Preface

Geometric configurations are a natural and attractive topic for students, teachers, and researchers. This text serves as a concise introduction to several important topics and the interesting interplay between them. Our unifying theme is graph theory: basic graph theory to introduce graphs themselves; algebraic graph theory to introduce groups; topological graph theory to introduce surfaces; and geometric graph theory to talk about geometries, in particular incidence geometries.

Incidence structures are seemingly uninteresting mathematical objects. They are too general to possess any deep algebraic structure. Lineal, semi-regular incidence structures, alias combinatorial configurations, usually result from an appropriate collection of points and lines in the plane. These arrangements of lines aroused considerable interest among leading mathematicians of the nineteenth century and much less during the rise of discrete mathematics of the twentieth century. As one puts different limitations on incidence structures, their number decreases and they become much more structured. This effect was observed in the theory of designs which became a successful and important piece of modern combinatorics.

In the second half of the twentieth century graph theory boomed. Since incidence geometries are essentially vertex-colored graphs, the tools from graph theory are now available for studying discrete geometrical objects. Symmetries of configurations establish an important link between group theory and combinatorics. There are several pathways between configurations and geometry, and certainly there are deep links between configurations and algebraic topology involving covering spaces and maps on surfaces. However, the reason for a success is the fact that configurations and incidence geometries can be considered as part of graph theory.

Shifting the paradigm from algebra and topology to graph theory puts new balls into the playing field of the discrete mathematician. We devote neither a chapter nor a section to incidence geometry. We use it throughout the book and it provides a wonderful viewpoint from which difficult topics involving graphs, groups, maps, and configurations become accessible.

The first book on configurations was written in 1929 by Friedrich Levi who already pointed out the interdisciplinary nature of configurations. The most recent book on configurations is *Configurations of Points and Lines* [44] by Branko Grünbaum. We would like to thank him for sharing his work with us. We adjusted terminology with that of Grünbaum in all feasible cases. Our term *Grünbaum calculus* is but a small tribute to his profound influence on the subject. The approach in [44] is geometric. We expect that our graph theoretical approach will be a welcome and timely complement.

Our book is nonstandard in the sense that it brings together a wide variety of mathematical concepts. It is not a monograph. It is a graduate textbook that is accessible to advanced undergraduate students. Each chapter contains a set of exercises. Problems denoted by (*) are more difficult and usually require material that was not covered up to this point. Problems denoted by (**) represent research problems or very difficult problems. The exercises are intended to give working knowledge of the subject and enable the reader to get as close to the research frontier as possible.

Parts of the book were tested in various graduate courses at the University of Ljubljana in Slovenia and at the Worcester Polytechnic Institute. Chapters 1–3 can be used as an invitation to algebraic graph theory, and Chaps. 1–4 give much more than an overview of algebraic and topological graph theory. Chapters 1, 2, 5, and 6 can serve as an introduction to the study of incidence geometries and configurations of points and lines.

In the course of writing the book several mathematicians read and commented on various parts of the manuscript. We are sincerely thanking everyone. We are listing those, who tested several exercises and proofread the manuscript in the last year, giving us invaluable suggestions for improvements. Special thanks go to: Nino Bašić, Leah Berman, Marko Boben, Jürgen Bokowski, Marston Conder, Maria del Rio Francos, Gábor Gévay, Branko Grünbaum, Martin Juvan, Matjaž Konvalinka, Aleksander Malnič, Martin Milanič, Marko Petkovšek, Primož Potočnik, Ivona Puljić, Tom Tucker, Gordon Williams, and Arjana Žitnik.

By far the most important contributor to this volume was the Canadian mathematician H. S. M. Coxeter, whose foundational 1950 paper "Self-dual configurations and regular graphs" in the *Bulletin of the American Mathematical Society* was the model for both the style and content of this volume. In that paper Coxeter coined the term "Levi graph" and established the combinatorial approach; described the connection to cages; examined the connection to Petersen and generalized Petersen graphs, their readily computable automorphism groups and their easy realization as unit distance graphs; exploited the symmetrical nature of small configurations, their classical roots, and their geometric connection to regular maps on surfaces; exposed the important combinatorial and algebraic descriptions of classical configurations which yield new perspectives onto their automorphism groups and their level of transitivity, as well as many other essential insights which have ever since guided the combinatorial approach to configurations. The reader who wishes to move beyond the material presented here can do no better than to consult this paper and those papers which cite it with the love and respect it deserves.

Tomaž Pisanski conceived the idea and started this project, while Brigitte Servatius insisted on the last word and is therefore prepared to take full responsibility for all the mathematical shortcomings of this volume.

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