Preface

Although stability is one of the most studied topics in the theory of time-delay systems, the corresponding chapters of classic works on time-delay systems (see, e.g., [3, 23, 44]) do not include a comprehensive study of a counterpart of the classic Lyapunov theory for linear delay-free systems. The principal aim of this volume is to fill this gap and provide the reader with a detailed treatment of the basic concepts of the Lyapunov–Krasovskii approach to the stability analysis of linear time-delay systems.

There are two types of stability results. Results of the first type are obtained in the following manner. First, a positive-definite functional is selected, then its time derivative along the solutions of a system is computed, and, finally, some negativity conditions for the derivative are proposed. This is how the majority of LMI type stability conditions have been obtained. A good account of such stability results can be found in [16, 17, 31, 54, 58, 64]. The scheme to obtain results of the second type is different. First, a desired time derivative is selected, and then a functional with this time derivative along the solutions is computed. Finally, one needs to check whether or not the functional is positive definite. Usually, functionals obtained in this way are more complex than those used to derive results of the first type. But since these functionals are adjusted to the system under consideration, they provide more complete information about system behavior. It would be naive to expect that the second scheme could be successfully applied to general classes of time-delay systems. Of course, such results should be available for the case of linear time-delay systems.

This book is divided into two parts. The first part, consisting of four chapters, considers the case of retarded type time-delay systems. The first chapter of this part is of the compilation character. The chapter discusses such basic notions as initial conditions and system state. In the exposition of the existence and uniqueness results presented in this chapter we follow [19]. Classical stability results based on the Lyapunov–Krasovskii approach are presented in a form inspired by [72].

In Chap. 2 the class of linear systems with one delay is studied. We start with a computation of the solutions of such systems. Then we explain in detail a general scheme used for the computation of Lyapunov functionals with a prescribed time derivative. Here matrix-valued functions that define these functionals are introduced. They are a counterpart of the classic Lyapunov matrices that appear as solutions of the classical Lyapunov matrix equation in the context of Lyapunov quadratic forms for the case of linear delay-free systems. We call them Lyapunov matrices for time-delay systems. A substantial part of this chapter is devoted to an analysis of the basic properties of Lyapunov matrices. Such issues as existence, uniqueness, and computation are treated. Next, we introduce Lyapunov functionals that admit quadratic lower and upper bounds. These are functionals of the complete type. Complete type functionals are then used to derive exponential estimates for the solutions of time-delay systems and robustness bounds for perturbed systems. The chapter ends with a brief historical survey, where the results of the principal contributors to the subject are presented.

The material presented in Chaps. 1 and 2 is recommended for an introductory course on the stability of time-delay systems. Such courses have been given for several years in the Department of Automatic Control at CINVESTAV in Mexico City and now in the Faculty of Applied Mathematics and Control Processes of Saint Petersburg State University in Russia.

In Chap. 3 we address the case of retarded type linear time-delay systems with multiple delays. Applying the scheme presented in the previous chapter, we obtain a general form of quadratic functionals with a prescribed time derivative along the solutions of such time-delay systems. A special system of matrix equations that defines the Lyapunov matrices is derived. It is shown that the special system admits a unique solution if and only if the spectrum of the time-delay system does not contain points arranged symmetrically with respect to the origin of the complex plane. This spectrum property is known as the Lyapunov condition. Two numerical schemes for the computation of Lyapunov matrices are presented. The first one is applicable to the case where time delays are multiple to a basic one. The other one allows one to compute approximate Lyapunov matrices in the case of general time delays. A measure that makes it possible to estimate the quality of an approximation is provided as well. Quadratic functionals of the complete type are defined, and several important applications of the functionals are presented in the final part of the chapter.

In Chap. 4 a linear retarded type system with distributed delay is studied. First, we introduce quadratic functionals and Lyapunov matrices for the system. Then we present the existence and uniqueness conditions for the matrices and provide some numerical schemes for the computation of the Lyapunov matrices. Finally, we derive a class of time-delay system with distributed delay for which Lyapunov matrices are solutions of a boundary value problem for an auxiliary system of linear delay-free matrix differential equations.

The second part of the book, comprising three chapters, is devoted to the case of neutral type time-delay systems. In Chap. 5 we extend the results presented in Chap. 1 to the case of neutral type time-delay systems. Issues of existence, uniqueness, and continuation of solutions of the initial value problem for such systems are discussed. Stability concepts and basic stability results obtained using the Lyapunov–Krasovskii approach, mainly in the form of necessary and sufficient conditions, are presented.

In Chap. 6 we consider the class of neutral type linear systems with one delay. We define the fundamental matrix of such a system and present the Cauchy formula for the solution of an initial value problem. This formula is used to compute a quadratic functional with a given time derivative along the solutions of the time-delay system. It is demonstrated that this functional is defined by a Lyapunov matrix for the time-delay system. A thorough analysis of the basic properties of this Lyapunov matrix is conducted. Complete type functionals are introduced, and various applications of the functionals are discussed.

The last chapter is dedicated to the case of neutral type linear systems with distributed delay. The structure of quadratic functionals with prescribed time derivatives along the solutions of such a system is defined, and the corresponding Lyapunov matrices are introduced. A system of matrix equations that defines the Lyapunov matrices is given. It is proven that under some conditions this system admits a unique solution. A class of systems with distributed delay for which Lyapunov matrices are the solutions of standard boundary value problems for an auxiliary system of linear matrix ordinary differential equations is presented. Complete type functionals are defined. It is shown that these functionals can be presented in a special form that is more convenient for the computation of lower and upper bounds for the functionals.

The book's bibliography does not pretend to cover all aspects of the stability analysis of time-delay systems. It includes entries that are closely related to the problems discussed in the book. More complete lists of literature can be found in [18, 23, 41, 43, 58].

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