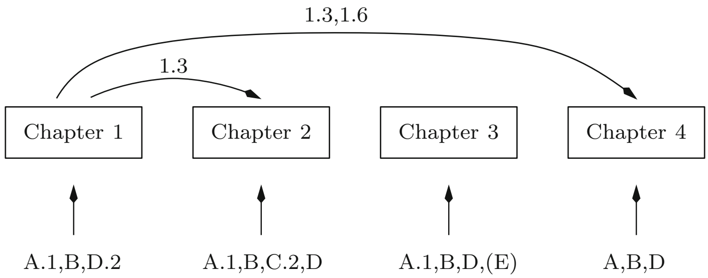
Preface

Topological combinatorics is a very young and exciting field of research in mathematics. It is mostly concerned with the application of the many powerful tools of algebraic topology to combinatorial problems. One of its early landmarks was Lovász's proof of the Kneser conjecture published in 1978. The combination of the two mathematical fields—topology and combinatorics—has led to many surprising and elegant proofs and results.

In this textbook I present some of the most beautiful and accessible results from topological combinatorics. It grew out of several courses that I have taught at Freie Universität Berlin, and is based on my personal taste and what I believe is suitable for the classroom. In particular, it aims for a clear and vivid presentation rather than encyclopedic completeness.

The text is designed for an advanced undergraduate level. Primarily it serves as a basis for a course, but is written in such a way that it just as well may be read by students independently. The textbook is essentially self-contained. Only some basic mathematical experience and knowledge—in particular some linear algebra—is required. An extensive appendix allows the instructor to design courses for students with very different prerequisites. Some of those designs will be sketched later on.

The textbook has four main chapters and several appendices. Each chapter ends with an accompanying and complementing set of exercises. The main chapters are mostly independent of each other and thus allow considerable flexibility for an individual course design. The dependencies are roughly as follows.



Suggested Course Outlines

For students with previous knowledge of graph theory and the basics of algebraic topology including simplicial homology theory. Use Chaps. 1–4. Whenever concepts and results on partially ordered sets and their topology from Appendix C or on group actions from Appendix D are missing, they should be included. Oliver's Theorem 3.17, which is proven in Appendix E, can easily be used as a black box. If the students are experienced with homology and if time permits, I recommend studying Appendix E after Chap. 3.

For students with previous knowledge of the basics of algebraic topology including simplicial homology theory only. Proceed as in the last case and provide the basics of graph theory from Appendix A along the way.

For students with previous knowledge of graph theory only. I recommend that the instructor introduces some basic topology with Sects. B.1 and B.3, and then presents Chap. 1, skipping the homological proofs. Before Sect. 1.6 I recommend giving a topology crash course with Sects. B.4–B.9. Proceed with Chaps. 2–4 and add concepts and results from Appendices C and D as needed. Apply Theorem 3.17 as a black box and use Appendix E as a motivation to convince students to study algebraic topology.

For motivated students with neither graph theory nor algebraic topology knowledge. Proceed as in the last case and provide the basics from graph theory from Appendix A along the way.

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