen has shown we can do with sequences of values as our semantic objects, and define dynamic notions of compositions on those. Actually, this is what we will do in the remainder of this chapter.

Before we really start let me briefly note on the status of the 'examples' in this chapter. The system of *PLA* is meant to eventually evolve into a formal language adequate for the formulation of the meanings, or structurally interesting aspects of meanings, of expressions of natural language. With this objective in mind, I suggest the basic language (i.e., that of *PLA*) to already model some such aspects, with a formal conjunction as a model for coordinating operations in natural language, and, significantly, an existential quantifier modeling the contribution of indefinite descriptions. Needless to say how poorly this is done with first order predicate logical means only. Even so, I will also use natural language paraphrases to illustrate such deliberately defective uses of our logical devices. In this chapter, these paraphrases only serve an expository purpose, to explain the basic devices. The examples, therefore, will be rather stilted.

Paying debt to the prehistory of *PLA*, and in order to set the stage, I will start with a very concise overview of the architecture of *DRT* , and the system of *DPL*. These systems have evidently served as the main inspiration for the system presented in this book and more in general they have been extremely influential in the relevant literature of the past 20 years or so.

2.1 Static and Dynamic Semantics

The system of *DRT* was first presented in the innovative (Kamp 1981), appropriately called 'a theory of truth and semantic representation'. It is a two-layered architecture in which, first, sentences from a natural language discourse are mapped into so-called *discourse representation structures* (*DRS* s), which represent the semantic contents of the current discourse, and extend it with the contents of the sentence next processed. On a second level these *DRS* s are assigned Tarski-style truth conditions, specifying under which circumstances the discourse representations are true, and also in what way.

I will not go into the way in which a discourse formulated in a stylized fragment of natural language gets mapped into these *DRS* s, and refer the reader to the handbooks (Kamp and Reyle 1993; Kamp et al. 2011) for an extensive treatment. I do want to say something about the language in which these *DRS* s are formulated, because that will enhance the comparison, technically, and philosophically, with the system presented later in this section.

In most run-of-the-mill applications of *DRT* the *DRS* -language is that of a special kind of a first order predicate logical language, in which atomic conditions, negations, disjunctions and implications, are always preceded by a (possibly empty) sequence of existential quantifiers. (Many practical applications employ a multi-sorted language, which not only quantify over a first order domain of individuals, but also over states, events, times, worlds, and what have you. More theoretically inspired implementations of *DRT* also have second and higher order reference and quantification, as well as lambda-abstraction. For the purposes of this monograph we can safely assume such generalizations.) These 'formulas', or structures, are very perspicuously and conveniently displayed, pictorially, as boxes, consisting of two parts, representing a domain of individuals under discussion, and a series of conditions imposed on them. They are typically (and schematically) displayed as follows:

- $$\begin{array}{|c|}\hline x_1, \ldots, x_n \\ \hline \phi_1 \\ \vdots \\ \phi_m \\ \hline \end{array}$$

In this pictorial representation, the $x_1, \ldots, x_n$ are 'discourse markers', which represent or keep track of the subjects of a discourse under consideration. These subject are also called 'discourse referents'. They are no ordinary referents, because their specific identity may be left undecided, and in more involved discourse representation structures they may become quantified 'from the outside'. (For a more logically oriented person, they are, hence, like variables.) The $\phi_1, \ldots, \phi_m$ are conditions on the values of these discourse markers, conditions on the discourse referents exposed in the discourse at issue. These conditions may atomically ascribe properties of and relations between the discourse referents, but they can also be compound, for instance the negation of an embedded *DRS* , or the statement of an implication relation between two *DRS* s. The *DRS* s are supposed to be partial models of reality, as construed on the basis of a discourse under construction, but the whole language is easily seen to be as expressive as first order predicate logic itself.

The statement of the *DRS* -language just now given is quite unspecific, but I hope sufficient for the remainder of this monograph. One thing, however, needs to be said about how it is used, especially in the original (Kamp 1981). The idea, in that paper, is that the interpretation of a discourse starts from an empty representation—because nothing has been said yet. (Obviously, it could be a representation of the shared assumptions of the participants in the discourse.) The idea about what happens is that any syntactically analyzed sentence uttered in a discourse gets added to the representation of the preceding discourse, and then decomposed into new discourse markers, atomic conditions, and compound ones. Thus the interpretation algorithm, also called 'discourse representation construction algorithm', yields a discourse representation structure of the contribution of the utterance relative to the preceding context, with an update (*DRS*) of the contents of the discourse. This of course sounds very natural, but there is a very interesting twist to it. By manipulating discourse representational structures, the system is capable of suitably establishing connections between subjects or discourse referents mentioned in one contribution, and subjects brought up earlier. Because the *DRS* s, as models, are partial, these connections can be efficiently established, even though the identity of the specific referents is still unresolved. Interestingly, this is the major, and sophisticated, innovation of *DRT* , and by the same token one of the main sources of criticism.

Jeroen Groenendijk and Martin Stokhof have argued that *DRT* commits itself, formally, to an indispensable level of representation in the interpretational architecture. (Hans Kamp has himself motivated this commitment, but does not seem to think it unconditionally indispensable for all purposes any longer.) The complaint, in their paper (Groenendijk and Stokhof 1991), is not that a level of representation is unrealistic, but that a semantic architecture should not be committed to it—or, more to the point, that the data that motivated the inception of *DRT* , do not force us to a representational architecture.

The paper by Groenendijk and Stokhof takes a constructivist stance. (The term 'constructivist' here is used in its ordinary, not in its logical or mathematical sense.) They present an interpretation of the language first order predicate logic, which is dynamic, but arguably not representational, and which accounts for the kind of discourse phenomena that *DRT* was originally meant to account for. The philosophical, or methodological, discussion will be postponed to Sect. 2.4. Here I want to briefly survey the main features of the system of *DPL* as presented in Groenendijk and Stokhof (1991).

Groenendijk and Stokhof combine, in a very interesting and sophisticated way, philosophical insights from Stalnaker (1978), with tools and results from the field of artificial intelligence, in particular, the semantics and verification of programming languages. The leading idea is that sentences, or better, descriptive utterances, are not just independent means to characterize the world as being a certain way. Rather, they are context dependent acts, which are meant to change the contexts. As in *DRT* , assertions are taken to be contributions to what is assumed to be the content of the discourse in which they occur, but unlike the way in which this is done in *DRT* , these contributions are not taken to be updates of representations of the content, but updates of the content itself. In order to settle the main theoretical point, and also guided by the preceding linguistic discussions, Groenendijk and Stokhof deliberately restrict themselves to the dynamics of establishing anaphoric relationships in discourse. However, as I have argued above, and will argue below, the dynamics of establishing these relationships can be taken to be paradigmatic for the dynamic establishment of structure in discourse in general.

One way of introducing the *DPL* concept of meaning consists in taking a procedural view upon the interpretation of ordinary first order predicate logic. Consider the following example, with its first order rendering.

(1) Mary borrowed a copy of *Naming and Necessity* from a professor in linguistics.
$\exists x(CNx \wedge \exists y(PLOyx \wedge BORmxy))$.

The translation key, simplified of course, runs as follows.

- $CNx :=$ x is a copy of *Naming and Necessity*;
  $PLOyx :=$ y is a professor in Linguistics who owns x;
  $BORmxy :=$ m borrowed x from y.

How can we evaluate this sentences in a model $M = \langle D, I\rangle$, where D is the domain of individuals under discussion, and I gives us an extensional interpretation of the individual and predicate/relational constants? We try and find a valuation of the variable x by means of a variable assignment g, so that it satisfies CNx; precisely, so that $g(x) \in I(CN)$. As long as we interpret x as something which is not a copy of *Naming and Necessity*, we try and find another value of x to see whether that is one that is such a copy. If we never find any such copy in our model, we report, "this does not work, the formula is false." However, if we can find a value for x which is such a copy, we continue. We then try and find a value for y in our model, such that is a professor in linguistics; moreover, one that owns the value of x, the copy of *Naming and Necessity*. If we do not find such a professor, we give up on the chosen value for x, and try and find another copy of *Naming and Necessity*, as a value for x, and a professor in linguistics, as a value for y, who owns that copy of the book. If all of this fails, we render the formula false again, but if we can get through, we continue. We finally test whether our value for x ($g'(x)$) is something that Mary ($I(m)$) borrowed from the professor ($g'(y)$). If it does not, we have to redo the whole procedure again, and if the procedure never succeeds, we, again, have to say, "Sorry, the formula is false." (In this model.) However, if we do succeed, we can happily report that the formula is true in the model, and that is all we need to hear—in our first order predicate logic. *DPL*, however, does not stop here with only returning the truth-value true. Rather, it remembers that our $g'(x)$ is that copy of *Naming and*

Necessity, which is owned by $g'(y)$, who is a professor in linguistics, and such that particular professor $g'(y)$ owns that copy $g'(x)$. Having these witnesses $g'(x)$ and $g'(y)$ is useful, because we can refer back to them. Consider a continuation of the discourse, with the following sentence, for instance.

(2) It$_x$ was covered with comments and exclamation marks. The professor$_y$ must have studied it$_x$ extensively.
($Cx \wedge SIyx$.)

(Cx rendering that x was covered with all these marks, and $SIyx$ that y must have studied x intensively.) Because *DPL*, in a sense, "remembers" the witnesses for x and y that satisfy the previous sentence, it can take up on that. It merely adds the condition that what we have found as satisfying values for x and y, also satisfies the next sentence. If they do not, we have refuted the continuation, but not without further ado. We might go back to find other values for x and y, which in the end do satisfy the continuation, and then we can go on, simply and happily, with subsequent discourse.

What distinguishes *DPL* from ordinary first order predicate logic, is that, first, it keeps track of the witnesses $g'(x), g'(y), \ldots$ of variables that have been quantified over in a previous discourse, and, second, and for good reasons, it keeps track of all of these possible satisfying values. Maybe this is all one needs to know about *DPL*, but for a proper understanding of the main contents of this monograph, it is useful to specify *DPL*'s formal details.

The language of *DPL* is that of first order predicate logic, and its semantics is specified as a relation between variable assignments g and h. Intuitively, a pairs of assignments $\langle g, h\rangle$ stands in that relation, for any given formula ϕ, iff g figure as a possible valuation of the free variables in ϕ, and h as a possible evaluation of the bound, existentially quantified, variables in ϕ. Given any such pair $\langle g, h\rangle$ in the interpretation of ϕ, the assignment g may count as a possible input for truthfully interpreting the formula ϕ, and h a possible output, satisfying, also, constraints on variables introduced in ϕ. Formally, the definition can be given as follows. Interpretation is stated relative to a standard model $M = \langle D, I\rangle$, with a domain D of individuals, or objects, and an interpretation function I for the individual and predicate logical constants of the language.

Definition 1 (*DPL Interpretation*)

- $g[\![Rx_1 \ldots x_n]\!]_M h$ iff $g = h$ & $\langle g(x_1), \ldots, g(x_n)\rangle \in I(R)$;
  $g[\![\neg\phi]\!]_M h$ iff $g = h$ & $\neg\exists h$: $g[\![\phi]\!]_M h$;
  $g[\![\exists x\phi]\!]_M h$ iff $\exists k$: $g[x]k[\![\phi]\!]_M h$;
  $g[\![\phi \wedge \psi]\!]_M h$ iff $\exists k$: $g[\![\phi]\!]_M k[\![\psi]\!]_M h$.
- A formula ϕ is true in a model M and relative to assignment g, $M, g \models \phi$, iff $\exists h$: $g[\![\phi]\!]_M h$

Apart from the revolutionary shift from (sets of) satisfying variable assignments, to (sets of) *pairs* of satisfying input-output variable assignments, nothing really freaky goes on in this definition. An input assignment, and the very same output, satisfies an atomic formula if the formula is satisfied in the classical way. Satisfying a negation $\neg\phi$ simply comes down to not being able to satisfy the negated sentence ϕ. A good input for $\neg\phi$ is a variable assignment relative to which (the interpretation of) ϕ is unable to render a satisfying output. Existentially quantified formulas $\exists x\phi$ are interpreted as usual, trying to find a satisfying instance of the variable x quantified over—but for the fact that the satisfying output assignment may remember which lucky choice of the interpretation of the variable x made ϕ get satisfied. (Let me emphasize the *may* in the previous sentence; it so happens in *DPL* that a variable x gets 'reintroduced', with the effect that previous knowledge about its possible values have to be discarded. This is an annoying technical issue, which will be readdressed below.) Since *DPL*'s interpretations are possible input/output pairs of variable assignments, conjunction cannot be considered other than as relation composition. If, given input g, ϕ may bring me to output k, and ψ, with input k, may produce output h, then, of course, with g as input, should produce h as output to $(\phi \wedge \psi)$.

It is not expedient to illustrate *DPL*'s interpretation function in full detail here—for this, consult (Groenendijk and Stokhof 1991; Dekker 2011)—but it is interesting to see the major difference with ordinary first order predicate logic. It can be argued that the following observation, Egli's Theorem, completely characterizes the main difference, and its corollary the useful result of that Egli (1979).

Observation 1 (Egli's Theorem)

- $(\exists x\phi \wedge \psi) \Leftrightarrow \exists x(\phi \wedge \psi)$.

Observation 2 (Egli's Corollary)

- $(\exists x\phi \rightarrow \psi) \Leftrightarrow \forall x(\phi \rightarrow \psi)$.

What is not prominently visible in observation (1), which is a standard equivalence, is that it *lacks* the proviso, that x should not be free in ψ. This is, however, *DPL*'s distinctive feature. The existential quantifier $\exists x$ in the first conjunct indeed, effectively, binds free occurrences of the variable x in the second conjunct ψ. The second observation is a direct consequence of this, if we employ the standard definition of $\rightarrow$ and $\forall x$ using negation and conjunction. Observation (1) can account for the following examples.

(3) A man is riding through the park. He is whistling.
(4) A man who is riding through the park is whistling.

Apart from the different ways in which information is presented in (3) and (4), the two examples seem to be (truth-conditionally) equivalent. But indeed, their fairly natural translation in a predicate logical language is, fully, equivalent as well in *DPL*. For Egli's Theorem gives us the following result.

- $(\exists x(Mx \wedge Rx) \wedge Wx) \Leftrightarrow \exists x((Mx \wedge Rx) \wedge Wx)$

Moreover, Egli's Corollary gives us the following result. Consider the following examples

- If a farmer owns a donkey, he beats it.

- Every farmer beats every donkey he owns.

These sentences had previously been argued to be equivalent as well, and this equivalence is rendered by the *DPL*-equivalence of the following formulas.

- $(\exists x(Fx \wedge \exists y(Dy \wedge Oxy)) \rightarrow Bxy) \Leftrightarrow$
  $\forall x((Fx \wedge \exists y(Dy \wedge Oxy)) \rightarrow Bxy) \Leftrightarrow$
  $\forall x(Fx \rightarrow (\exists y(Dy \wedge Oxy) \rightarrow Bxy)) \Leftrightarrow$
  $\forall x(Fx \rightarrow \forall y((Dy \wedge Oxy) \rightarrow Bxy))$

DPL thus deals with a number of anaphoric relationships, which had puzzled logicians and philosophers for years. In addition, it can be argued that *DPL*'s distinctive feature is that it *only* does this: rendering Egli's Theorem a real theorem. For there is a reduction algorithm that takes any predicate logical formula as input, and possibly changes that formula into one with the illuminating property that, while it has exactly the same meaning as the original formula in *DPL*, the reduced formula has also exactly the same truth-conditions in first order predicate logic and *DPL*. The steps in the algorithm thus reveal what is 'standard', and what is 'new' in *DPL*, and, interestingly, the step where conjunctions are reduced along the lines of Egli's algorithm is the only non-standard step. [For a full specification of the algorithm, see (Groenendijk and Stokhof 1991; Dekker 2011).]

The above characterization of *DPL* is of course not all there is to say about the system. Of course, changing things in a logic system have repercussions. Some of them maybe might have been expected, but it is worth mentioning them. For one thing, conjunction is not commutative and not idempotent. That is, it is not in general the case that $(\phi \wedge \psi) \Leftrightarrow (\psi \wedge \phi)$ or that $\phi \Leftrightarrow (\phi \wedge \phi)$. Thinking from a classical perspective this may sound strange, but from a dynamic perspective it makes sense. If interpretation is dynamic, that is, if it is both context dependent *and* context altering, then of course one can expect that the interpretation of one occurrence of a formula ϕ may affect a second, subsequent, occurrence of it. Thus, if one formula ϕ is repeated, as in $(\phi \wedge \phi)$, the second occurrence of that formula may have an interpretation different from that of the first, so that the whole may mean something different from what a single occurrence of the formula means. As for commutativity, if a formula ψ occurs after a formula ϕ has occurred, as in $(\phi \wedge \psi)$, its interpretation may be different from the one that it is obtained without first establishing ϕ, as in $(\psi \wedge \phi)$; moreover, the interpretation of ϕ itself may be different if it 'occurs out of the blue', as in $(\phi \wedge \psi)$, or after ψ, as in $(\psi \wedge \psi)$.

DPL also employs a dynamic notion of entailment, which we will not formally specify here, but which is easily seen to lack the structural properties of reflexivity, monotonicity, and transitivity. It is dynamic, because it builds on the idea that an entailment is valid if the conclusion is true, not always *in case*, but always *after* the premises have been accepted. Entailments come as an ordered sequence of formulas, in which (anaphoric) dependencies may get established in a dynamic way. I will not go into the (non-)structural properties of this entailment here, because very much the same holds of the entailment relation of *PLA*, extensively discussed in Sect. 2.3. For now, it suffices to observe that if a formula ϕ is accepted, it may as well have changed the context of interpretation, also if that is a context in one wants to conclude that ϕ. Since, due to the premise occurrence of ϕ, it may have changed the context of interpretation of the conclusion, the same formula but in a different context of interpretation, it may make the conclusion, on its second interpretation, no longer guaranteed, or even acceptable. Similar arguments can be given to show that because of the possibility of context change, and the (im-)possibility of establishing anaphoric relationships, the *DPL*-entailment relation is not monotone or transitive either.

2.2 First Order Satisfaction in PLA

It is time now to turn the main subject of this chapter, the system of *PLA*. I will argue that most of the findings and results achieved in *DRT* and *DPL* can be achieved in a more classical fashion by means of an arguably static semantics, a Tarskian satisfaction semantics, which accommodates dynamic interpretation as the dynamic composition of meanings. The language with which we will be concerned from now on is literally that of a predicate logic with anaphora, or anaphoric pronouns. It is a minimal first order predicate logical language that, apart from employing a primitive category of pronouns as terms, is built up like a language of ordinary predicate logic. As I said above, I will assume throughout that the reader is familiar with predicate logic.

Definition 2 (*PLA Language*) The *PLA*-language $\mathcal{L}_{PLA}$ is built up from relational constants $R^n \in \mathcal{R}^n$, individual constants $c \in C$, variables $x \in V$ and pronouns $\mathtt{p}_i$, for $i \in \mathbb{N}$.

- $t ::= c \mid x \mid \mathtt{p}_i$;
  $\phi ::= R^n t_1 \ldots t_n \mid \neg\phi \mid \exists x\phi \mid (\phi \wedge \phi)$.

(Later on we will also assume identity formulas $(t_1 = t_2)$ to be among the atomic formulas.) As can be seen from the definition, any constant c, variable x, and pronoun $\mathtt{p}_i$ counts as a term t. Terms are those things which fill argument positions of predicates in atomic formulas. An individual constant simply names an individual as the argument; a variable indeed is a variable ranging over possible interpretations of it, to be controlled by corresponding variable binding quantifiers; and a pronoun refers back to something which has been mentioned before it. Together with n such terms, an n-place predicate may be saturated in an atomic formula, and it says, when it occurs, that the n terms have referents which stand in the corresponding n-place relation.

Constants and variables will be interpreted like they are in ordinary first order logics; the new category of pronouns will be interpreted in the style of "de Bruijn indices" (de Bruijn 1972). The interpretation of a pronoun $\mathtt{p}_i$ totally depends on the context of its occurrence. In the style of a variable free semantics (Jacobson 1999; Szabolcsi 1989) it is interpreted like the identity function, which take a contextually supplied individual as an argument, and delivers the very same individual as a value.

Pronouns in general are assumed to be a bit more flexible, though. In *PLA* they are functions taking a sequence of individuals as an argument selecting a designated one of them as a value. The sequences of individuals here are the possible values of terms in previous discourse, and any selective pronoun p_i will serve to refer to the ith term occurring before the pronoun's occurrence, in a manner detailed below.

The categories of constants, variables, and pronouns are, thus, all dependent on context, but in different manners. A constant's interpretation depends on the model, or the world of interpretation, if you want, in which a name has been defined, or an object has been baptized; a variable's interpretation depends on the quantifier (or other operator) which controls it, or binds it; variables are very useful technical devices, which are eliminable, though; a pronoun is a device to correlate actual pieces of discourse and its interpretation serves to establish coreference between actually occurring linguistic material. Eventually, pronouns are eliminable, too, but in the practice of structured exchange of information they are indispensable. We will come back in more detail on the relation between, especially, variables and pronouns, in Sect. 2.4.

To keep generalizations within easy reach, the language consists of formulas ϕ, built up from atomic formulas using negation ($\neg$), existential quantification ($\exists x$) and conjunction ($\wedge$). Other useful operators like the universal quantifier, and other connectives like disjunction ($\vee$) and (material) implication ($\rightarrow$) can be defined in the usual way. I.e., $\forall x \phi$ can be taken to be short for $\neg\exists x \neg\phi$, $(\phi \vee \psi)$ for $\neg(\neg\phi \wedge \neg\psi)$ and $(\phi \rightarrow \psi)$ for $\neg(\phi \wedge \neg\psi)$. With some of the required hand-waving, the operators of the language can be taken to correspond to natural language expressions like 'not', 'some', 'and', 'all', 'or', and 'if ... then'; predicates can be taken to stand in for nouns and verbs, individual constants for proper names, and pronouns, indeed, for pronouns. This, in short, is the layman's translation manual for expressing natural language meanings in first order logic. The only difference with textbook first order logic is the use of pronouns. As we will see, they are eliminable, but useful as well.

Pronouns are essentially indexical. Like I said, pronouns are a device to refer to contextually given entities. We cannot state out of the blue what the referent of a pronoun is, because one and the same pronoun may have different referents in different contexts. In natural language such a context is always a context of use, and this means a context of an occurrence of a pronoun. In *PLA*, for the time being, I identify the uses of a pronoun with its occurrences, and thereby employ the formulas in which they occur as their contexts of utterance. It is through these formulas that pronouns may find their referents. And when I say that a pronoun refers to a contextually given entity, I mean it relates to something that is 'given' at its point of occurrence, something given by an expression that literally occurs to the left of the pronoun's occurrence in a formula. (We do make generalizations here, which is clear from the fact that we can talk about the *occurrences* of a formula in which a pronoun *occurs* at a certain place; and these formulas may occur in larger constructions which have their own occurrences in even larger ones. The notion of an 'occurrence', therefore, needs to be understood relativistically, or contextually, or, indeed, indexically.) Moreover, the specific choice of the previous 'antecedent' is determined in an entirely indexical way: it is determined by the index i on a pronoun p_i by counting back from the location of the occurrence of the pronoun in a formula ϕ, to the ith antecedent term before that occurrence of p_i. This is exactly how de Bruijn's indices in the lambda calculus work: they are indices i, who indicate the $lambda$-occurrence that binds them, by counting up, from the location of the occurrence of i, to the ith λ that binds them.

Pronouns take their input, more specifically, from previously used existentially quantified formulas, typically the expressions that formally render the interpretation of indefinite noun phrases in natural language. Existentially quantified formulas are selected for this purpose, partly for theoretical and partly for practical reasons. Like I said, the formal language in which I want to start to state our investigations is kept as simple as possible. The very first level to stage discussions of reference and coreference then is at the level of sentences in what must be at least a first order logic and the very first operators which we find there are the first order quantifiers; among these the existential quantifier lends itself best for the present purposes. (From a philosophical and an historical perspective one could in principle choose among four quantifiers: one saying that a formula is satisfied by everything, one saying that it has a satisfying instance, one saying that it has an exception, and one saying that it is not instantiated; for practical reasons I take the second option as the basic one, even though philosophically I'd prefer the third, in the spirit of Herakleitos.) There is also an historical reason. Most of the formal semantic work on the interpretation of pronouns in *DRT* , *FCS* and *DPL*, has concentrated on anaphoric relationships of pronouns and preceding indefinite noun phrases. The reason is that these posed the greatest challenge for both syntactic and semantic treatments of anaphora. As we will see, this is not a bad motivation, since anaphoric relationships with terms other than indefinite noun phrases can be easily modeled after those with indefinite noun phrases.

Before I start with the semantics of the above language, I need to introduce two syntactic notions with semantic impact. The following definition first spells out how many potential antecedents a formula may provide for subsequent pronouns yet to come, and next how many of such antecedents it presupposes itself. The first is called the *domain* $n(\phi)$ of a formula, which equals the number of 'active' existential quantifier occurrences in ϕ; the second is called its *range* $r(\phi)$, which is determined by occurrences of pronouns in ϕ.

Definition 3 (*Domain* $n(\phi)$ *and Range* $r(\phi)$ *of a Formula* ϕ)

- $n(Rt_1 \ldots t_m) = 0$; $\quad n(\neg\phi) = 0$;
  $n(\phi \wedge \psi) = n(\phi) + n(\psi)$; $\quad n(\exists x\phi) = n(\phi) + 1$.
- $r(Rt_1 \ldots t_m) = MAX\{j \mid \mathrm{p}_j \in \{t_1, \ldots, t_n\}\}$; $\quad r(\neg\phi) = r(\phi)$;
  $r(\phi \wedge \psi) = MAX\{r(\phi), (r(\psi) - n(\phi))\}$; $\quad r(\exists x\phi) = r(\phi)$.

If $n(\phi) = 0$, ϕ is called *closed*; if $r(\phi) = 0$, ϕ is called *resolved*.

(Later in this chapter we will also talk about the domain and range of sequences of formulas $n(\phi_1, \ldots, \phi_n)$ and $r(\phi_1, \ldots, \phi_n)$. The sequences then are conceived of as their conjunction.)

The domain of a formula originates from existentially quantified formulas. Each quantifier occurrence contributes one item, and in a conjunction they are summed up. There is one proviso: an existential quantifier should not occur in the scope of a negation. Negations annul the discourse contribution of existential quantifiers, as is generally assumed. So $n(\neg\phi) = 0$ no matter what $n(\phi)$ is. The reason is that, in general, a negation has the effect of denying the existence of witnesses for existentially quantified variables in its scope. In this sense, ironically, existentially quantified variables behave like free variables, as Heim in the spirit of Quine has rightly claimed. The same effect is observed with indefinite noun phrases in the scope of a negation in natural language.

The range of a formula originates from pronouns. A pronoun p_i literally claims coreference with the ith existential quantifier before its occurrence, so that it requires a pre-existing discourse where at least i items have been introduced. From an atomic formula, the loudest, or most far reaching claim will be heard. If the most demanding pronoun in a formula is honoured, which requires at least i previous existentials, then the claims of less demanding pronouns are automatically satisfied.

As one can see from the definition, the range of $\neg\phi$ and $\exists x\phi$ equals that of ϕ, which means that pronominal claims are preserved under negation and quantification. (Notice that if a pronoun is in the scope of an existential quantifier $\exists x$, then that occurrence of $\exists x$ does not count as a term which *precedes* the pronoun; for one thing, this is correct because the pronoun itself contributes to establishing the possible witnesses for the variable quantified over. We will learn more about this in due course.) A conjunction, however, allows pronominal demands from the second conjunct to be weakened. For instance, if the second conjunct ψ in a conjunction $(\phi \wedge \psi)$ requires a number of existential terms before it, and if the first conjunct ϕ actually contributes such terms, then the demand of the whole conjunction may be weakened or even annulled. Of course, pronouns in the first conjunct ϕ may impose further, and even stronger, constraints on the discourse preceding the conjunction. The range of a conjunction, therefore, equals the strongest demand among that of the first conjunct $r(\phi)$ and that of the second conjunct $r(\psi)$, as it is weakened by the contribution of the first conjunct, so minus $n(\phi)$. Here already we can see that the dynamics of our system of interpretation originates from its notion of conjunction. Pronominal demands may get satisfied, in a conjunction, where else? And if they do, that is, if any pronouns at all occur in a formula, and if they are, where they occur, satisfied by existentials preceding them in the very same formula, then these pronouns counts as resolved, and the formula counts as resolved as well then.

With the definition of the range of a formula, in terms of the occurrence of existential quantifiers, and with their use in resolving pronouns, in whatever way this is going to be spelled out, we face again the issue already mentioned. I could have chosen to allow individual constants (proper names) and pronouns to contribute to the domain of a formula, because, for one thing, they are most natural antecedents for pronouns themselves. For the moment I have chosen not to do so. For expository reasons it is better to start with one, typical, category of introductory terms only. Besides, the anaphoric potential of names and pronouns is easily dealt with by similar means. [A simple, quite crude, but effective, way of implementing this potential is to assume that names and pronouns always come with an existential quantifier themselves, so that, after all, they *do* introduce items. Instead of rendering *Anne/She walks* as Wa or $W\mathrm{p}_1$, one could render it as $\exists x(x = a \wedge Wx)$ or $\exists x(x = \mathrm{p}_1 \wedge Wx)$. Actually this is also how it works, roughly, in, for instance, (Kamp and Reyle 1993).]

We may now turn to the semantics of *PLA*. Existentially quantified expressions (like indefinite descriptions) are evaluated relative to possible satisfying witnesses for their eventual truth. In standard logic $\exists x\phi$ is deemed true iff ϕ is true under at least one evaluation of the variable x. That is, if for some individual d from the domain, $\phi[x/d]$ is true under the usual rendering of $\phi[x/d]$. The same will be required to be the case in *PLA*, but for the fact that such a witness d of the truth of $\exists x\phi$ is remembered, so to speak. If $\exists x\phi$ is true because a couple of instantiations of x make ϕ true, then, in *PLA*, $\exists x\phi$ is satisfied relative to each of these witnesses. The witnesses associated with (occurrences of) existentially quantified formulas will figure as the intended referents of the (occurrences of) coreferential pronouns *later* in that formula. Notice that, since any formula may introduce any number of items (that is, $n(\phi)$ may be any number m), we cannot just speak of 'a possible witness' of a formula, but we have to speak of 'sequences of possible witnesses', as Tarski has observed. If $n(\phi) = 0$, a possible witness is the empty sequence. These things being said, we may now turn to the definition itself.

Formally, the semantics of *PLA* is spelled out as a Tarskian satisfaction relation, in which the formulas of our language are said to be 'satisfied' (or not satisfied) relative to the relevant parameters. It is defined, first, relative to the usual first order models $M = \langle D, I\rangle$, consisting of a domain of individuals D and an interpretation function I for the relational and individual constants and such that $I(R^n) \in \mathcal{P}(D^n)$ and $I(c) \in D$. Satisfaction is defined also relative to variable assignments g such that $g(x) \in D$ for all variables x, and relative to sequences of witnesses $\widehat{e} = e_1 \dots e_n \in D^n$. The values of variables, given by assignment functions g, are controlled by variable binding operators, the existential quantifier in the first place, and this is done in the usual way known from predicate logic. The witness sequences $\widehat{e}$ establish the interpretation of pronouns; they are controlled by the linguistic environment of the pronouns, and essentially keep track of the witnesses of terms (read here: existentially quantified phrases) which have, or may have, occurred before these sequences are actually employed. (It may surprise, or even worry, the reader that I propose a separate treatment of variables and pronouns, and do this in even in formally distinct ways: the first in terms of variable assignments g; the other in terms of sequences of objects $\widehat{e}$. I will come back to this issue in Sect. 2.4 where I give formal, theoretical, and methodological arguments for proceeding this way.)

In the definition below, and throughout the monograph, I employ the following conventions. Formulas ϕ are in general evaluated relative to sequences $\widehat{ce}$, or $\widehat{ace}$, or $b\widehat{ce}$, where:

- $\widehat{ce}$ is the concatenation of the sequences $\widehat{c}$ and $\widehat{e}$,
  $\widehat{ace}$ the concatenation of the sequences $\widehat{a}$, $\widehat{c}$ and $\widehat{e}$,
  and $b\widehat{ce}$ the concatenation of an individual $b \in D$ and the sequence $\widehat{ce}$.

All sequences $\widehat{a}$, $\widehat{c}$ and $\widehat{e}$ may be empty; their length depends on the formal properties of the formula ϕ at issue.

- If ϕ is evaluated relative to a sequence $\widehat{ce}$ (or $\widehat{ace}$), $\widehat{e}$ has *at least* length $r(\phi)$; the sequence is long enough to satisfy pronominal demands;
- $\widehat{c}$ (or $\widehat{ac}$) is supposed to have *exactly* length $n(\phi)$; the sequence supplies exactly the right number of witnesses for existentially quantified formulas in ϕ.
- For the evaluation of a conjunction $(\phi \wedge \psi)$ the sequences $\widehat{c}$ and $\widehat{a}$ are used to distinguish the contributions of ϕ, and ψ, respectively, so that $\widehat{c}$ is a sequence of length $n(\phi)$ and $\widehat{a}$ a sequence of length $n(\psi)$; thus $\widehat{ac}$, the concatenation of $\widehat{a}$ and $\widehat{c}$ has length $n(\phi \wedge \psi)$;
- $\widehat{ce}$ has length at least $r(\psi)$, and $\widehat{e}$ has length at least $r(\phi \wedge \psi)$, so it is at least $r(\phi)$ elements long and $r(\psi) - n(\phi)$ elements long; thus, $\widehat{e}$ directly satisfies the pronominal demands of ϕ, and, together with witnesses for existentials in ϕ, it indirectly satisfies the pronominal demands of ψ.

Here is a specific simple example to illustrate these conventions:

(5) A Canadian farmer, whose horse was ill, went to see his veterinarian. She lent him her donkey.

Neglecting many details, the whole sequence can be translated using the following abbreviations. 'CFx' stands short for 'x is a Canadian farmer', '$OHxy$' for 'x owns horse y', 'Iy' for 'y is ill', '$SVxu$' for 'x see x's veterinarian u', '$ODuv$' for 'u owns donkey v', and '$Luvx$' for 'u lends v to x'; the choice of the indices on the pronouns are explained below.

- $(\exists x((CFx \wedge \exists y(OHxy \wedge Iy)) \wedge \exists uSVxu) \wedge \exists v(OD\mathrm{p}_2 v \wedge L\mathrm{p}_2 v\mathrm{p}_1))$

This formula is a conjunction $(\phi \wedge \psi)$, the first conjunct of which:

- $\phi := \exists x((CFx \wedge \exists y(OHxy \wedge Iy)) \wedge \exists uSVxu)$

contains no pronouns, and the second one

- $\psi := \exists v(OD\mathrm{p}_2 v \wedge L\mathrm{p}_2 v\mathrm{p}_1))$

contains three occurrences of two pronouns. So $r(\phi) = 0$, and $r(\psi) = 2$, the highest index of a pronoun here. We also see that ϕ contains three existential quantifiers, not in the scope of a negation, so that $n(\phi) = 3$; and ψ contains one, so that $n(\psi) = 1$, and $n(\phi \wedge \psi)$, the domain of the whole formula is 4. The range of the whole formula, $r(\phi \wedge \psi)$ is the largest among $r(\phi) = 0$ and $(r(\psi) - n(\phi)) = (2 - 3) = -1$ which is 0.

Putting things together, I will, according to the stated conventions, evaluate such a conjunction relative to a sequence $\widehat{ace}$, where:

- the length of $\widehat{e}$ is a least 0; the length of $\widehat{c}$ is 3; the length of $\widehat{a}$ is 1.

It will turn out that the above formula, for such a sequence $\widehat{ace}$ of length 4, of course, imposes no requirements on $\widehat{e}$; it will require $\widehat{c}$ to consist of 3 elements, a Canadian Farmer c, a veterinarian v that the farmer c sees, and an ill horse h that the farmer owns; the sequence $\widehat{a}$, a one element sequence only, is furthermore required to consists of a donkey d which the veterinarian owns, and which she lends to the farmer. If one tries to evaluate a formula ϕ relative to a sequence $\widehat{e}$ which cannot be split along the above lines, that is, a sequence that is too short, then interpretation crashes. Let us now turn to the definition which actually gives us this notion of satisfaction.

Definition 4 (*PLA Satisfaction and Truth*)

- $[c]_{M,g,\widehat{e}} = I(c)$; $[x]_{M,g,\widehat{e}} = g(x)$; $[\mathrm{p}_i]_{M,g,\widehat{e}} = \widehat{e}_i$;
- $M, g, \widehat{e} \models Rt_1 \ldots t_n$ iff $\langle [t_1]_{M,g,\widehat{e}}, \ldots, [t_n]_{M,g,\widehat{e}} \rangle \in I(R)$;
  $M, g, \widehat{e} \models \neg\phi$ iff there is no $\widehat{c} \in D^{n(\phi)}$: $M, g, \widehat{ce} \models \phi$;
  $M, g, b\widehat{ce} \models \exists x\phi$ iff $M, g[x/b], \widehat{ce} \models \phi$;
  $M, g, \widehat{ace} \models (\phi \wedge \psi)$ iff $M, g, \widehat{ce} \models \phi$ and $M, g, \widehat{ace} \models \psi$;
- ϕ is true relative to M, g and $\widehat{e}$ iff there is a $\widehat{c} \in D^{n(\phi)}$: $M, g, \widehat{ce} \models \phi$.

(In case ϕ is true relative to M, g and $\widehat{e}$, I also write $M, g, \widehat{e} \models \phi$. This notation is the same as the one used for the satisfaction of atomic formulas and negations; this should not be problematic since for these formulas truth and satisfaction coincide.)

Atomic formulas are evaluated in the Tarskian way, relative to sequences of individuals. A pronoun p_i selects the ith individual in the sequence $\widehat{e}$, claiming, and as a matter of fact effectuating, coreference with a variable existentially quantified over i-terms ago. With an atomic formula $Rt_1 \ldots t_n$ the references of the sequence of terms $t_1 \ldots t_n$ are required to stand in the said relation R. (With an identity formula $t_1 = t_2$ the references of t_1 and t_2 are required to be identical.)

An existentially quantified formula $\exists x\phi$ behaves like an ordinary quantifier in the sense that it binds free occurrences of the variable x in its scope. In addition, the first element of the sequence parameter is required to be a 'witness' of x for the satisfaction of the embedded formula ϕ. We may understand this best in a constructive, or 'dynamic' way. If ϕ can be satisfied relative to a sequence $\widehat{ce}$ with b as a witness for x, then $\exists x\phi$ can be said to contribute b to the sequence of possible witnesses, thus delivering $b\widehat{ce}$ as a witness sequence. With the notation conventions employed this means that $b\widehat{c}$ is a sequence of witnesses contributed by $\exists x\phi$, relative to M and g, and relative to a sequence of witnesses $\widehat{e}$ brought up by preceding discourse. Notice that the witness b for the existential quantifier is put in the front of the sequence $\widehat{ce}$. This reflects the fact that the item introduced most recently is the first to be selected for pick-up afterwards. If the next formula ψ uses a pronoun p_1 to refer to the item introduced last, i.e., to the witness of the last term which has occurred before the pronoun, it will indeed pick up the witness b from the sequence $b\widehat{ce}$ relative to which ψ is evaluated. The combined interpretation of existential quantifiers and pronouns thus displays the truly indexical nature of the pronouns. A pronoun's reference can only be determined from the location in the formula where it occurs.

Here is an elementary example in which two typical formulas are combined in a conjunction. (Conjunction will be explained in more detail below.)

(6) There is a boy in the garden. He sneezes.
$(\exists x BGx \wedge S\mathrm{p}_1)$.

In a model M, and relative to a sequence $\widehat{e}$ (which turns out to be irrelevant), the first conjunct is satisfied by an individual b iff b is a boy_in_the_garden in the model, i.e., iff $b \in I(BG)$. For, $M, g, \widehat{be} \models \exists x BGx$ iff $M, g[x/b], \widehat{e} \models BGx$ iff $[x]_{M,g[x/b],\widehat{e}} = g[x/b](x) = b \in I(BG)$. Relative to this sequence $\widehat{be}$ the second conjunct is true iff the witness b for the first conjunct sneezes, i.e., iff $b \in I(S)$. For, $M, g, \widehat{be} \models S\mathrm{p}_1$ iff $[\mathrm{p}_1]_{M,g,\widehat{be}} = (\widehat{be})_1 = b \in I(S)$. In a model M the whole conjunction is, thus, satisfied by a sequence $\widehat{be}$ iff b is a boy_in_the_garden who sneezes in M, and the conjunction is true iff there is such a sequence, that is, iff there is a boy_in_the_garden who sneezes in M, i.e., iff $I(BG) \cap I(S) \neq \emptyset$.

Real quantificational effects in *PLA* are obtained by negation (and by the corresponding notion of truth). The negation of a formula ϕ requires that there are no witnesses for the existentially quantified variables in ϕ, thus obtaining the usual effect of denying the quantified variables a satisfying instantiation. (In the definition $D^{n(\phi)}$ is the set of $n\phi$)-tuples of individuals, the set of sequences that might potentially be witnesses for existentially quantified terms in ϕ.) A formula like $\neg\exists x Fx$ thus means, as usual, that no individual has the property F. Saying that a formula ϕ is true is claiming that there *are* witnesses for the existentially quantified variables in ϕ, thus stating that ϕ *can* be satisfied. The only difference with plain satisfaction is that in stating the truth of a formula one does not keep track of the witnesses by means of which it is satisfied.

Let us now inspect the clause defining satisfaction of conjunctions. If such a conjunction $(\phi \wedge \psi)$ is evaluated relative to a sequence $\widehat{e}$, the first conjunct ϕ is evaluated relative to $\widehat{e}$ and may contribute a sequence $\widehat{c}$, so that $\widehat{ce}$ satisfies ϕ. The second conjunct ψ is then evaluated relative to $\widehat{ce}$, that is the original context plus the contribution $\widehat{c}$ from ϕ, and it may contribute its own sequence $\widehat{a}$ next. Such a conjunction is thus taken to be satisfied by a sequence $\widehat{ace}$ where $\widehat{a}$ satisfies ψ relative to $\widehat{ce}$, and ϕ is satisfied by $\widehat{c}$ relative to $\widehat{e}$. It may be noticed, as example (6) already illustrates, that the interplay between existential quantifiers and pronouns only comes off the ground through the conjunction of the respective elements. (This is interesting, because we may as well choose to correlate the conjuncts in a different way. Nothing stands in the way of a notion of conjunction in which pronouns in the first conjunct get related to quantifiers in the second, as it also may happen in natural language by the way. Notice that such an alternative way of conjoining information contents is hardly conceivable in rigid systems of dynamics semantics, in which the left to right interpretation is taken to be constitutive of the meanings of formulas.)

Let us consider a slightly more involved example in some detail.

(7) There once was a king. He lived in a castle.
$(\exists x Kx \wedge \exists y(Cy \wedge LI\mathrm{p}_1 y))$.

The whole formula $(\phi \wedge \psi)$ is a conjunction of two formulas ϕ and ψ each of which contains one existential quantifier. Neither of them occur in the scope of a negation, so $n(\phi \wedge \psi) = (n(\phi) + n(\psi)) = (1 + 1) = 2$. The second conjunct contains a pronoun, and its range $r(\psi) = 1$, but this is met by the item introduced by the first conjunct, because $n(\phi) = 1$, so the range $r(\phi \wedge \psi)$ of the whole is 0. Unlike the second conjunct, the whole conjunction counts as resolved. The conjunction is satisfied by two individuals c and k iff k satisfies the first conjunct $(\exists x Kx)$ and ck satisfies the second conjunct $(\exists y(Cy \wedge LI\mathrm{p}_1 y))$. This is the case if k is a king and c a castle which the king lives in. Formally this is spelled out as follows.

- $M, g, ck \models (\exists x Kx \wedge \exists y(Cy \wedge LI\mathrm{p}_1 y))$ iff
  $M, g, k \models \exists x Kx$ and $M, g, ck \models \exists y(Cy \wedge LI\mathrm{p}_1 y)$ iff
  $M, g[x/k] \models Kx$ and $M, g[y/c], k \models (Cy \wedge LI\mathrm{p}_1 y)$ iff
  $M, g[x/k] \models Kx$ and $M, g[y/c], k \models Cy$ and $M, g[y/c], k \models LI\mathrm{p}_1 y$ iff
  $k \in I(K)$ and $c \in I(C)$ and $\langle k, c\rangle \in I(LI)$.

The conjunction of the two formulas, with an existential quantifier in the first and a pronoun in the second, is true if we can find a witness sequence for this example. That is, the conjunction is true in a model M iff that model hosts a king k (an element of $I(K)$) and a castle c (an element of $I(C)$) such that the first lived in the second ($\langle k, c\rangle \in I(LI)$). The connection between the existential quantifier in the first conjunct and the pronoun in the second is properly taken care of.

The observations on *PLA*'s notion of conjunction bring to bear on the *PLA* notion of material implication. Recall the classical definition of $(\phi \to \psi)$ as $\neg(\phi \wedge \neg\psi)$. Observe that, according to this definition, the implication inherits the combinatory effects of *PLA*'s dynamic conjunction, but with a different force. Working through the clauses for negation and conjunction, an implication is seen to be satisfied under the following conditions.

Observation 3 (Implication Satisfaction)

- $M, g, \widehat{e} \models (\phi \to \psi)$ iff for all $\widehat{c} \in D^{n(\phi)}$ there is $\widehat{a} \in D^{n(\psi)}$:
  if $M, g, \widehat{ce} \models \phi$ then $M, g, \widehat{ace} \models \psi$.

A formula $(\phi \to \psi)$ is satisfied in a context $\widehat{e}$ iff the consequent clause ψ is satisfied relative to *all* witness sequences satisfying the antecedent ϕ in that context. In effect, this implies that variables existentially quantified over in the antecedent are read with universal force. Consider the following simple example.

(8) If someone is a king, he lives in a castle. $(\exists x Kx \to \exists y(Cy \wedge LI\mathrm{p}_1 y))$.

The implication requires that for every witness of the antecedent, i.e., for every king, there is a witness for the consequent, a castle which he lives in. So for the implication to be satisfied, every king must live in some castle.

From the explanatory remarks so far it may be clear that (1) *PLA*'s semantics is a proper extension of a classical semantics in both form and content, and (2) that the dynamics resides in the actual conjunction of information. The format of the clauses in the above definition is classical, *but* for the use of an additional parameter, that of witness sequences; and the contents are classical as well, *but* for the additional requirements (dynamically) imposed on these witness sequences. These two features are persistent. In the extensions proposed below the basic definitions remain classical and the only additional machinery relates to the additional parameter of witnessing sequences.

Let us state the first feature mentioned in a precise manner. Let *PL* stand for ordinary first order predicate logic.

Observation 4 (PLA and PL) For all formulas ϕ without pronouns

- for all M, g: $M, g \models_{PL} \phi$ iff $M, g \models_{PLA} \phi$.

Since pronouns, the additional category of terms in *PLA*, relate to existential quantifiers, but do not interfere with them, the usual laws of quantification remain valid. In particular, α-conversion is a meaning preserving substitution operation under the usual conditions, and throughout the *PLA*-language.

Observation 5 (α-conversion) If y is free for x in ϕ, then

- for all $M, g, \widehat{ce}$: $M, g, \widehat{ce} \models \exists x\phi$ iff $M, g, \widehat{ce} \models \exists y[y/x]\phi$,

where $[y/x]\phi$ is obtained from ϕ by replacing all free occurrences of x in ϕ with y.

This fact may rightly surprise only those who are familiar with *DPL*, in which this fact fails to obtain. Since pronouns are not conflated with variables in *PLA*, α-conversion holds unconstrained, and this fact will turn out to be important when we turn to *PLA*'s resolving binding forms below.

Conjunction and Resolution in PLA

In this section I describe in more detail how anaphoric connections are established in *PLA*, so as to pave the ground for a fully general comparison with other, static and dynamic, systems of interpretation. I consider a number of constructions which display the dynamic properties of *PLA*'s conjunction and implication in a relatively natural way. Conjunctions (or implications) with pronouns in them are shown to be equivalent to certain quantified constructions which are pronoun free. Consider the following example.

(9) A man is walking in the park. ($\exists x(Mx \wedge Wx)$.) There is a dog. ($\exists y Dy$.) It frightens him and he chases it. ($(F\mathtt{p}_1\mathtt{p}_2 \wedge C\mathtt{p}_2\mathtt{p}_1)$.)

As noted above, I use rather stilted examples, and for the purpose of exposition I neglect all temporal and situational aspects of the interpretation of these sentences. The above sequence of sentences, with the given translation, fits the equation below. I use $\Leftrightarrow$ to indicate that two formulas are the same in meaning, that is, by definition, $\phi \Leftrightarrow \psi$ iff for all $M, g, \widehat{ce}$: $M, g, \widehat{ce} \models \phi$ iff $M, g, \widehat{ce} \models \psi$.

- $((\exists x(Mx \wedge Wx) \wedge \exists y Dy) \wedge (F\mathtt{p}_1\mathtt{p}_2 \wedge C\mathtt{p}_2\mathtt{p}_1)) \Leftrightarrow$
  $\exists y \exists x(((Mx \wedge Wx) \wedge Dy) \wedge (Fyx \wedge Cxy))$.

The equivalence can be easily calculated. The second formula requires valuations of the variables y and x as d and m, respectively, by means of which all of the formulas Mx, Wx, Dy, Fyx, and Cxy are satisfied; if these requirements are met, then dm constitutes a witness sequence for the formula. The first formula, the one from example (9), achieves precisely the same effect, but in a step by step manner. First it seeks a valuation of x by means of which Mx and Wx are satisfied; any such valuation m counts as a witness; then it seeks a valuation of y by means of which Dy is satisfied, and such a witnessing dog d is paired with the man m in the sequence dm; finally it is required that the first of dm (i.e., the dog d) frightens the second (i.e., the man m) and that the man m chases the dog d. If the tests succeed, then the sequence dm satisfies the whole conjunction. The satisfaction conditions are, thus, identical to those of the existentially quantified formula. The equivalence above shows that the same can be expressed with a pronoun free formula, or, rather, that what can be expressed by a rather involved existentially quantified formula can be expressed in a constructive, step by step, manner as well.

The next example is a variant of the previous one with an implication.

(10) If a man is walking in the park ($\exists x(Mx \wedge Wx)$), and there is a dog ($\exists y Dy$), then ($\rightarrow$) it frightens him and he chases it ($F\mathtt{p}_1\mathtt{p}_2 \wedge C\mathtt{p}_2\mathtt{p}_1$).

- $((\exists x(Mx \wedge Wx) \wedge \exists y Dy) \rightarrow (F\mathtt{p}_1\mathtt{p}_2 \wedge C\mathtt{p}_2\mathtt{p}_1)) \Leftrightarrow$
  $\forall y \forall x(((Mx \wedge Wx) \wedge Dy) \rightarrow (Fyx \wedge Cxy))$.

As we have seen in Sect. 2.2, an implication requires its consequent clause to be satisfied relative to all witness sequences for its antecedent clause. The above implication, thus, is satisfied if relative to *all* pairs dm of a dog d and a man m who is walking in the park, the consequent clause ($F\mathtt{p}_1\mathtt{p}_2 \wedge C\mathtt{p}_2\mathtt{p}_1$) is satisfied. That is, it requires of *any* such pair dm that the first (the dog d) frightens the second (the man m), and that the man m chases the dog d. These truth conditions are typically rendered by the (equivalent) formula $\forall y \forall x(((Mx \wedge Wx) \wedge Dy) \rightarrow (Fyx \wedge Cxy))$, which is pronoun free.

Notice that the pronouns $\mathtt{p}_1$ and $\mathtt{p}_2$ in the examples (9) and (10) refer to the last and the penultimate introduced term with reference to the location where these pronouns occur. Thus, $\mathtt{p}_1$ picks up the dog there, introduced *last* in these examples, and $\mathtt{p}_2$ the man, introduced *earlier*. The order of appearance is the reverse of the order of salience so to speak, due to the essentially indexical interpretation of the pronouns. In the next two examples this pattern is different.

(11) A diver found a pearl. ($\exists x(Dx \wedge \exists y(Py \wedge Fxy))$.) She lost it again. ($L\mathtt{p}_1\mathtt{p}_2$.)

We find two existentially quantified formulas again, both of which are picked up by a subsequent pronoun. This time, however, the quantified expressions do not properly succeed one another, but one of them is embedded in the other. The two existential quantifiers require a pair of witnesses, of a diver d and a pearl p the diver found. If one carefully inspects the satisfaction clause for existentially quantified formulas, this pair of witnesses will appear in the order of presentation, as dp. For, first $(Dy \wedge Oxy)$ will have to be evaluated relative to $g[x/d][y/p]$ for some diver d and pearl p, and if this is successful p is added to the contextual sequence as a witness for $\exists y(Py \wedge Fxy)$. Only then $(Dx \wedge \exists y(Py \wedge Fxy))$ can be evaluated relative to $g[x/d]$ and if this is successful indeed the diver d is added to the sequence creating dp as a witness for $\exists x(Dx \wedge \exists y(Py \wedge Fxy))$. For this reason, the next conjunct

$L\mathrm{p}_1\mathrm{p}_2$ states that the last witness constructed, which is the diver d, not the pearl p, lost the one constructed earlier, i.e., the pearl, not the diver. For this reason the sequence of sentences turns out to be equivalent to "A diver lost a pearl she found."

- $(\exists x(Dx \wedge \exists y(Py \wedge Fxy)) \wedge L\mathrm{p}_1\mathrm{p}_2) \Leftrightarrow$
  $\exists x \exists y((Dx \wedge (Py \wedge Fxy)) \wedge Lxy).$

Although the terms 'A diver' and 'a pearl' literally occur in this order, and that one first hears of a diver, and then of a pearl, the witnesses they introduce are raised in reverse order, because we have to have a pearl first, to make up the property of finding that pearl, before we can set up a witness for a diver, who has that property of finding that pearl. Since, in this sense, the diver comes *after* the pearl here, it is the first in prominence when the pronouns are interpreted.

With the witness order in mind, we may now inspect the museum-piece donkey sentence, which is an implicative variant of example (11).

(12) If a farmer owns a donkey, he beats it.
$(\exists x(Fx \wedge \exists y(Dy \wedge Oxy)) \rightarrow B\mathrm{p}_1\mathrm{p}_2).$

The antecedent of the implication $(\exists x(Fx \wedge \exists y(Dy \wedge Oxy)))$ is satisfied by any pair of witnesses fd, where f is a farmer who owns a donkey d. By the satisfaction clause for implications (Observation 3), the formula requires $B\mathrm{p}_1\mathrm{p}_2$ to be satisfied by any such sequence. That is, for any such pair fd the first (the farmer) is required to beat the second (the donkey). The truth-conditions of this example then are adequately rendered by the equivalent sentence "Every farmer beats every donkey he owns."

- $(\exists x(Fx \wedge \exists y(Dy \wedge Oxy)) \rightarrow B\mathrm{p}_1\mathrm{p}_2) \Leftrightarrow$
  $\forall x \forall y((Fx \wedge (Dy \wedge Oxy)) \rightarrow Bxy).$

The preceding observations are summed up in the following example, in which three items are introduced.

(13) Once there was a queen. ($\exists x Qx$.) Her son fell in love with a frog. ($\exists y(Sy \wedge \exists z(Fz \wedge Lyz))$.) He kissed it, and she got mad. ($(K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3)$.)

The first conjunct may be satisfied by a (any) (former) queen q. The second is satisfied by any pair sf of a son s and a frog f the son fell in love with. The first two conjuncts thus will be satisfied, if by anything, by triples sfq of a son, a frog and a queen, the mother of the son. In this order these contextually given entities are addressed by the last conjunct $K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3$, requiring the first (the son) to have kissed the second (the frog), to the effect that the third (the mother / queen) got mad. Again the order of the witnesses is the reverse of the order of appearance. The truth conditions are captured in an equivalent existentially quantified formula.

- $((\exists x Qx \wedge \exists y(Sy \wedge \exists z(Fz \wedge Lyz))) \wedge (K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3)) \Leftrightarrow$
  $\exists y \exists z \exists x((Qx \wedge (Sy \wedge (Fz \wedge Lyz))) \wedge (Kyz \wedge Mx)).$

It is interesting to see, then, that the contents of rather involved existentially quantified formulas can always be displayed in a step by step manner using the pronominal devices of *PLA*.

So far we have been looking at some examples of sentences or sequences of sentences in which pronouns occur, and which are equivalent to ordinary pronoun free first order formulas. These 'normalized', or 'binding' alternatives can be defined fully generally. I will present a 'normalization' algorithm below, which essentially draws from the following equation presented first. (Here, and in what follows, if $\widehat{x}$ is a sequence of variables $x_1 \ldots x_n$, then $\exists\widehat{x}\phi$ abbreviates $\exists x_1 \ldots \exists x_n \phi$.) (The notion of a (normal) binding form is given in the normalization algorithm.)

Observation 6 (Binding Conjunctions) If $\widehat{x}$ is a sequence of variables $x_1 \ldots x_n$, and ϕ and ψ are closed and in binding form, and if the variables in $\widehat{y}$ respectively $\widehat{x}$ do not occur free in ϕ respectively ψ, then:

- $(\exists\widehat{x}\phi \wedge \exists\widehat{y}\psi) = \exists\widehat{y}\exists\widehat{x}(\phi \wedge [\widehat{x}/\mathrm{p}_i]\psi)$, where
  - $[x_1 \ldots x_n/\mathrm{p}_i]\psi = [\mathrm{p}_{r(\psi)-n}/\mathrm{p}_{r(\psi)}] \ldots [\mathrm{p}_1/\mathrm{p}_{n+1}][x_n/\mathrm{p}_n] \ldots [x_1/\mathrm{p}_1]\psi$, and
  - the variables x_i are free for the occurrences of p_i in ψ.

The conditions on this equation are that ϕ and ψ themselves are closed and in binding form, i.e., they do not contain 'active' existential quantifiers, and no locally resolved pronouns. (Cf., the binding algorithm below.)

Even though the conditions may appears rather complicated at first glance, the idea behind them is conceptually and computationally pretty simple. Observation (6) shows that quantifiers $\exists x_i$ from the left conjunct may take scope over the right conjunct, *if* pronouns p_i coreferential with them are replaced by variables x_i now bound by the quantifier. (The details of this are explained in a second.) When the scope of these existential quantifiers $\exists\widehat{x}$ is, thus, extended, we have to make sure that existential quantifiers $\exists\widehat{y}$ from the second conjunct, which were introduced later, still make their contribution after the $\exists\widehat{x}$ have done so. This is why the $\exists\widehat{y}$ from the second conjunct now also are assigned scope over the existential quantifiers $\exists\widehat{x}$. In order to make sure that no further existential quantifiers and resolved pronouns are left in the embedded formulas ϕ and ψ, they are required to be closed and in binding form before this resolution takes place.

In order for this binding reformulation to be equivalent, two kinds of things have to happen in the second conjunct ψ. Since ψ is assumed to be closed and resolved, there are no internally resolved pronouns there. So all pronouns either relate back to terms in the first conjunct ϕ, or they relate back to terms before the conjunction. Pronouns of the first kind, viz., those pronouns p_i with $i \leq n(\phi)$, are eliminated and replaced by the variable x_i bound by the ith existential quantifier from $\exists\widehat{x}$, which comes to take scope over ψ. Pronouns of the second kind, viz., those pronouns $\mathrm{p}_{n(\phi)+j}$ lose $n(\phi)$ preceding terms as an antecedent (the $n(\phi)$ existential quantifiers from ϕ) so their index reduces to j in p_j. They thus remain coreferential with the jth term before the whole conjunction. I will supply examples of the above observation after I have presented the *PLA*-binding algorithm.

Observation (6) can be used to produce the (normal) binding forms of a *PLA*-formula, which is an equivalent formula from with resolved pronouns removed. We obtain the binding form of a formula ϕ by questioning it as $[\phi]^?$, which will return an answer $[\psi]^!$ to the effect that ψ indeed is the binding form of ϕ, also written as $\phi^\bullet$.

Definition 5 (*Binding Algorithm*) The (normal) binding form $\phi^\bullet$ of a formula ϕ is the formula ψ such that $[\phi]^? \mapsto [\psi]^!$, where:

- $[(\phi \wedge \psi)]^? \mapsto ([\phi]^? \wedge [\psi]^?)$; $([\exists\widehat{x}\phi]^! \wedge [\exists\widehat{y}\psi]^!) \mapsto [\exists\widehat{y}\exists\widehat{x}(\phi \wedge [\widehat{x}/\mathrm{p}_i]\psi)]^!$ [1];
  $[\exists x\phi]^? \mapsto \exists x[\phi]^?$; $\exists x[\phi]^! \mapsto [\exists x\phi]^!$;
  $[\neg\phi]^? \mapsto \neg[\phi]^?$; $\neg[\phi]^! \mapsto [\neg\phi]^!$;
  $[Rt_1 \ldots t_m]^? \mapsto [Rt_1 \ldots t_m]^!$.

The questioning procedure moves us directly, top down, to the atomic subformulas of a formula ϕ, in order to observe that these are in binding form: $[Rt_1 \ldots t_m]^? \mapsto [Rt_1 \ldots t_m]^!$. This reflects the fact that in a solitary atomic formula pronouns cannot get resolved. Once a formula ϕ is in binding form, its negation $\neg\phi$ is as well, and also the existentially quantified $\exists x\phi$. In a negation and in an existentially quantified formula there is nothing to be resolved besides what can be resolved in the embedded (negated or quantified) formula. This means that the feature of being in binding form percolates bottom up through quantifiers and negations, until it reaches a conjunction. And such a conjunction, then, will always be of the form $(\exists\widehat{x}\phi \wedge \exists\widehat{y}\psi)$, where ϕ and ψ are closed and in binding form. The resulting translation is like the one given in Observation (6). From this definition it is directly clear where the resolution/binding does happen, that is, in a conjunction, in which pronominal relations are after all established. This happens in the way we have seen in Observation (6).

Observe that, of course, it may happen that the side conditions on observation (6) fail to hold, if variables in $\widehat{y}$ or $\widehat{x}$ *do* occur free in ϕ or ψ, respectively, or if an x_i is not free for a p_i in ψ. However, in all of these cases we can always use an α-converted variant of the formula to be resolved, which, by observation (5) above, is fully equivalent.

I now present an application of the Binding Algorithm to example (13), repeated here for convenience:

(13) Once there was a queen. Her son fell in love with a frog. He kissed it, and she got mad. $((\exists x Qx \wedge \exists y(Sy \wedge \exists z(Fz \wedge Lyz))) \wedge (K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3))$.

The algorithm applies as follows.

- $[((\exists x Qx \wedge \exists y(Sy \wedge \exists z(Fz \wedge Lyz))) \wedge (K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3))]^? \mapsto$
  $((\exists x[Qx]^? \wedge \exists y([Sy]^? \wedge \exists z([Fz]^? \wedge [Lyz]^?))) \wedge ([K\mathrm{p}_1\mathrm{p}_2]^? \wedge [M\mathrm{p}_3]^?)) \mapsto$
  $((\exists x[Qx]^! \wedge \exists y([Sy]^! \wedge \exists z([Fz]^! \wedge [Lyz]^!))) \wedge ([K\mathrm{p}_1\mathrm{p}_2]^! \wedge [M\mathrm{p}_3]^!)) \mapsto$
  $(([\exists x Qx]^! \wedge \exists y([Sy]^! \wedge \exists z[(Fz \wedge Lyz)]^!)) \wedge [(K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3)]^!) \mapsto$

[1] provided that the conditions in observation (6) obtain.

$$
\begin{aligned}
&(([\exists x Qx]^! \wedge \exists y([Sy]^! \wedge [\exists z(Fz \wedge Lyz)]^!)) \wedge [(K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3)]^!) \mapsto \\
&(([\exists x Qx]^! \wedge \exists y[\exists z(Sy \wedge (Fz \wedge Lyz))]^!) \wedge [(K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3)]^!) \mapsto \\
&(([\exists x Qx]^! \wedge [\exists y \exists z(Sy \wedge (Fz \wedge Lyz))]^!) \wedge [(K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3)]^!) \mapsto \\
&([\exists y \exists z \exists x(Qx \wedge (Sy \wedge (Fz \wedge Lyz)))]^! \wedge [(K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3)]^!) \mapsto \\
&[\exists y \exists z \exists x((Qx \wedge (Sy \wedge (Fz \wedge Lyz))) \wedge (Kyz \wedge Mx))]^!.
\end{aligned}
$$

All in all, we find that:

- $((\exists x Qx \wedge \exists y(Sy \wedge \exists z(Fz \wedge Lyz))) \wedge (K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3))^\bullet =$
  $\exists y \exists z \exists x((Qx \wedge (Sy \wedge (Fz \wedge Lyz))) \wedge (Kyz \wedge Mx))$.

Before we carry on, it is interesting to observe that indeed all the action in the binding algorithm takes place in the rule dealing with conjunctions—because conjunctions are the place where connections get established and pronouns may get resolved. Also, this is quite the same as what happens in the translation of *DPL* into a static predicate logic as given in Cresswell (2002). There, as well, the crucial work is done in the clause for conjunctions, in which, too, major substitutions take place. Notice, however, that Cresswell's specification of the *DPL* notion of conjunction, in terms of a number of substitutions, is a way of rendering a dynamic interpretation of discourse in terms of a static first order translation. The translation, however, is stipulated, and it is not based on an independently specified semantics for the discourse itself. Distinctively, the substitutions we find in Observation (6) are sound with respect to the semantics of *PLA*, which has been independently specified above. The *PLA*-semantics itself, of course, is in no need of any substitutions.

The following observation shows the resolution to be correct.

Observation 7 (Resolution Correctness)

- For all formulas ϕ: $\phi^\bullet \Leftrightarrow \phi$.

This observation enables a straightforward comparison of *PLA* with kindred systems.

Observation 8 (PLA, PL, DRT and DPL) For all resolved formulas ϕ

1. for all M, g: $M, g \models_{PL} \phi^\bullet$ iff $M, g \models_{PLA} \phi$;
2. a *DRS* representing the contents of ϕ is isomorphic to $\phi^\bullet$, up to alphabetic equivalence;
3. for all M, g: $g[\![\phi]\!]_M g[\widehat{x}/\widehat{c}]$ iff $M, g, \widehat{c} \models \phi$,

where $\phi^\bullet = \exists\widehat{x}\psi$ with ψ closed, and there are no repetitions of quantifiers $\exists x$ in ϕ.

The first observation is a direct consequence of the Observations (4) and (7). The resolution of any resolved formula is pronoun free, and, hence, classical.

The second observation builds on the fact that ordinary discourse representation structures (*DRS* s) are of the form $\langle D, C\rangle$, where D is a set or sequence of discourse referents, and C a set or sequence of conditions, and the conditions are atomic formulas, or negations *DRS* s (or, for that matter, disjunctions of, or implications between *DRS* s). The resolution of a *PLA*-formula consists of a sequence of existential quantifiers $\exists\widehat{x}$ followed by a sequence of conjunctions of atomic formulas and negations, where in each negation $\neg\phi$ the formula ϕ is in resolved form, i.e., a *DRS* . Notice that while the architecture reaching normal binding forms of *PLA*-formulas is not stated in a compositional fashion, neither is the construction algorithm yielding *DRS* for natural language, or natural discourse. However, the interpretation function for the *PLA*-formulas themselves gives us the very same results, but in a direct and compositional way. If we put these facts in a illustrative row, for instance for the previous example (13), we get the following picture. First we repeat the sentence; then we translate in a fairly standardly fashion into the language of predicate logic (with anaphora); next we show the result of applying the normal binding algorithm, which we have seen before, and finally we get the corresponding *DRS* that would have come out of the *DRS* construction algorithm for example (13).

(13) Once there was a queen. Her son fell in love with a frog. He kissed it, and she got mad.
(13′) $((\exists x Qx \wedge \exists y(Sy \wedge \exists z(Fz \wedge Lyz))) \wedge (K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3))$.
(13′•) $\exists y \exists z \exists x((Qx \wedge (Sy \wedge (Fz \wedge Lyz))) \wedge (Kyz \wedge Mx))$.
(DRC(13))

xzy
Qx Sy Fz Lyz Kyz Mx.

Notice, first, that the order of discourse markers in (*DRC*(13)) is opposite to that of the existential quantifiers in (13′•), but this is immaterial; second, that the round brackets, superfluous here, have disappeared—but they are pieces of structure that are not omitted in more sophisticated versions of *DRT* . For the rest we can easily construct isomorphisms, and the reader is invited to do so with examples of his own, using the definitions stated above.

The comparison with *DPL* in Observation (8) is a bit more involved. *DPL*'s input variable assignment g plays the ordinary role of an assignment in *PL*, and *PLA*, of the free variables in a formula. A so-called *DPL*-'output' assignment h, which is $g[\widehat{x}/\widehat{c}]$ in the case above, encodes possible values of variables quantified over in ϕ, viz., the variables in the sequence $\widehat{x}$. Their possible values are the witnesses $\widehat{c}$ of the corresponding quantified constructions in *PLA*, whence the equivalence in observation (8).

Notice that the stated equivalence cannot be maintained if a formula does not have a binding form itself and we have to resort to α-conversion in *PLA*. Since α-conversion is typically not allowed in *DPL*, the correspondence between *DPL* and *PLA* breaks down here. Notice that it is precisely in these cases that *DPL*'s elegance breaks down as well. In these cases where we would have to resort to α-conversion in *PLA*, we find so-called 'variable-clashes' in *DPL*, where information about items introduced in a discourse gets destroyed because a variable in use is re-used for another purpose. This nasty and well-known problem in *DPL* basically originates from the choice to equate pronouns with variables.

It appears that Vermeulen (1993) provides an interesting alternative motivation for moving from *DPL* to *PLA*. In order to solve the mentioned problem with variable clashes, Vermeulen argues that one can model the repeated use of one and the same variable if different uses are taken to invoke *stacks of values*, instead of singular ones. Variables indicate whether they relate to the last, or any earlier element of the stack associated with the variable. As it happens, *DPL* can thus be modeled by stacking all discourse referents on one variable, so that, as a matter of fact, the variable is useless, and we are left with stacks alone, as in *PLA*.

A final note concerns the restriction in Observation (8), that we only deal with resolved formulas ϕ, which have no unresolved pronouns. For a logical system, which cannot itself look outside its context of use, this is a natural restriction. For, similarly, a classical system of predicate logic cannot say much about the resolution or interpretation of free variables, besides assuming they have some value.

2.3 Logical Properties of PLA

The last part of this section is devoted to the general logical properties of *PLA*. Not much would be gained if *PLA* did not fail *some* classical logical properties, and it fortunately does do so. *PLA*-conjunction, the typically 'dynamic' *PLA*-operation, fails two characteristic properties of its standard kin: idempotence and commutativity.
Observation 9 (Non-idempotence and Non-commutativity)

- *PLA* conjunction is not idempotent and not commutative.

Since the idea embodied in a system of dynamic semantics is that the occurrence of a formula is not only context dependent, but also context changing, clearly these two properties are under attack. If a formula ϕ contains pronouns (and its interpretation is, hence, dependent on context) and if it also contributes items (and, hence, changes the context), ϕ may have a different impact before and after it has occurred, i.e. in $(\phi \wedge \phi)$. Let us label ϕ_1 the first occurrence of ϕ in $(\phi \wedge \phi)$, and ϕ_2 the second. Then we can say that the context for ϕ_2 is the context for ϕ_1 *plus* what ϕ_1 has contributed to the context in the meantime; and the contribution of ϕ_1 is present, but put behind what ϕ_2 contributes. The conjunction $(\phi \wedge \phi)$ thus may be stronger than ϕ itself. Here is a stilted example.

(14) She is seeing a woman. She is seeing a woman.
$(\exists y(Wy \wedge S\mathrm{p}_1 y) \wedge \exists y(Wy \wedge S\mathrm{p}_1 y)) \Leftrightarrow$
$\exists y \exists x((Wx \wedge S\mathrm{p}_1 x) \wedge (Wy \wedge Sxy))$.

The effect of the first occurrence of $\phi = \exists y(Wy \wedge S\mathrm{p}_1 y)$ consists in the contribution of a woman seen by *she*, and this creates a context in which that woman, not the original antecedent of *she*, figures as a target referent for the use of *she* in the second occurrence of ϕ. The repetition of the sentence, *She is seeing a woman*, requires that she, whoever she is, is seeing a woman who is (also) seeing a woman, something which was not asserted by one occurrence of the sentence ϕ alone. Therefore, conjunction is not idempotent.

By the same token, if two formulas ϕ and ψ are both context dependent and context changing, then $(\phi \wedge \psi)$ and $(\psi \wedge \phi)$ are also deemed to be different as well. The context relevant for ψ in $(\phi \wedge \psi)$ is whatever is the context for ψ in $(\psi \wedge \phi)$ *with* the contribution of ϕ added, and the same goes *mutatis mutandis* for the context for ϕ. Moreover, the contributions of ϕ and ψ are ordered differently in both conjunctions. So, conjunction is not commutative.

Of course, this dynamic extension of *PL* affects the accompanying notion of entailment. As we will see, however, besides solicited effects, there are no unsolicited ones. Let me first state the required notion of entailment. (For completeness I add the standard static notion of entailment, which will become useful later.) In the following a sequence of formulas is naturally understood as its conjunction.

Definition 6 (*PLA Entailment*) A sequence of formulas $\phi_1, \ldots, \phi_m$ dynamically entails a formula ψ, denoted by $\phi_1, \ldots, \phi_m \models \psi$, iff

- for all $M, g, \widehat{e} \in D^{r(\phi_1,\ldots,\phi_n,\psi)}$ and $\widehat{c} \in D^{n(\phi_1,\ldots,\phi_m)}$ there is $\widehat{a} \in D^{n(\psi)}$: if $M, g, \widehat{ce} \models (\phi_1, \ldots, \phi_m)$, then $M, g, \widehat{ace} \models \psi$.

Naturally, *PLA* accommodates a notion of entailment involving sequences, not sets, of premises, which, together with the conclusion, are evaluated as to logical consequence. It is required, relative to any model M and variable assignment g, and relative to any potentially required sequence $\widehat{e}$ for unresolved pronouns in $\phi, \ldots, \phi_n, \psi$, in that order, that if any sequence of witnesses satisfies the premises, in the order given, then it provides a context in which ψ is true—always. The idea is classical, and the only difference is that possible witnesses may be passed through in the order of premises to serve as target reference points for subsequent pronouns. Obviously, when restricted to the pronoun free fragment the *PLA*-entailment relation is classical.

Observation 10 (Conservative Entailment) The *PLA* entailment relation $\models$ is classical relative to the pronoun free formulas of $\mathcal{L}_{PLA}$.

Behaving classically in all classical cases does not mean that *PLA* entailment is classical in all respects. The notion is dynamic not only in the rather trivial sense that anaphoric connections may get established between terms in different premises; anaphoric connections are also possible between terms in premises and pronouns in the conclusion. The following inference is modeled after Heim (1982); the translation is mine.

(15) If a man is from Athens, he is not from Rhodes. There is a man from Athens here. So, he is not from Rhodes.
$(\exists x(Mx \wedge Ax) \rightarrow \neg R\mathrm{p}_1), \exists x(Mx \wedge Ax) \models \neg R\mathrm{p}_1$.

The translation, as entailment, is valid in *PLA*. A similar, more natural, example Geach (1962) has taken from Peter Strawson.

(16) A: A man has just drunk a pint of sulphuric acid.
B: Nobody who drinks sulphuric acid lives through the day.
A: Very well then, *he* will not live through the day.
$\exists x(Mx \wedge DPSAx), \neg\exists z(DPSAz \wedge LTDz) \models \neg LTD\mathrm{p}_1$.

The transcription is a valid entailment in *PLA*. We observe natural entailments here, with pronouns in the conclusion, which relate back to terms figuring in the premises, and which are valid entailments.

In the examples from Heim and Strawson we observe, not surprisingly perhaps, a strong connection between premises and conclusion. Heim's man from Athens can be concluded to be not from Rhodes because *no* man from Athens is from Rhodes, at least, this is what one of the premises claims. Likewise, Geach's man cannot live through the day, because he has just a drunk a pint of sulphuric acid, and an additional premise has it that no man who does so will live through the day. The inferential step from an arbitrary man to a conclusion about *him* can only be made because the step is universal, by the logic, and/or by additional premises. This strong (universal) connection between terms in the premises of an entailment, and pronouns in its conclusion, is reminiscent of the strong (universal) connection between terms in donkey sentences. This, again, need not come as a surprise, since it is easily seen that a valid entailment directly corresponds with an unconditionally valid implication.

Observation 11 (Deduction Theorem)

- $\phi_1, \ldots, \phi_m, \chi \models \psi$ iff $\phi_1, \ldots, \phi_m, \models (\chi \rightarrow \psi)$.

Heim's inference directly follows from the following, valid, identity inference.

- $(\exists x(Mx \wedge Ax) \rightarrow \neg R\mathrm{p}_1) \models (\exists x(Mx \wedge Ax) \rightarrow \neg R\mathrm{p}_1)$.

Geach's inference requires some more work, but it can be remodeled to the same form.

The fact that $\models$ subsumes all classical validities (Observation 10), does not imply that it obeys classical structural laws. Obviously it does not, since, as we have seen, entailment is dynamic, and it will not generally be preserved under permutation of the premises. Order matters. Moreover, like notions of entailment from other systems of dynamic interpretation, the *PLA*-entailment relation fails some other structural properties, characteristic of, and sometimes deemed essential for, standard logical systems. Entailment is not a reflexive, monotone, and transitive relation. Good reason exist for these failures though, as I will argue, and I will next show that the consequences are not that bad after all, in *PLA* that is.

Let us inspect the rationale behind a non-reflexive, non-monotone, and non-transitive entailment relation. Like I said before, interpretation is context dependent and context changing. From this it directly follows that, in principle, a formula which is satisfied in a certain context, may also change the context into one which no longer satisfies it. This observation relates to the fact that conjunction is not idempotent. A clear, but artificial, example is the following.

(17) He is an Irish boy, and he wrote a non-Irish friend. So$^?$, *he* (i.e., the friend) is an Irish boy and he wrote a non-Irish friend.
$(IB\mathrm{p}_1 \wedge \exists y(\neg IBy \wedge W\mathrm{p}_1 y)) \not\models (IB\mathrm{p}_1 \wedge \exists y(\neg IBy \wedge W\mathrm{p}_1 y))$.

The formula at issue (or the rendering of it under the intended interpretation) does not entail itself, because that would require a non-Irish friend to be Irish. (Worse, the conjunction of the contingent formula with itself is inconsistent.) Non-monotonicity comes about for basically the same reason. A conclusion may follow from a sequence of premises, because the premises always set up a context in which the conclusion is satisfied, but then an additional premise may undo precisely the relevant contextual effects. Consider the previous example again. From *He is an Irish boy* it follows that

He is an Irish boy. The conclusion, however, does not follow from *He is an Irish boy, and he wrote a non-Irish friend.*

(18) $IB\mathrm{p}_1 \models IB\mathrm{p}_1$, but $IB\mathrm{p}_1, \exists y(\neg IBy \wedge W\mathrm{p}_1 y)) \not\models IB\mathrm{p}_1$.

The non-transitive aspects of entailment already can be witnessed from Johan van Benthem's "dynamic" counterexample to Aristotle's prime example of a valid syllogism, Barbara. Barbara relies on the transitivity of the universal quantifier, and in van Benthem's counterexample the cutting of Barbara's middle term causes the break down of an anaphoric connection.

(19) All men who have a garden sprinkle it on Saturdays.
All men who have a house are men who have a garden.
So$^?$, all men who have a house sprinkle it on Saturdays.

The example has the impact of a practical joke, but it does show a serious problem, which we have to take to heart. For, inference schemes like Barbara, like basically all logical schemes, are supposed to be valid because of their form, and the form of van Benthem's example is impeccably Barbarian. With the following example we see essentially the same problem arise for our notion of entailment.

(20) If Jane has a garden, she sprinkles it right now and if Jane owns a house, she has a garden. Now Jane actually owns a house. So, she has a garden, and, so, she sprinkles it right now.
$((\exists y(Gy \wedge Hjy) \rightarrow Sj\mathrm{p}_1), (\exists x(Hx \wedge Ojx) \rightarrow \exists y(Gy \wedge Hjy)), \exists x(Hx \wedge Ojx) \models (\exists y(Gy \wedge Hjy) \wedge Sj\mathrm{p}_1).)$

In this example the second and the third premise yield the first conclusion, that Jane has a garden, and this conclusion together with the first premise yields the goal conclusion that she sprinkles it. By 'cutting' the inference, taking out the intermediate conclusion that Jane has a garden, we would like to conclude directly from the three premises that she sprinkles it.

(21) If Jane has a garden, she sprinkles it right now and if Jane owns a house, she has a garden. Now Jane actually owns a house. So, she sprinkles it right now.
$((\exists y(Gy \wedge Hjy) \rightarrow Sj\mathrm{p}_1), (\exists x(Hx \wedge Ojx) \rightarrow \exists y(Gy \wedge Hjy)), \exists x(Hx \wedge Ojx) \models Sj\mathrm{p}_1.)$

This obviously sounds absurd and indeed the inference is invalid in *PLA*. By cutting the middle term in the inference we lose the appropriate anaphoric connection gets lost, and the pronoun appears to resolve with the house. The conclusion then, which is *not* entailed, would be that Jane sprinkles the house. Naturally, this inference comes out wrong in *PLA*, but then it shows that it is not in general allowed to 'cut' inferences this way.

Even though the *PLA* entailment relation fails the three mentioned properties, for good reasons I say, it nevertheless retains the valuable aspects of these properties. That entailment is reflexive and monotone is attractive, or even pertinent, because an assumption should, if anything, at least entail itself, and a conclusion ought to remain valid once it is obtained. In *PLA*, *these* facts do remain beyond doubt, however. If a conclusion is established as an assumption or conclusion at a certain point in an argument, then it remains as a valid assumption or conclusion throughout the whole argument; the point is that the same conclusion may have to be reformulated, in order to adjust it to the fact that the context may have changed in the meantime. [The point is nicely put by Frege: "Wenn jemand heute dasselbe sagen will, was er gestern das Wort 'heute' gebrauchend ausgedrückt hat, so wird er dieses Wort durch 'gestern' ersetzen." (Frege 1918, p. 62)] As we will see shortly, such a reformulation can always be given, a possibility which is not obvious in a system like, for instance, *DPL*. Transitivity is also a useful property, because it allows us to re-use inferences made earlier, and not to have to redo every inference time and again. In *PLA* we can transport the results of one inference from one context to another, as long as we keep track of the various changes in context and make suitable reformulations.

In order to state the right structural rules in *PLA*, we need a general device to adapt formulas to the fact that the context around it may have been expanded or reduced. The relevant facts pertain to the number of existentials that have occurred, and that may have increased or decreased. Obviously, such changes in the context should be reflected by corresponding increases and decreases of the index on the relevant pronouns. I therefore define the increase $[^{n+}_{j}]$ with n and a decrease $[^{-n}_{j}]$ with n of the index of pronouns which are selected by the auxiliary device j. The definition is a bit tedious, yet easily computable.

Definition 7 (*Pronoun Update and Downdate*) $[^{n+}_{j}]\phi$ and $[^{-n}_{j}]\phi$ are recursively defined for variables $\widehat{x}$ free for the pronouns in ϕ.

- $[^{n+}_{j}]c = c = [^{-n}_{j}]c$; $[^{n+}_{j}]\mathtt{p}_i = \mathtt{p}_{n+i}$, if $(j < i)$;
  $[^{n+}_{j}]x = x = [^{-n}_{j}]x$; $[^{-n}_{j}]\mathtt{p}_i = x_{i-j}$, if $(j < i \leq (j+n))$;
  $[^{n+}_{j}]\mathtt{p}_i = \mathtt{p}_i = [^{-n}_{j}]\mathtt{p}_i$ if $(i \leq j)$; $[^{-n}_{j}]\mathtt{p}_i = \mathtt{p}_{i-n}$, if $((j+n) < i)$;
- $[^{n+}_{j}]Rt_1 \dots t_n = R[^{n+}_{j}]t_1 \dots [^{n+}_{j}]t_n$; $[^{n+}_{j}]\neg\phi = \neg[^{n+}_{j}]\phi$;
  $[^{-n}_{j}]Rt_1 \dots t_n = R[^{-n}_{j}]t_1 \dots [^{-n}_{j}]t_n$; $[^{-n}_{j}]\neg\phi = \neg[^{-n}_{j}]\phi$;
  $[^{n+}_{j}](\phi \wedge \psi) = ([^{n+}_{j}]\phi \wedge [^{n+}_{j+n(\phi)}]\psi)$; $[^{n+}_{j}]\exists x\phi = \exists x[^{n+}_{j}]\phi$;
  $[^{-n}_{j}](\phi \wedge \psi) = ([^{-n}_{j}]\phi \wedge [^{-n}_{j+n(\phi)}]\psi)$; $[^{-n}_{j}]\exists x\phi = \exists x[^{-n}_{j}]\phi$.

The instruction $[^{n+}_{j}]$ on ϕ updates ϕ to the fact that n more terms have been used, and $[^{-n}_{j}]$ indicates the update of ϕ to the fact that n less terms have been used. The j indicates *where* in the past n more or n less terms have been used, the break-even point so to speak. This means that pronouns are left untouched if they relate to witnesses up to j terms back. Pronouns, however, that refer to witnesses beyond that point will either have to gain a wider reach (increase their index by n) or lower it (decrease their index by n). If less terms have been used than before, and if a pronoun initially targeted its referent from among the terms that have gone, then it will be removed and replaced by a variable. (A variable which eventually is existentially closed.)

In Definition (7), the first half states the required substitutions on terms. In the left column nothing happens, because it concerns individual constants, variables, and pronouns with an index up to j, which are not affected. In the right column the upgrade and downgrade get accounted for, in the manner just sketched. The second half of the definition states the required effects for any formula in a recursive manner. Except for the second conjunct of a conjunction, this effect involves a simple matter of distribution. In a conjunction $(\phi \wedge \psi)$, however, the instruction on the second conjunct ψ has to adapt to the fact that $n(\phi)$ more terms have occurred between ψ and the break-point j, which means that the break-even point has to be updated to $j + n(\phi)$.

Armed with this notational device we can state acute versions of identity, monotonicity and a cut rule, the rules which motivate the reflexive, monotone and transitive nature of entailment.

Observation 12 (Acute Identity and Monotonicity)

- $\phi_1, \ldots, \phi_i, \ldots, \phi_m \models [^{k+}_{0}]\phi_i$
  - for $k = n(\phi_i, \ldots, \phi_m)$, and
- if $\phi_1, \ldots, \phi_m \models \psi$, then $\phi_1, \ldots, \phi_{i-1}, \chi, \phi_i, \ldots, \phi_m \models [^{k+}_{j}]\psi$,
  - for $k = n(\chi)$ and $j = n(\phi_i, \ldots, \phi_m)$.

Any formula ϕ entails itself, or can be repeated, provided that, when it is used as a conclusion, it is updated with the information that more terms have occurred than when it was used before. The conclusion drawn from ϕ is the one obtained from ϕ by replacing unresolved pronouns p_i by p_{k+i}, with k the number of terms that have occurred in the meantime. In the example (17) discussed above, indeed $(IB\mathrm{p}_1 \wedge \exists y(\neg IBy \wedge W\mathrm{p}_1 y))$ does not entail itself:

- $(IB\mathrm{p}_1 \wedge \exists y(\neg IBy \wedge W\mathrm{p}_1 y)) \not\models (IB\mathrm{p}_1 \wedge \exists y(\neg IBy \wedge W\mathrm{p}_1 y))$.

However, it does entail $[^{1+}_{0}](IB\mathrm{p}_1 \wedge \exists y(\neg IBy \wedge W\mathrm{p}_1 y))$ which is $(IB\mathrm{p}_2 \wedge \exists y(\neg IBy \wedge W\mathrm{p}_2 y))$:

- $(IB\mathrm{p}_1 \wedge \exists y(\neg IBy \wedge W\mathrm{p}_1 y)) \models (IB\mathrm{p}_2 \wedge \exists y(\neg IBy \wedge W\mathrm{p}_2 y))$.

Also, if ψ is entailed by a sequence of premises, then the same sequence with an additional premise χ still entails ψ, but after the conclusion is updated with the fact that $n(\chi)$ more terms have occurred. So every pronoun p_i in ψ which has to 'bridge' the interfering terms in χ, is replaced by $\mathrm{p}_{n(\chi)+i}$.

The statement of the cut-inference pattern is a bit more involved, because a mediating conclusion which serves as a premise gets cut out, even though it may have supplied witnesses for the goal conclusion. For this reason, we have to use the $[^{-n}_{j}]$ instruction which deletes pronouns which get rid of their antecedent witness, and replace these with existentially bound variables.

Observation 13 (Acute Cut Rule)

- If $\phi_1, \ldots, \phi_{m-1}, \phi_m \models \psi$ and $\chi_1, \ldots, \chi_l \models \phi'_m$,
  then $\phi_1, \ldots, \phi_{m-1}, \chi_1, \ldots, \chi_l \models \psi'$, where

– $\phi'_m = [^{n(\chi)+}_{0}]\phi_m$, with $\chi = \chi_1, \ldots, \chi_l$ and
  $\psi' = [^{n(\chi)+}_{0}]\exists\widehat{x}[^{-n(\phi_m)}_{0}]\psi$, with $\widehat{x} = x_1 \ldots x_{n(\phi_m)}$.

In the second condition for the cut rule the conclusion ϕ' says what the premise ϕ says in the first condition if $\chi_1, \ldots, \chi_l$ had not occurred. The goal conclusion ψ', is the original conclusion ψ updated to the fact that the premise ϕ_m has been cut out, and $\chi_1, \ldots, \chi_l$ have been added. Pronouns which related back to antecedents in ϕ_m are replaced by variables which get bound by $\exists\widehat{x}$. Here is how the van Benthem-style inference gets cured. We find that (a) and (d) entail (e), and that (b) and (c) entail (d). But we do not find that (a) and (b) and (c) entail (e), but (e′).

(a) If Jane has a garden, she sprinkles it right now. $(\exists y(Gy \wedge Hjy) \rightarrow Sj\mathrm{p}_1)$;
(b) If Jane owns a house, she has a garden. $(\exists x(Hx \wedge Ojx) \rightarrow \exists y(Gy \wedge Hjy))$;
(c) Jane actually owns a house. $(\exists x(Hx \wedge Ojx))$;
(d) Jane has a garden. $(\exists y(Gy \wedge Hjy))$;
(e) She sprinkles it right now. $(Sj\mathrm{p}_1)$;
(e′) She sprinkles something right now. $([^{1+}_{0}]\exists x[^{-1}_{0}]Sj\mathrm{p}_1 \equiv \exists xSjx)$.

Clearly the (derived) cut rule will never fail for any practical purpose, since the conditions on identity or cut never fail. The standard structural properties of logical consequence are thus preserved in a suitably adapted form, and the logic remains well-behaved.

2.4 On the Representation of Information

The achievements of *DRT* , *FCS*, and *DPL*, and all of their offspring, can be characterized as follows. Agents have representations of the world, or of stories, dreams, or impressions they have made up, or have been told, or have experienced. These representations are very often rather large structured wholes and communicating them involves cutting them into pieces. When these wholes have been cut into pieces, and other agents have to glue the pieces together, the structural relations between (the parts of) the pieces have to be re-established of course. The three frameworks mentioned present typical ways in which this may be done, for the typical kind of relationships which, in natural language, we encounter as identity anaphora.

The way in which this task is achieved in classical *DRT* , that of (Kamp 1981; Kamp and Reyle 1993), may be pictured as follows. I will, again, use a rather stilted example, but it displays the essential ingredients. Suppose someone has a picture of the following situation, in which a man, who was walking in the park, ran away from a dog he saw there. To simplify things a bit more we neglect the park and all temporal aspects and then the description, as given, can be represented by means of the following predicate logic formula:

(22) $\exists x((Mx \wedge Wx) \wedge \exists y((Dy \wedge Sxy) \wedge Rxy))$. (A man is walking and there is a dog he sees and which he runs away from.)

In *DRT*, the corresponding representation is the following discourse representation structure (*DRS*):

(23) $\begin{array}{|l|}\hline x \quad y \\ \hline Mx \;\; Dy \\ Wx \;\; Sxy \;\; Rxy \\ \hline \end{array}$

Surely, there is a way of communicating this little 'story' in one sentence, like I did above, but it can be cut in parts, as in the following little discourse.

(24) A man is walking in the park.

(25) He sees a dog.

(26) He runs away from it.

The most interesting part of this little discourse is the sentence in the middle. The sentence contains a pronoun ('he'), which needs to be resolved with something mentioned in the previous sentence, and it contains an indefinite ('dog') which may license subsequent anaphoric pronouns. Classical *DRT* deals with this sequence of sentences in the following way. The first sentence (24) gives rise to a preliminary *DRS*:

(27) $\begin{array}{|c|}\hline x \\ \hline Mx, Wx \\ \hline \end{array}$

This *DRS* represents the information that some man was walking in the park, and the *representation* serves as the context of interpretation for the second and subsequent sentences. The second sentence (25) is then literally *plugged* into this representation, yielding the *DRS*:

(28) $\begin{array}{|l|}\hline x \quad y \\ \hline Mx \;\; Dy \\ Wx \;\; Sxy \\ \hline \end{array}$

This *DRS* represents the information that some man who was walking through the park saw a dog, and the representation again serves as the context of interpretation for the final sentence (26). This third sentence is also plugged into the current discourse representation and yields the final *DRS*, which is the same as the original one (23), and thus captures the information we started out with. In this way, a structured representation has been cut into pieces, and reconstructed, in a sound and information preserving way. All more involved structured representations which agents may have, with structural relationships other than identity, and which cannot that easily be formulated into one sentence, can similarly be decomposed and reconstructed along essentially similar lines.

While any occurring discourse representation in the example above comes with its own content, or meaning, specified in terms of its Tarskian truth-conditions if you want, one of the major criticisms of *DRT* has been that almost none of the sentences of natural language themselves are assigned a meaning under this approach. In the example above, the middle sentence (25), does not have a meaning of its own, but it is associated with an instruction to changes one *representation*, *DRS* (27), into another one, *DRS* (28). And although each of these *DRS* 's have their own truth conditions, the change from the one into the other cannot be given a truth-conditional interpretation. For this reason, many authors hold that, although meaning may in the end be truth-conditional, practically a level of representation is necessary for the actual interpretation of structured discourse, and, hence, of meaning in general.

However, one of the merits of the *DRT* interpretation procedure, and also a reason for its success, is that this representational level is, or can be, extremely rich, much richer and more fine-grained than a corresponding domain of meanings, so that a lot of interpretational effects can be modeled there. *DRT* indeed lends itself most naturally as an architecture in which to formulate an account of phenomena which seem to require quite a lot of computation, such as those involving plurals, tense, ellipsis, and many other phenomena. This wealth, however, also carries a risk.

While it may be obvious to many that this level of representation is called for, as Groenendijk and Stokhof remark, such a conclusion does not come without philosophical pitfalls (Groenendijk and Stokhof 1991, p. 97). For if such representationalist conclusions are implemented in the formal semantic architecture, it may become vulnerable to cognitive psychologist findings about how agents *actually* represent things. If natural language meanings are spelled out, partly, in terms of how they interact with given discourse representation structures, then these ought to be to some extent realistic structures, in order for the analyses to be tenable, or at least so it seems. Furthermore, if it is not the expressions of an arguably *public* language, like a natural language, whose meanings we learn in life, but if these are eventually, equally arguably, *private* representations, then how at all could we learn them, or how could there be anything to learn about them at all?

Groenendijk and Stokhof therefore conceive of the decomposition and reconstruction task differently, in *DPL*, and as a matter of fact, recent formulations of *DRT* , as in van Eijck and Kamp (1997) and Kamp et al. (2011), appear to agree with their conception. This alternative conception is nicely displayed as follows. In order to deconstruct a representation like (23), we can suggestively cut it apart as follows:

(29)
$$\begin{array}{|lll|} x & y & \\ \hline Mx & Dy & \\ Wx & Sxy & Rxy \end{array}$$

The first part in (29) indicates that we start the story with a man who is walking in the park, and suggests there may be more to be told about such a man, which is indicated by omitting the right- and bottom-lines from (23). [Actually, this is the type of representation employed by Pieter Seuren (1985).] The second part states the condition that *he* (x) sees a dog (y), where the omitted top- and left-lines indicate that x seeks a resolution in previous discourse, and the omitted right- and bottom-lines indicate that both x (here: the man) and y (the dog) may be elaborated further upon in subsequent discourse. The third, and final part, adds the condition that x runs away from y, that both need to be resolved by previous discourse, and the concluding right- and bottom-line indicate that this is the end of the story.

Notice that the elegant simplicity of *DPL* may obscure the remarkable achievement that it succeeds in associating all three parts of (29), that is each of the sentences (24), (25), and (26), with an independent and uniform type of meaning. As we indicated above, this is done in terms of input- output-conditions. Most significant is again the middle part. It requires as an input an assignment which associates x with someone who sees a dog, and renders as output an assignment with the same value for x, and which associates y with a dog x sees. Basically, this is the technique *DPL* employs to model the decomposition and reconstruction of structured representations, in terms of a truly compositional semantics.

It can be argued, though, that *DPL* does not really answer the anti-representationalist challenge, mentioned above. For while *DPL* can model the interpretations of *DRS* parts as in (29), and, thus, of the type of representations employed in recent versions of *DRT* like that in Kamp et al. (2011), it inherits some of *DRT* 's representational nature. For one thing, taking the *DPL* metaphor of interpretation literally, a (pseudo-)formula like $\exists y((Dy \wedge Sxy)$, which corresponds to the second part of (29), carries the information, or presupposition, that something has been previously mentioned under the label, or by means of the variable x. It establishes "a fact about the conversation, and not about the subject matter," as Stalnaker (1998, p. 13) puts it. In the words of Groenendijk et al. (1996, p. 183): "When one is engaged in a linguistic information exchange, one (…) has to store *discourse information*. …. Discourse information of this type looks more like a book-keeping device, than like real information." Of course, there is no point in denying that many devices of natural language have a typical discourse role to play, and whose meaning partly or even entirely consists in its function in whole-scale discourses. But it is quite debatable that such should constitute the core-idea of linguistic meaning, as displayed in the typically dynamic slogan that meanings are context change potentials.

With Frege, the early Wittgenstein, and Tarski, it seems we can make some sense of a truth-conditional concept of meaning, or at least of truth-conditional roots of meaning, and with the radical translations and interpretations of Quine and Davidson, a truth-conditional methodology in the theory of interpretation may stand up against relativistic threats. Against such a background, however, the idea that meanings are context change potentials is hard to hold. The contexts employed in dynamic semantics, and the changes brought about in them, are very abstract objects, and not just because they belong to the linguist's theoretical ontology, but also if they are conceived of as real objects which these abstracts objects are supposed to model. As abstract objects, they will not provide the radical translator, or the language learner, any input. And if they are concrete, say representations or information states of the individuals involved in a conversation, they are again subject to the anti-representationalist challenge, and, indeed, provide little or no input to the radical translator.

How does *PLA* stand in the face of these conceptual qualms? In the abstract model of *PLA* the witnesses, together with a dynamic notion of conjunction, are used to establish anaphoric connections. If we generalize over structural relationships other than identity anaphora only, represented connections have to be established at the level of meaning, in the real relations between witnesses. Indeed all of this is done in terms of the models, worlds, and its ingredients relative to which interpretation takes place, and it makes no representational commitments. In addition to this, as we have seen, everything that does not belong to establishing meaningful connections, everything that does not contribute to decomposing and reconstructing meanings, is totally classical. So while I do not want to deny *DRT* or *DPL* any of their respective merits and benefits, I claim that *PLA* may serve the same purposes without deviating from conceptually well-established paradigms.

Variables and Pronouns Another issue different from but also related to representation and information, is the use of (free) variables to hang discourse information upon, or pronouns, or neither. The literature on discourse, quantification, and anaphora displays a variety of positions one may take in this issue. We can ultimately deal only with bound variables (classical logic, *DPL*); we can deal only with free variables (Kamp, Heim, *FCS*); we can do without variables (Quine, Jacobson); and we can deal with bound variables and pronouns (*PLA*). Mixed logical and theoretical motivations are given for either of these stances.

First observe that anything that can be said in *PLA*, with resolved formulas, can be said by (quantified) formulas which are pronoun-free. This is essentially what Observation (4) tells us. The converse, however, holds as well. Anything that can be said in *PL*, with formulas without free variables, can be said with *PLA*-formulas without any (distinct) variables. For, the resolved form $\exists x_1 \ldots \exists x_n \phi$ of any such formula can be written, equivalently, as $((\ldots(\exists x_n(x_n = x_n) \wedge \exists x_{n-1}(x_{n-1} = x_{n-1}))\ldots) \wedge \phi')$ where ϕ' is obtained from ϕ by replacing all the x_i's with suitable pronouns. The proof is left to the reader. Notice that the variables $x_1 \ldots x_n$ in the resulting formula are completely immaterial, so that we might have written $((\ldots(\exists \wedge \exists)\ldots) \wedge \phi)$ instead. The trade-off between variables and pronouns sheds an interesting light on several discussions about variables and pronouns in the logico-linguistic literature. The differences among the various approaches must lie in the details of implementing the various types of semantics coming with them.

The claim that the variables can be dispelled with has been convincingly argued for in Quine (1960), Jacobson (1999) and Szabolcsi (1989). Without going into details, the main moral here consists in the fact that the meanings of sentences can be conceived of in full generality as formulas with n open places, so that their meanings are functions from sequences of n individuals to the truth values the sentences obtain when they ate evaluated when the n open places are filled with the n individuals in these sequences. As a matter of fact, this is how Alfred Tarski conceived of spelling out the truth conditions of a formal language (Tarski 1923, 1956).

Similarly, while considering variables and pronouns in a dynamic semantic setting, we can identify, on methodological grounds for instance, the use of variable assignments and that of sequences of individuals. In the framework of a dynamic semantics, Irene Heim first cashed out the trade-off between variable assignments on the one hand (assuming an enumeration of the variables), and sequences of individuals, on the other (Heim 1982). Kees Vermeulen and Jan van Eijck proceeded along similar lines (Vermeulen 1993; van Eijck 2001). Quite recently, Cresswell (2002) has argued for a reformulation of *DPL* in terms of static first order predicate logic. It is worthwhile to inspect his reformulation in some detail.

Max Cresswell builds his *DPL*-interpretation of a sentence "A man entered the house." on the insight that it is supposed to mean something like, "A man, namely y, entered the house." Even though y may be left unspecified, it serves the purpose of anaphorical pick-up in a continuation "He was broke," meaning, that he, viz. y, was broke. Notice that this "namely y" interpretation of uses of indefinite noun phrases is very much like its *PLA* interpretation, where we employ the witness d, which can be supposed to be the interpretation of y. As we mentioned above already, Cresswell's main moral is like the one argued for in this monograph. Dynamic semantic observations do not force us to adjust our notion of meaning, but, as Cresswell also shows, that they require a more involved, or dynamic notion of conjunction, and of other coordinating expressions. Notice, too, that the output value of variables in the static reformulation suggested by Cresswell, is stored under a name (the variable y) which is required not to belong to the language of *DPL* itself. Like our witnesses, one might say, it is 'alien'.

The difference between Cresswell's approach and the one offered in this monograph, which one may classify as a formal semantic implementation of Cresswell's insights, bears on the present discussion. While the title suggests otherwise, Cresswell does not really supply a static semantics for dynamic discourse. What he eventually offers is a translation of *DPL* into a static first order language, where, like in *FCS* and *DRT* , and *PLA*, the terms that seems to be existentially bound variables are treated as free variables, or free variable copies. The translation is effective, but it is not stated in a compositional way, though. For instance, the (crucial) translation of a conjunction $(\phi \wedge \psi)$ from *DPL*, is stated, among other things, in terms of the translation of ϕ along with a substitution of that translation, of certain variables that ϕ—not the translation of ϕ!—has in common with ψ. To define the interpretation of the modified translation of ϕ from the interpretation of the translation of ϕ itself i surely not easy.

Nevertheless, the task seems to succeed, but arguably in a rather unrevealing manner. The main technical result of Cresswell's paper can be phrased, in more mundane words, as follows. Where a pair $\langle g, h\rangle$ of assignments to the variables in a *DPL*-language satisfies a certain formula ϕ, the standard predicate logical interpretation of Cresswell's translation of the formula is an assignment μ, which encodes both $g(x)$'s possible input value of a variable x, and $h(x)$'s possible output value of that variable. In formal semantic terms, it translates a set of pairs of assignments $\langle g, h\rangle \in [\![\phi]\!]_{dpl} \subseteq (D^V \times D^V)$ into a set of assignments $\mu \in [\![\phi']\!] \subseteq D^{V'}$, where V' is the disjoint union of V with itself. The two domains are isomorphic, as is easily seen. Furthermore, I have given the semantic implementation of Cresswell's ideas in Dekker (1998, 2000), where the 'static' meanings are assignments of both input- and output values of variables.

While very similar in spirit the approach with *PLA* is more attractive than the one suggested by Cresswell. In the first place, *PLA* does employ a direct and ordinary interpretation of conjunction, as intersection, whereas Cresswell's reformulation of

DPL conjunction requires a translation algorithm involving elaborate substitution and quantification over the input- and output-values of variables.

As may be obvious from the preceding deliberations, we can easily deal with variables in terms of sequences of individuals, like Tarski did, or in terms of variable assignments, which may be the standard way now, and which is extensively studied and motivated by Theo Janssen (1986). The same holds for the interpretation of pronouns, which can be dealt with by sequences of witnesses of terms that have occurred previously, as in *PLA*, or by assignments to variables that name these previous occurrences, like, e.g., in Groenendijk et al. (1996), among many others. The fact that variables and pronouns are dealt with by means of different techniques in *PLA* is totally immaterial. That they are handled separately, I believe, has some substance.

For as far as I can make sense of them, variables are theoretical devices. They are a logician's or a theoretical linguist's invention, used to indicate argument places, binding open slots of predicates and relation expressions, but do not seem to be realized in natural language. In contrast, pronouns, as I think of them, really occur in natural language, even if invisible. Sure enough, obvious connections exist between the two, because what can be formulated, in a formal language, using variables, can be formulated, in a natural language, using pronouns, and vice versa, as most of the literature on the subjects presupposes. But there is no a priori, or self-evident, justification that the theorist's variables really are the natural language user's pronouns. The very, valuable, existence of a variable free semantics bear witness to that fact: we can have a logic without variables, but we cannot deny that natural languages do accommodate pronouns.

A distinction between pronouns and variables helps to explain some fundamental features of syntax, having to do with locality constraints on reflexives, and non-locality constraints on pronouns. [The following observation are taken from Butler (2003).] Consider the following examples, typically attributed to conditions A and B of Binding Theory:

(30) David/Every boy shaved himself.

(31) David/Every boy shaved him.

In example (30), the reflexive "himself" must be interpreted as, or bound by, David, or every boy, in its most local domain. In example (31), such a reading is hardly/not possible for the pronoun "him." The pronoun can be interpreted as David, if it can be construed as being anaphorically dependent on David when David has been mentioned before, but it cannot get a bound reading as in example (30). The reason is that "him" really is a pronoun, which cannot be bound by a controlling term; it can only be co-valuated with terms mentioned before it. "Himself" is not a pronoun, or variable, at all, but it is best conceived of as an operator which turns a relation into a reflexive predicate. Indeed, this is something we can represent very well by using variables in a language with lambdas. For any relation R, we can take "R him/her-self" to translate as $\lambda x\ Rxx$. In this way, the argument position occupied by "himself" gets bound by the term this predicate is ascribed to. Likewise, consider:

(32) Only David voted for himself.

(33) Only David voted for him.

In example (32) we get the reading that only David is a self-shaver, nobody, except David, is said to shave himself. Conversely, in example (33) we only get the reading that nobody else but David only shaved “him”, a figure that needs an antecedent from the context—and maybe this is David himself. Even in the latter case, the reading is that nobody else but David shaved David.

The above observations are accounted for if we maintain that pronouns cannot be bound by governing quantifiers, but that they are there in order to pick up a subject already established in the wider context of their occurrence. Variables, if there are any [cf., (Szabolcsi 1989)], must be bound and they are, hence, eliminable in their local context.

Pronouns behave like bound variables in two types of cases:

(34) Every boy thinks he is smart.

(35) No boy brought his umbrella.

However, Butler (2003) argues that sentential complements as in (34) and possessive constructions as in (35) involve a shift to a structural subdomain where information about variables is stored on the contextual sequence parameter. Thus, after all, in the relevant *quantified* context, it is again contextual information, and not (directly) the quantifier, which determines the interpretation of the pronoun—*not* a variable. The very shift is brought about by a BAR-operator, which, Butler convincingly shows, can be used to account for condition C effects, locality constraints on different types of movement (A-bar and A), and strong crossover violations.

In this monograph I do not want to maintain or defend certain syntactic principles, but at least Butler’s work supports a firm distinction between pronouns and variables. Maybe it does not so much provide an argument for making the distinction, but it seems a harmless thing indeed. There may be a methodological argument, so certainly not a knock-down argument, for preceding the way we do, to distinguish the two categories of terms. Adding pronouns to the classical, static machinery does not interfere with it, and not with its quantificational apparatus. Whatever holds with respect to quantifiers, variables, and binding, continues to hold after we have introduced our new category of pronouns.

In this context it is interesting to see de Bruijn’s motivation for his variable free notation, which employs indexical de Bruijn indices.

> Manipulations in the lambda calculus are often troublesome because of the need for re-naming bound variables. (…) It seems to be worth-while to try to get rid of the re-naming, or, rather, to get rid of names altogether. Consider the following criteria for a good notation: (i) easy to write and easy to read for the human reader; (ii) easy to handle in metalingual discussions; (iii) easy for the computer and for the computer programmer. The system we shall develop here is claimed to be good for (ii) and good for (iii). It is not claimed to be very good for (i) …. (de Bruijn 1972, pp. 381–382)

The kind of complications mentioned under (i) may certainly obtain for λ-terms his calculus generates, and the same goes for Quine (1960)'s rendering of the sentence "Some student admires no professor which we saw in the introduction.". $(\mathcal{E}(\mathcal{R}(STU \times \mathcal{N}(\mathcal{E}(\mathcal{R}(\mathcal{I}(PRO \times ADM)))))))$ It may be a prejudice due to my specific logical training, but I believe the average natural language user would very much prefer to read a first order predicate logical rendering of the sentences, *with* (bound) variables, indeed.

When it comes to pronouns, one may feel slightly uneasy if we interpret them indeed in the de Bruijn fashion. Reformulating formulas into their binding forms requires, motivated, but tedious, substitution operations, and pronouns which carry the very same index only occasionally select the same antecedent. One of the major attractive features of *DPL* is indeed that it is prima facie obvious from the logical form of a formula which quantifier semantically binds which variable, and whether the quantifier syntactically binds the variable or not. The worrying down-side of this feature is that it severely damages structural logical properties of the system, e.g., it does not license α-conversion. In response to such effects, Jan van Eijck has resorted to a variable free notation for pronouns, in terms of inverse de Bruijn indices, which select an antecedent by counting from the start of a discourse.

Even so, van Eijck's inverse de Bruijn indices are not well motivated intuitively. For one thing, such a treatment neglects the inherently indexical nature of pronouns. Related to this, it obscures the obvious fact that what is most salient is introduced last in a discourse. In *PLA* this is always the first and foremost element of a sequence that a pronoun may take a value from. A default pronoun, roughly, is p_1. In van Eijck's system of incremental dynamics, the default value of a pronoun has to be found by first counting how many items have been introduced in the discourse, and then determining for such a number n, the individual e_n which is the last element of a contextually supplied sequence.

References

Barwise, J., & Perry, J. (1983). *Situations and Attitudes*. Cambridge, MA: MIT Press.

Butler, A. (2003). Predicate logic with barriers and its locality effects. In M. Weisgerber (Ed.) *Proceedings of Sinn und Bedeutung VII*. FB Sprachwissenschaft (pp. 70–80).

Cresswell, M. J. (2002). Static semantics for dynamic discourse. *Linguistics and Philosophy, 25*, 545–571.

de Bruijn, N. G. (1972). Lambda calculus notation with nameless dummies: A tool for automatic formula manipulation, with application to the Church–Rosser theorem. *Indagationes Mathematicae, 34*, 381–392.

Dekker, P. (1994). Predicate logic with anaphora (seven inch version). In L. Santelmann & M. Harvey (Eds.), *Proceedings of SALT IV* (p. 95). Ithaca: DMLL Publications.

Dekker, P. (1998). Speaker's reference, descriptions and information structure. *Journal of Semantics, 15*(4), 301–330.

Dekker, P. (2000). The semantics of dynamic conjunction. In N. Braisby, L. Cavedon, P. Blackburn, & Atsushi Shimojima (Eds.), *Logic, language and computation*, vol. III (pp. 107–127). Stanford, CA: CSLI.

Dekker, P. (2011). Dynamic semantics. In C. Maienborn, K. von Heusinger, & P. Portner (Eds.), *Semantics: An international handbook of natural language meaning, volume 1 of HSK Handbook series*. Mouton de Gruyter: Berlin, New York.

Egli, U. (1979). The Stoic concept of anaphora. In R. Bäuerle, U. Egli, & A. von Stechow (Eds.), *Semantics from different points of view* (pp. 266–283). Berlin: Springer.

Frege, G. (1918). Der Gedanke. Eine logische Untersuchung. *Beiträge zur Philosophie des deutschen Idealismus, 2*(1), 58–77.

Geach, P. T. (1962). *Reference and generality. An examination of some medieval and modern theories*. Ithaca: Cornell University Press.

Groenendijk, J., & Stokhof, M. (1991). Dynamic predicate logic. *Linguistics and Philosophy, 14*(1), 39–100.

Groenendijk, J., Stokhof, M., & Veltman, F. (1996). Coreference and modality. In L. Shalom (Ed.), *The handbook of contemporary semantic theory* (p. 213). Oxford: Blackwell.

Heim, I. (1982). The semantics of definite and indefinite noun phrases. (Ph.D. thesis, University of Massachusetts, Amherst, 1982). Published in 1988 by Garland, New York.

Hintikka, J. (1983). *The game of language*. Dordrecht: Reidel.

Jacobson, P. (1999). Towards a variable-free semantics. *Linguistics and Philosophy, 22*, 117–184.

Janssen, T. M. V. (1986). *Foundations and application of montague grammar. Part 1: Philosophy, framework, computer science*. CWI Tract 19, Amsterdam: CWI (Centre for Mathematics and Computer Science).

Kamp, H., 1981. A theory of truth and semantic representation. In J. Groenendijk, T. Janssen & M. Stokhof (Eds.) *Formal Methods in the Study of Language* (pp. 277–322). Amsterdam: Mathematical Centre. Reprinted in J. Groenendijk, T. Janssen & M. Stokhof, (Eds.). (1984). *Truth, interpretation and information* (pp. 1–41). Dordrecht: Foris.

Kamp, H., & Reyle, U. (1993). *From discourse to logic*. Dordrecht: Kluwer.

Kamp, H., Van Genabith, J., & Reyle, U. (2011). Discourse representation theory. In D. Gabbay & F. Guenthner (Eds.), *Handbook of philosophical logic* (Vol. 17, pp. 125–394). Dordrecht: Reidel.

Quine, W. V. (1960). Variables explained away. In *Proceedings of the American Philosophical Society*. vol. 104 (pp. 343–347). Reprinted in Quine, W. V. (1966). *Selected Logic Papers* (pp. 227–235).

Seuren, P. A. M. (1985). *Discourse semantics*. Oxford: Blackwell.

Stalnaker, R. (1978). Assertion. In P. Cole (Ed.), *Syntax and semantics 9—pragmatics* (p. 332). New York: Academic Press.

Stalnaker, R. (1998). On the representation of context. *Journal of Logic, Language and Information, 7*(1), 3–19.

Sundholm, G. (1984). Proof theory and meaning. In D. Gabbay & F. Guenthner (Eds.), *Handbook of philosophical logic* (Vol. III, pp. 471–506). Dordrecht: Reidel.

Szabolcsi, A. (1989). Bound variables in syntax (are there any?). In R. Bartsch, J. van Benthem, & P. van Emde Boas (Eds.), *Semantics and contextual expression* (pp. 294–318). Dordrecht: Foris.

Tarski, A. (1923). O wyrazie pierwotnym logistyki. *Przegld Filozoficzny, 26*, 68–89. A translation appeared as "On the primitive term of logistic" in Tarski, A. (1956). *Logic, metamathematics, semantics* (pp. 1–23). Oxford: Clarendon Press.

Tarski, A. (1956). The concept of truth in formalized languages. In *Logic, semantics, metamathematics* (pp. 152–278). Oxford: Clarendon Press. Originally published as "Der Wahrheitsbegriff in der formalisierten Sprachen" (1936). *Studia Philosophica, 1*, 261–405.

van Eijck, J. (2001). Incremental dynamics. *Journal of Logic, Language and Information,11*(3), 319–351.

van Eijck, J., & Kamp, H. (1997). Representing discourse in context. In M. ter Alice (Ed.), *Handbook of logic and language* (p. 237). Amsterdam: Elsevier.

Vermeulen, C. F. M. (1993). Sequence semantics for dynamic predicate logic. *Journal of Logic, Language and Information, 2*, 217–254.

Zeevat, H. (1989). A compositional approach to discourse representation theory. *Linguistics and Philosophy, 12*(1), 95–131.

http://www.springer.com/978-94-007-4868-2

Dynamic Semantics
Dekker, P.J.E.
2012, VIII, 128 p., Hardcover
ISBN: 978-94-007-4868-2