

## Chapter 2

# Improved Multiple Attribute Decision Making Methods

The improved multiple attribute decision making methods for decision making in the manufacturing environment are described in this chapter.

### 2.1 Improved Analytic Hierarchy Process Method

Analytic hierarchy process (AHP) is one of the most popular analytical techniques for complex decision making problems [1, 2]. An AHP hierarchy can have as many levels as needed to fully characterize a particular decision situation. A number of functional characteristics make AHP a useful methodology. These include the ability to handle decision situations involving subjective judgments, multiple decision makers, and the ability to provide measures of consistency of preferences [3]. Designed to reflect the way people actually think, AHP continues to be the most highly regarded and widely used decision making method. AHP can efficiently deal with objective as well as subjective attributes. In this method, a pairwise comparison matrix is constructed using a scale of relative importance. The judgments are entered using the fundamental scale of the AHP. The method determines the consistent weights and evaluates the composite performance score of alternatives to get the rank the alternatives. Higher the composite performance scores of the alternative, higher the rank of that alternative.

In this book, the AHP method is improved by proposing a systematic way of normalizing the values of the attributes and the conversion of subjective values into objective values. In the original version of AHP, the method requires pairwise comparison of various alternatives with respect to each of the attributes and a pairwise comparison of attributes themselves. The size and number of the comparison matrices increases rapidly as the number of alternatives and/or attributes increases. The AHP method is improved by eliminating the comparison matrices

**Table 2.1** Pairwise comparison scale of attributes

Degree of importance	Definition
1	Equal importance (no preference)
2	Intermediate between 1 and 3
3	Moderately more important
4	Intermediate between 3 and 5
5	Strongly more important
6	Intermediate between 5 and 7
7	Very strongly important
8	Intermediate between 7 and 9
9	Extremely strongly more important
1/2, 1/3, 1/4, 1/5, 1/6, 1/7, 1/8, 1/9	Reciprocals of 2, 3, 4, 5, 6, 7, 8, and 9

required for alternatives. Also, by normalizing the values of attributes by a systematic way, the rank reversal problem is removed in the improved AHP method. The steps of the improved AHP method are explained below:

**2.1.1 Formulating the Decision Table**

Step 1: Identify the selection attributes for the considered decision making problem and short-list the alternatives on the basis of the identified attributes satisfying the requirements. A quantitative or qualitative value or its range may be assigned to each identified attribute as a limiting value or threshold value for its acceptance for the considered application. An alternative with each of its attribute, meeting the requirements, may be short-listed. The short-listed alternatives may then be evaluated using the proposed methodology. The values associated with the attributes for different alternatives may be based on the available data or may be the estimations made by the decision maker [4].

**2.1.2 Deciding Weights of the Attributes**

Step 2: Find out the relative importance of different attributes with respect to the objective. To do so, one has to construct a pairwise comparison matrix using a scale of relative importance. An attribute compared with it is always assigned the value 1 so the main diagonal entries of the pairwise comparison matrix are all 1. The numbers 3, 5, 7, and 9 correspond to the verbal judgments ‘moderate importance’, ‘strong importance’, ‘very strong importance’, and ‘absolute importance’ (with 2, 4, 6, and 8 for compromise between the previous values). Table 2.1 presents the relative importance scale used in the AHP method.

Assuming M attributes, the pairwise comparison of attribute *i* with attribute *j* yields a square matrix  $A_{M \times M}$  where  $r_{ij}$  denotes the comparative importance of

**Table 2.2** Random index (RI) values

$n$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0	0	0.52	0.89	1.11	1.25	1.35	1.4	1.45	1.49	1.51	1.54	1.56	1.57	1.59

attribute  $i$  with respect to attribute  $j$ . In the matrix,  $r_{ij} = 1$  when  $i = j$  and  $r_{ji} = 1/r_{ij}$ .

$$A_{M \times M} = \begin{matrix} & \text{Attribute} & 1 & 2 & 3 & \dots & \dots & M \\ \begin{matrix} 1 \\ 2 \\ 3 \\ \dots \\ \dots \\ M \end{matrix} & \left[ \begin{array}{cccccc} r_{11} & r_{12} & r_{13} & \dots & \dots & r_{1M} \\ r_{21} & r_{22} & r_{23} & \dots & \dots & r_{2M} \\ r_{31} & r_{32} & r_{33} & & & r_{3M} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ r_{M1} & r_{M2} & r_{M3} & \dots & \dots & r_{3M} \end{array} \right] & \end{matrix} \quad (2.1)$$

- Find the relative normalized weight ( $w_j$ ) of each attribute by (1) calculating the geometric mean of  $i$ th row and (2) normalizing the geometric means of rows in the comparison matrix. This can be represented as,

$$GM_j = \left\{ \prod_{j=1}^M r_{ij} \right\}^{1/M} \quad \text{and} \quad (2.2)$$

$$w_j = GM_j / \sum_{j=1}^M GM_j \quad (2.3)$$

The geometric mean method of AHP is used in the present work to find out the relative normalized weights of the attributes because of its simplicity and easiness to find out the maximum Eigen value and to reduce the inconsistency in judgments.

- Calculate matrix  $A3$  and  $A4$  such that  $A3 = A1 \times A2$  and  $A4 = A3/A2$ , where  $A2 = [w_1, w_2, \dots, w_M]^T$ .
- Find out the maximum Eigen value  $\lambda_{\max}$  (i.e. the average of matrix  $A4$ ).
- Calculate the consistency index  $CI = (\lambda_{\max} - M)/(M - 1)$ . The smaller the value of  $CI$ , the smaller is the deviation from the consistency.
- Obtain the random index (RI) for the number of attributes used in decision making [2]. Table 2.2 presents the RI values for different number of attributes.
- Calculate the consistency ratio  $CR = CI/RI$ . Usually, a  $CR$  of 0.1 or less is considered as acceptable and it reflects an informed judgment that could be attributed to the knowledge of the analyst about the problem under study.

### 2.1.3 Calculating Composite Performance Scores

Step 3: The next step is to obtain the overall or composite performance scores for the alternatives by multiplying the relative normalized weight ( $w_j$ ) of each attribute (obtained in Step 2) with its corresponding normalized weight value for each alternative (obtained in Step 1) and making summation over all the attributes for each alternative.

$$P_i = \sum_{j=1}^M w_j (m_{ij})_{\text{normal}} \quad (2.4)$$

Where  $(m_{ij})_{\text{normal}}$  represents the normalized value of  $m_{ij}$ .  $P_i$  is the overall or composite score of the alternative  $A_i$ . The alternative with the highest value of  $P_i$  is considered as the best alternative.

## 2.2 Improved Technique for Order Preference by Similarity to Ideal Solution Method

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method was developed by Hwang and Yoon [5]. This method is based on the concept that the chosen alternative should have the shortest Euclidean distance from the ideal solution and the farthest from the negative ideal solution. The ideal solution is a hypothetical solution for which all attribute values correspond to the maximum attribute values in the database comprising the satisfying solutions; the negative ideal solution is the hypothetical solution for which all attribute values correspond to the minimum attribute values in the database. TOPSIS thus gives a solution that is not only closest to the hypothetically best, that is also the farthest from the hypothetically worst [4].

The main procedure of the improved TOPSIS method for the selection of the best alternative from among those available is given below:

### 2.2.1 Formulating the Decision Table

Step 1: The first step is to determine the objective and to identify the pertinent evaluation attributes. This step represents a matrix based on all the information available on attributes. Each row of this matrix is allocated to one alternative and one attribute to each column. Therefore, an element  $m_{ij}$  of the decision table gives the value of the  $j$ th attribute in original real values, that is, non-normalized form and units, for the  $i$ th alternative.

In the case of a subjective attribute (i.e. objective value is not available), a ranked value judgment on a scale is adopted. Once a subjective attribute is represented on a scale then the normalized values of the attribute assigned for different alternatives are calculated in the same manner as that for objective attributes. Normalized decision matrix,  $R_{ij}$ , is obtained using the following expression.

$$R_{ij} = m_{ij} / \left[ \sum_{j=1}^M m_{ij}^2 \right]^{1/2} \quad (2.5)$$

### 2.2.2 Deciding Weights of the Attributes

Step 2: A set of weights  $w_j$  (for  $j = 1, 2, \dots, M$ ) such that  $\sum w_j = 1$  may be decided. The relative importance weights of the attributes can be assigned arbitrarily by the decision maker based on his/her preference. In this book, AHP method is suggested for helping the decision maker to decide the relative importance weights of attributes in a systematic manner. The relative importance weights using AHP can be calculated as explained in the [Sect. 2.1.2](#).

### 2.2.3 Calculating Composite Performance Scores

Step 3:

- Obtain the weighted normalized matrix  $V_{ij}$ . This is obtained by the multiplication of each element of the column of the matrix  $R_{ij}$  with its associated weight  $w_j$ . Hence, the elements of the weighted normalized matrix  $V_{ij}$  are expressed as:

$$V_{ij} = w_j R_{ij} \quad (2.6)$$

- Obtain the ideal (best) and negative ideal (worst) solutions in this step. The ideal (best) and negative ideal (worst) solutions can be expressed as:

$$\begin{aligned} V^+ &= \left\{ \left( V_{ij}^+ / j \right), \left( V_{ij}^- / j' \right) / i = 1, 2, \dots, N \right\}, \\ &= \{ V_1^+, V_2^+, V_3^+, \dots, V_M^+ \} \end{aligned} \quad (2.7)$$

$$\begin{aligned} V^- &= \left\{ \left( V_{ij}^- / j \in J \right), \left( V_{ij}^+ / j' \in J' \right) / i = 1, 2, \dots, N \right\}, \\ &= \{ V_1^-, V_2^-, V_3^-, \dots, V_M^- \} \end{aligned} \quad (2.8)$$

Where,

$J = (j = 1, 2, \dots, M)/j$  is associated with beneficial attributes and

$J' = (j = 1, 2, \dots, M)/j$  is associated with non-beneficial attributes.

- $V_j^+$  indicates the ideal (best) value of the considered attribute among the values of the attribute for different alternatives. In case of beneficial attributes (i.e. whose higher values are desirable for the given application),  $V_j^+$  indicates the higher value of the attribute. In case of non-beneficial attributes (i.e. whose lower values are desired for the given application),  $V_j^+$  indicates the lower value of the attribute.
- $V_j^-$  indicates the negative ideal (worst) value of the considered attribute among the values of the attribute for different alternatives. In case of beneficial attributes (i.e. whose higher values are desirable for the given application),  $V_j^-$  indicates the lower value of the attribute. In case of non-beneficial attributes (i.e. whose lower values are desired for the given application),  $V_j^-$  indicates the higher value of the attribute.
- Obtain the separation measures. The separation of each alternative from the ideal one is given by Euclidean distance by the following Eqs.

$$S_i^+ = \left\{ \sum_{j=1}^M (V_{ij} - V_j^+)^2 \right\}^{0.5}, i = 1, 2, \dots, N \quad (2.9)$$

$$S_i^- = \left\{ \sum_{j=1}^M (V_{ij} - V_j^-)^2 \right\}^{0.5}, i = 1, 2, \dots, N \quad (2.10)$$

- The relative closeness of a particular alternative to the ideal solution,  $P_i$ , can be expressed in this step as follows.

$$P_i = S_i^- / (S_i^+ + S_i^-) \quad (2.11)$$

- A set of alternatives is made in the descending order in this step, according to the value of  $P_i$  indicating the most preferred and least preferred feasible solutions.  $P_i$  may also be called as overall or composite performance score of alternative  $A_i$ .

### 2.3 Data Envelopment Analysis Method

Data envelopment analysis (DEA), occasionally called frontier analysis, was first put forward by [6]. It is a performance measurement technique which can be used for evaluating the relative efficiency of alternatives for given decision making situation. After the initial study by [6], DEA has got rapid growth and widespread acceptance.

Hashimoto [7] addressed a ranked voting system to determine an ordering of candidates in terms of the aggregate vote by rank for each candidate. Sarkis [8] carried out the evaluation of environmentally conscious manufacturing programs

using a method involving the syn book of the analytic network process (ANP) and DEA was carried out. Sarkis [9] provided an empirical evaluation of various DEA ranking approaches and MADM techniques, which include outranking and multi-attribute utility techniques, using case study information. Tone [10] proposed a slacks-based measure (SBM) of efficiency based on input excesses and output shortfalls. Each decision making unit can be improved and become more efficient by deleting the input excess and augmenting the output shortfalls. Sun [11] reported on an application of DEA to evaluate computer numerical control (CNC) machines in terms of system specification and cost. The methodology proposed for the evaluation of CNC machines is based on the combination of the Banker, Charnes, and Cooper (BCC) model and cross-efficiency evaluation. Liu [12] developed a fuzzy DEA/AR method that is able to evaluate the performance of FMS alternatives when the input and output data are represented as crisp and fuzzy data. Wang et al. [13] proposed a DEA model with assurance region (AR) for priority derivation in the AHP to overcome the shortcomings of the DEAHP such as illogical local weights, over insensitivity to some comparisons, information loss and overestimation of some local weights, and provide better priority estimate and better decision conclusions than the DEAHP. Cooper et al. [14] described various DEA models in their book.

DEA is an extreme point method and compares each alternative with only the “best” alternative. A fundamental assumption behind an extreme point method is that if a given alternative,  $A$ , is capable of producing  $Y(A)$  units of output with  $X(A)$  inputs, then other alternatives should also be able to do the same if they were to operate efficiently. Similarly, if alternative  $B$  is capable of producing  $Y(B)$  units of output with  $X(B)$  inputs, then other alternatives should also be capable of the same production schedule. Alternatives  $A$ ,  $B$  and others can then be combined to form a composite or virtual alternative with composite inputs and composite outputs. The heart of the analysis lies in finding the “best” virtual alternative for each real alternative. If the virtual alternative is better than the original alternative then the original alternative is “inefficient”.

For a given MADM problem, alternatives ( $A_1, A_2, \dots$  and  $A_N$ ) and different attributes affecting the selection of an alternative are identified. Attributes are divided into two groups: (1) outputs: attributes for which higher values are desirable or beneficial attributes and (2) inputs: attributes for which lower values are desirable or non-beneficial attributes.

Suppose  $s$  input items and  $t$  output items are selected. Let the input and output data for alternative  $A_j$  be  $(x_{1j}, x_{2j}, \dots, x_{sj})$ , and  $(y_{1j}, y_{2j}, \dots, y_{tj})$ , respectively. The input data matrix  $X$  and the output data matrix  $Y$  can be prepared as follows,

$$\text{For non-beneficial attributes, } X = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1N} \\ x_{21} & x_{22} & \dots & x_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ x_{s1} & x_{s2} & \dots & x_{sN} \end{pmatrix}$$

$$\text{and for non-beneficial attributes, } Y = \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1N} \\ y_{21} & y_{22} & \cdots & y_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ y_{t1} & y_{t2} & \cdots & y_{tN} \end{pmatrix} \quad (2.12)$$

where,  $X$  is a  $(s \times N)$  matrix and  $Y$  is a  $(t \times N)$  matrix.

### 2.3.1 The Basic CCR Model

This is one of the most basic DEA models. Suppose the data is given in form of matrices  $X$  and  $Y$ , the efficiency of each alternative is measured once. Hence  $N$  optimizations are needed, one for each alternative  $A_j$ , to completely solve the MADM problem. Let the  $A_j$  to be evaluated on any trial be designated as  $A_o$ , where  $o$  ranges over  $1, 2, \dots, N$ . The following fractional programming problem is solved to obtain the values of the input “weights” ( $v_i$ ) ( $i = 1, 2, \dots, s$ ) and output “weights” ( $u_r$ ) ( $r = 1, 2, \dots, t$ ) as variables.

$$(FP_o) \quad \max \quad \theta = \frac{u_1 y_{1o} + u_2 y_{2o} + \cdots + u_t y_{to}}{v_1 x_{1o} + v_2 x_{2o} + \cdots + v_s x_{so}} \quad (2.13)$$

$$\text{subject to, } \frac{u_1 y_{1j} + \cdots + u_t y_{tj}}{v_1 x_{1j} + \cdots + v_s x_{sj}} \leq 1 \quad (j = 1, \dots, N) \quad (2.14)$$

$$v_1, v_2, \dots, v_s \geq 0, \quad u_1, u_2, \dots, u_t \geq 0$$

The constraints mean that the ratio of “virtual output” vs. “virtual input” should not exceed 1 for every alternative. The objective is to obtain weights ( $v_i$ ) and ( $u_r$ ) that maximize the ratio of the alternative  $A_o$ , being evaluated. By virtue of the constraints, the optimal objective value  $\theta^*$  is at most 1. Based on the matrix  $(X, Y)$ , the CCR model is formulated in as an LP problem with row vector  $v$  for input multipliers and row vector  $u$  as output multipliers. These multipliers are treated as variables in the following LP problem. The above CCR model can be replaced in matrix form by the following model,

$$(LP_o) \quad \max \quad u y_o \\ \text{subject to, } v x_o = 1 \quad (2.15)$$

$$\begin{aligned} -v X_1 + u X_2 &\leq 0 \\ v &\geq 0, \quad u \geq 0 \end{aligned} \quad (2.16)$$

The dual problem of  $LP_o$  is expressed with a real variable  $\theta$  and a nonnegative vector  $\lambda = (\lambda_1, \dots, \lambda_N)^T$  of variables as follows:



$$\begin{aligned} (\text{DLP}_o) \quad & \min \quad \theta \\ \text{subject to, } & \theta x_0 - X\lambda \geq 0 \end{aligned} \quad (2.17)$$

$$\begin{aligned} & Y\lambda \geq y_o \\ & \lambda \geq 0 \end{aligned} \quad (2.18)$$

DLP<sub>o</sub> has a feasible solution  $\theta = 1$ ,  $\lambda_o = 1$ ,  $\lambda_j = 0$  ( $j \neq o$ ). Hence the optimal  $\theta$ , denoted by  $\theta^*$ , is not greater than 1. On the other hand, due to the nonzero (i.e. semipositive) assumption for the data,  $\lambda$  will be nonzero because  $y_o \geq 0$  and  $y_o \neq 0$ . Hence,  $\theta$  must be greater than zero. Putting this all together, we have  $0 < \theta^* \leq 1$ .

### 2.3.2 Strengths and Limitations of Basic CCR Model

Strengths:

DEA can be a powerful tool and a few of the characteristics that make it powerful are:

- DEA utilizes techniques such as mathematical programming which can handle large numbers of variables and relations (constraints). DEA can handle multiple inputs and multiple outputs.
- It also does not require prescribing the functional forms that are needed in statistical regression approaches to find efficiency of alternatives.
- Inputs and outputs can have very different units. For example, one attribute could be in units of lives saved and the other could be in units of dollars without requiring an a priori tradeoff between the two.

Limitations:

The same characteristics that make DEA a powerful tool can also create problems. Limitations of DEA are listed below:

- DEA is good at estimating “relative” efficiency of an alternative but it cannot give absolute efficiency. In other words, it can tell how well an alternative is performing compared to peers but not compared to a “theoretical maximum” of that particular alternative.
- Since DEA weights of attributes are decided by the method itself such that the efficiency of the alternative under consideration is maximized, decision maker’s opinion is not considered for the final ranking.
- Since a standard formulation of DEA creates a separate linear program (LP) for each alternative, large problems can be computationally intensive.

### 2.3.3 Reduced CCR Model

Solving the basic CCR model gives the efficiencies of alternatives which are then used to rank the alternatives. The maximum efficiency obtained for any alternative by this model is 1. In many cases two or more alternatives get efficiencies equal to 1 upon ranking by the basic CCR model. In this situation it is not possible to rank the alternatives completely using DEA scores. To overcome this situation a variation of the CCR model is proposed by Andersen and Petersen [15] that allows the use of DEA efficiency scores for complete ranking of alternatives. In their model, they simply eliminate the test unit from the constraint set. The new formulation is known as Reduced CCR (RCCR) model. The new formulation is represented as follows:

$$\text{Maximize } uy_o \quad (2.19)$$

$$\begin{aligned} \text{Subject to, } & Vx_0 = 1 \\ & -vX + uY \leq 0, \quad \text{excluding the } o\text{th constraint} \\ & v \geq 0, u \geq 0 \end{aligned} \quad (2.20)$$

### 2.3.4 Improved RCCR/Assurance Region Model

It should be noted that efficiencies in RCCR model of DEA also depend on accuracy of the values of the attributes available for the comparison of alternatives. There are some cases in which the judgment by RCCR model is not adequate. In other words, in some cases an alternative is not necessarily judged to be inefficient by the RCCR model even though it is inefficient. So, it may not be representing the true ranking of various alternatives. This is because of the fact that there is no provision in RCCR model of DEA to add the information about the importance of one attribute over the other. AR approach can be used to provide the information about the comparative importance of attributes.

AR constraints for two input weights  $v_i$  and  $v_j$  is initiated by setting lower (LB) and upper bounds (UB) on each weight [8]. These LB and UB may be ranges for preference weights for each of the attributes as defined by the decision makers. The AR constraints relate the weights and they bounds to each other. The generalized AR constraint sets that are derived from LB and UB data for non-beneficial attributes are:

$$v_i \geq \frac{LB_i}{UB_j} v_j \quad \text{and} \quad v_i \leq \frac{UB_i}{LB_j} v_j \quad (2.21)$$

or

$$v_j \cdot LB_i - v_i \cdot UB_j \leq 0 \quad \text{and} \quad v_i \cdot LB_j - v_j \cdot UB_i \leq 0 \quad (2.22)$$

Similar relations can be developed for all other non-beneficial and beneficial attributes and they can be arranged in form of matrices  $P$  and  $Q$  for non-beneficial and beneficial attributes, respectively. The basic CCR model is improved by introducing an AR to add the decision maker's perception in calculating the efficiencies and written as given by Eqs. (2.23–2.24).

$$\begin{aligned} &\text{Maximize} && uy_o \\ &\text{Subject to,} && vx_o = 1 \\ &&& -vX + uY \leq 0 \end{aligned} \quad (2.23)$$

$$\begin{aligned} &vP \leq 0 \\ &uQ \leq 0 \\ &v \geq 0, u \geq 0. \end{aligned} \quad (2.24)$$

This model of DEA can be effectively used for multiple attribute decision making situations.

The RCCR/AR model helps the decision maker to restrict the attribute weights within a range. But, here also the weights of attributes are determined by the DEA method itself. So, the DEA efficiency scores obtained in this case are also not reliable. In this book, it is suggested to set same values of lower bound and upper bound in order to assign the exact weights to the attributes.

## 2.4 Improved Preference Ranking Organization Method for Enrichment Evaluations

PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluations) was introduced by Brans et al. [16] and belongs to the category of outranking methods. In this section, the focus is put on the PROMETHEE method and is improved by incorporating a fuzzy conversion scale to convert the qualitative attribute into a quantitative attribute and AHP method is incorporated for deciding the attributes' weights.

The improved PROMETHEE method involves a pairwise comparison of alternatives on each single attribute in order to determine partial binary relations denoting the strength of preference of an alternative 'a1' over alternative 'a2'. In the evaluation table, the alternatives are evaluated on different attributes. These evaluations involve mainly quantitative data. The implementation of improved PROMETHEE requires additional types of information, namely:

- information on the relative importance that is the weights of the attributes considered,
- information on the decision maker preference function, which he/she uses when comparing the contribution of the alternatives in terms of each separate attribute.

It may be added here that the original PROMETHEE method can effectively deal mainly with quantitative attributes. However, there exists some difficulty in the case of qualitative attributes. In the case of a qualitative attribute (i.e. quantitative value is not available); a ranked value judgment on a fuzzy conversion scale is adopted. By using fuzzy set theory, the value of the attributes can be first decided as linguistic terms, converted into corresponding fuzzy numbers and then converted to the crisp scores. The presented numerical approximation system systematically converts linguistic terms to their corresponding fuzzy numbers. A 11-point scale fuzzy conversion scale is presented in this book, as shown in Fig. 2.1, to help the users in assigning the quantitative values to the qualitative terms [17].

Values corresponding to the conversion scale are represented in Table 2.3 for better understanding. Appendix I describes the development of the 11-point fuzzy conversion scale shown in Fig. 2.1. Once a qualitative attribute is represented on a scale then the alternatives can be compared with each other on this attribute in the same manner as that for quantitative attributes.

The improved PROMETHEE methodology for decision making in the manufacturing environment is described below:

#### ***2.4.1 Formulation of Decision Table***

Step 1: This step is similar to step 1 of the improved AHP method.

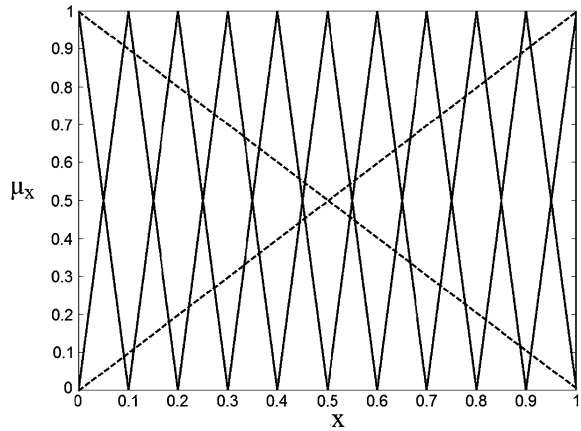
#### ***2.4.2 Deciding Weights of the Attributes***

Step 2: In the PROMETHEE method suggested by Brans et al. [16], there is no systematic way to assign weights of relative importance of attributes. Hence, in the improved PROMETHEE method AHP method is suggested for deciding the weights of relative importance of the attributes [18]. The procedure for the same is as explained in the step 2 of the improved AHP method.

#### ***2.4.3 Improved PROMETHEE Calculations***

Step 3: After calculating the weights of the attributes using AHP method, the next step is to have the information on the decision maker preference function, which he/she uses when comparing the contribution of the alternatives in terms of each attribute. The preference function ( $P_j$ ) translates the difference between the evaluations obtained by two alternatives ( $a_1$  and  $a_2$ ) in terms of a particular attribute, into a preference degree ranging from 0 to 1. Let  $P_{j, a_1 a_2}$  be the preference function associated to the attribute  $b_j$ .

**Fig. 2.1** Linguistic terms to fuzzy numbers conversion (11-point scale) ([17]; Reprinted with permission from © Elsevier 2010)



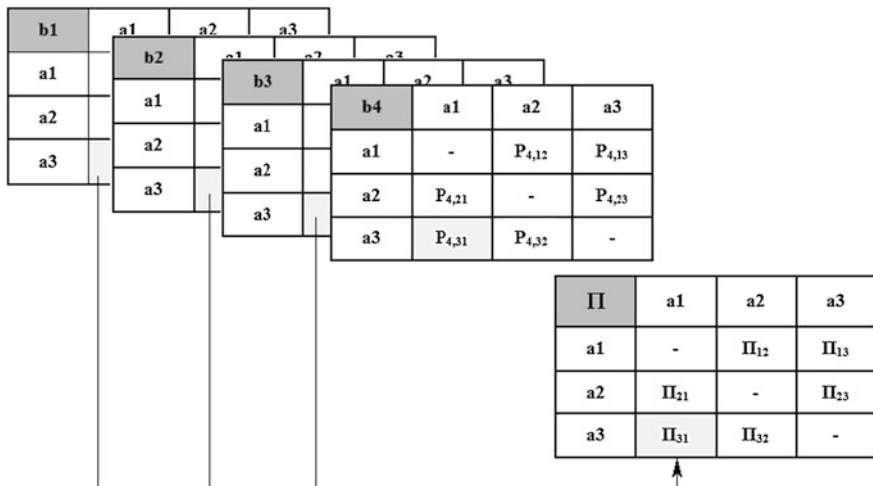
**Table 2.3** Values of selection attributes

Qualitative measures of selection attribute	Assigned value
Exceptionally low	0.0455
Extremely low	0.1364
Very low	0.2273
Low	0.3182
Below average	0.4091
Average	0.5000
Above average	0.5909
High	0.6818
Very high	0.7727
Extremely high	0.8636
Exceptionally high	0.9545

$$P_{j,a1a2} = G_j[b_j(a1) - b_j(a2)] \quad (2.25)$$

$$0 \leq P_{j,a1a2} \leq 1 \quad (2.26)$$

Where  $G_i$  is a non-decreasing function of the observed deviation (d) between two alternatives 'a1' and 'a2' over the attribute ' $b_j$ '. In order to facilitate the selection of a specific preference function, six basic types were proposed [16, 19, 20]. Preference "usual function" is equal to the simple difference between the values of the attribute ' $b_j$ ' for alternatives 'a1' and 'a2'. For other preference functions, not more than two parameters (threshold q, p, or s) have to be fixed [21]. Indifference threshold 'q' is the largest deviation to consider as negligible on that attribute and it is a small value with respect to the scale of measurement. Preference threshold 'p' is the smallest deviation to consider decisive in the preference of one alternative over another and it is a large value with respect to the scale of measurement. Gaussian threshold 's' is only used with the Gaussian preference function. It is usually fixed as an intermediate value between indifference and a preference threshold.



**Fig. 2.2** Preference indices for a problem consisting of 3 alternatives and 4 attributes ([20]; Reprinted with permission from © Taylor and Francis 2012)

If the decision maker specifies a preference function  $P_i$  and weight  $w_i$  for each attribute ' $b_j$ ' ( $j = 1, 2, \dots, M$ ) of the problem, then the multiple attribute preference index  $\Pi_{a1a2}$  is defined as the weighted average of the preference functions  $P_j$ :

$$\Pi_{a1a2} = \sum_{j=1}^M w_j P_{j,a1a2} \quad (2.27)$$

$\Pi_{a1a2}$  represents the intensity of preference of the decision maker of alternative 'a1' over alternative 'a2', when considering simultaneously all the attributes. Its value ranges from 0 to 1. This preference index determines a valued outranking relation on the set of alternatives. As an example, the schematic calculation of the preference indices for a problem consisting of 3 alternatives and 4 attributes is given in Fig. 2.2.

For improved PROMETHEE outranking relations, the leaving flow, entering flow, and the net flow for an alternative ' $a$ ' belonging to a set of alternatives  $A$  are defined by the following Eqs.:

$$\Phi^+(a) = \sum_{x \in A} \Pi_{xa} \quad (2.28)$$

$$\Phi^-(a) = \sum_{x \in A} \Pi_{ax} \quad (2.29)$$

$$\Phi(a) = \Phi^+(a) - \Phi^-(a) \quad (2.30)$$

$\Phi^+(a)$  is called the leaving flow,  $\Phi^-(a)$  is called the entering flow, and  $\Phi(a)$  is called the net flow.  $\Phi^+(a)$  is the measure of the outranking character of ' $a$ '

(i.e. dominance of alternative 'a' over all other alternatives) and  $\Phi^-(a)$  gives the outranked character of 'a' (i.e. degree to which alternative 'a' is dominated by all other alternatives). The net flow,  $\Phi(a)$ , represents a value function, whereby a higher value reflects a higher attractiveness of alternative 'a'. The net flow values are used to indicate the outranking relationship between the alternatives. For example, for each alternative 'a', belonging to the set A of alternatives,  $\Pi_{a1a2}$  is an overall preference index of 'a1' over 'a2', taking into account all the attributes,  $\Phi^+(a)$  and  $\Phi^-(a)$ . Alternative 'a1' outranks 'a2' if  $\Phi(a1) > \Phi(a2)$  and 'a1' is said to be indifferent to 'a2' if  $\Phi(a1) = \Phi(a2)$ .

$$\Pi_{31} = \sum_{j=1}^4 w_j P_{j,31}$$

The proposed decision making framework using the improved PROMETHEE method provides a complete ranking of the alternatives from the best to the worst one using the net flows. A computer program is developed in the present work in MATLAB environment that can be used for the improved PROMETHEE calculations. Any number of alternatives and the attributes can be considered and the time required for computation is less as compared to DEA method.

## 2.5 Improved EElimination Et Choix Traduisant la REalité Method

ELECTRE is one of the widely accepted methods for multiple attributes decision making in various fields of science and technology. However, only a few applications are found in the field of manufacturing, such as manufacturing system selection [22], facility location selection [23], material selection [24, 25], and vendor selection [13, 26, 27]. Furthermore, the researchers had mainly focused upon the quantitative attributes and had not effectively considered the fuzzy and/or linguistic attributes.

The outranking method EElimination Et Choix Traduisant la REalité (ELECTRE) i.e. EElimination and Choice Expressing the Reality, was developed by Roy [28]. Like all outranking methods, ELECTRE proceeds to a pairwise comparison of alternatives in each single attribute in order to determine partial binary relations denoting the strength of preference of one alternative over the other. The ELECTRE method is a highly efficient multiple attribute decision making method, which takes into account the uncertainty and vagueness, which are usually inherent in data produced by predictions and estimations. Three different threshold values are to be defined for this purpose. The thresholds of preference (p), indifference (q), and veto (v) have been introduced in the ELECTRE method, so that outranking relations are not expressed mistakenly due to differences that are less important. These three thresholds can be defined as follows:

Preference threshold (p):—Preference threshold (p) is a difference of objective values of an attribute above which the decision maker strongly prefers an alternative over other for the given attribute. Alternative b is strictly preferred to alternative a in terms of attribute  $i$  if,  $f_i(b) \geq f_i(a) + p$ .

Indifference threshold (q):—Indifference threshold (q) is a difference of attribute values beneath which the decision maker is indifferent between two alternatives for the given attribute. Alternative b is indifferent to alternative a in terms of attribute  $i$  if,  $f_i(b) < f_i(a) + q$ .

Veto threshold (v):—Veto threshold (v) blocks the outranking relationship between alternatives for the given attribute. Alternative a cannot outrank alternative b if the performance of b exceeds that of a by an amount greater than the veto threshold, i.e.  $f_i(b) \geq f_i(a) + v$ .

ELECTRE method in its basic form can successfully deal with quantitative attributes. However, to deal with a qualitative attribute (i.e. quantitative value is not available); a ranked value judgment on a fuzzy conversion scale (Table 2.3) is used. Once a qualitative attribute is represented on a scale then the alternatives can be compared with each other on this attribute in the same manner as that for quantitative attributes.

The methodology for decision making in the manufacturing environment using improved ELECTRE method can be described as below:

### ***2.5.1 Construction of the Decision Table***

Step 1: Form a decision table using the information available regarding the alternatives and attributes. This step is similar to step-1 of the improved AHP method discussed in the Sect. 2.1.1. In this book, ELECTRE method is improved by incorporating a fuzzy conversion scale because the basic ELECTRE method cannot deal with the qualitative attributes.

### ***2.5.2 Calculating the Weights of the Attributes Using AHP***

Step 2: The basic ELECTRE method also lacks in systematic method of deciding relative importance weights of the attributes. Hence, in this book, AHP procedure is suggested for deciding the relative importance weights as explained in Sect. 2.1.2.

### ***2.5.3 Calculations Using ELECTRE for Final Ranking***

Step 3: After calculation of weights, the operational implementation of the outranking principles of improved ELECTRE is now described, assuming that all



attributes are to be beneficial (i.e. higher value is desired). If  $f_j(a1)$  is defined as the score of alternative “a1” on attribute  $j$  and  $w_j$  represents the weight of attribute  $j$ , the concordance index  $C(a1,a2)$  is defined as follows:

$$C(a1,a2) = \frac{1}{W} \sum_{j=1}^M w_j c_j(a1,a2), \text{ where } W = \sum_{j=1}^M w_j \quad (2.31)$$

where,

$$c_j(a1,a2) = \begin{cases} 1, & \text{if } f_j(a1) + q_j \geq f_j(a2) \\ 0, & \text{if } f_j(a1) + p_j \leq f_j(a2), \\ \frac{p_j + f_j(a1) - f_j(a2)}{p_j - q_j} & \text{otherwise.} \end{cases} \quad j=1,2,\dots,M \quad (2.32)$$

The concordance index  $C(a1,a2)$  indicates relative dominance of alternative “a1” over alternative “a2”, based on the relative importance weightings of the relevant decision attributes. In case, if any attribute is non-beneficial, negative of the objective values can be considered. To calculate discordance, a threshold, called the veto threshold, is defined. The veto threshold ( $v_j$ ) allows for the possibility of alternative “a1” outranking “a2” to be refused totally if, for any one attribute  $j$ ,  $f_j(a2) \geq f_j(a1) + v_j$ . The discordance index for each attribute  $j$ ,  $d_j(a1,a2)$  is calculated as:

$$d_j(a1,a2) = \begin{cases} 0, & \text{if } f_j(a1) + p_j \geq f_j(a2) \\ 1, & \text{if } f_j(a1) + v_j \leq f_j(a2), \\ \frac{f_j(a2) - f_j(a1) - p_j}{v_j - p_j} & \text{otherwise} \end{cases} \quad j=1,2,\dots,M \quad (2.33)$$

The discordance index  $d_j(a1,a2)$  measures the degree to which alternative “a1” is worse than “a2”. The essence of the discordance index is that any outranking of “a2” by “a1” indicated by the concordance index can be overruled if there is any attribute for which alternative “a2” outperforms alternative “a1” by at least the veto threshold. The final step in the model building phase is to combine these two measures to produce a measure of the degree of outranking; that is, a credibility index which assesses the strength of the assertion that “a1 is at least as good as a2”. The credibility degree for each pair  $(a1,a2) \in A$  is defined as:

$$S(a1,a2) = \begin{cases} C(a1,a2), & \text{if } d_j(a1,a2) \leq C(a1,a2), \forall j \\ & \text{where, } j \in J(a1,a2) \text{ is the set of criteria} \\ & \text{such that } d_j(a1,a2) > C(a1,a2) \\ C(a1,a2) \cdot \prod_{j \in J(a1,a2)} \frac{1-d_j(a1,a2)}{1-C(a1,a2)} & \text{otherwise} \end{cases} \quad (2.34)$$

This concludes the construction of the outranking model. The next step in the outranking approach is to create the hierarchy of the alternative solutions from the elements of the credibility matrix. The determination of the hierarchy rank is achieved by calculating the superiority ratio for each alternative. This ratio is calculated from the credibility matrix and is the fraction of the elements' sum of every alternative's respective column. The numerator represents the total dominance of the specific alternative over the remaining alternatives and the denominator the dominance of the remaining alternatives over the former. The numerator for each alternative is also known as concordance credibility is calculated as follows:

$$\phi^+(a1) = \sum_{a2 \in A} S(a1, a2) \quad (2.35)$$

The denominator for each alternative i.e. discordance credibility is calculated as follows:

$$\phi^-(a1) = \sum_{a2 \in A} S(a2, a1) \quad (2.36)$$

Finally, the superiority ratio is obtained as:

$$R(a1) = \phi^+(a1)/\phi^-(a1) \quad (2.37)$$

The alternatives are then arranged in ascending order of their superiority ratio. The alternatives with higher values of superiority ratio are preferred over the others.

## 2.6 Improved COMplex PROportional ASsessment Method

This section describes another decision making method known as COPRAS (COMplex PROportional ASsessment) method for decision making in the manufacturing environment.

COPRAS is one of the MADM methods for decision making in various fields of science and technology. The COPRAS method uses a stepwise ranking and evaluating procedure of the alternatives in terms of significance and utility degree. The success of the methodology is basically due to its simplicity and to its particular friendliness of use. However, only a few successful applications of COPRAS method have been reported in literature in various fields for decision making, such as construction [29], sustainability evaluation [30], buildings construction [31, 29], road design [32], and education [33]. However, the researchers had mainly focused upon the quantitative attributes and had not effectively considered the qualitative attributes.

The various steps of improved COPRAS method presented in this book for decision making in the manufacturing environment are described below:

### 2.6.1 Construction of the Decision Table

Step 1: Similar to step 1 of the improved AHP method discussed in [Sect. 2.1.1](#), prepare a decision table which shows data of various available alternatives and attributes affecting their selection. The COPRAS method is improved by including a fuzzy scale for conversion of converting qualitative data into quantitative data.

### 2.6.2 Calculating the Weights of the Attributes Using AHP

Step 2: This step of weight calculation is also similar to step 2 of the improved PROMETHEE method discussed in [Sect. 2.4.2](#). The AHP method is suggested in the improved COPRAS method for systematically deciding the relative importance weights of the attributes.

### 2.6.3 COPRAS Calculations for Final Ranking

Step 3: The procedure of the COPRAS method consists of the following steps:

- Preparing of the decision making matrix  $X$ :

$$X = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1M} \\ m_{21} & m_{22} & \cdots & m_{2M} \\ \vdots & \vdots & \cdots & \vdots \\ m_{N1} & m_{N2} & \cdots & m_{NM} \end{bmatrix} \quad (2.38)$$

where,  $N$  is the number of alternatives and  $M$  is the number of attributes.

- Normalization of the decision making matrix  $\bar{X}$ . The normalized values of this matrix are calculated using following formula.

$$\bar{x}_{ij} = \frac{m_{ij}}{\sum_{i=1}^N m_{ij}} ; i = 1, 2, \dots, N \text{ and } j = 1, 2, \dots, M. \quad (2.39)$$

where,  $j$  refers to the attribute and  $i$  to the alternative. After this step, the normalized decision making matrix can be presented as:

$$\bar{X} = \begin{bmatrix} \bar{x}_{11} & \bar{x}_{12} & \cdots & \bar{x}_{1M} \\ \bar{x}_{21} & \bar{x}_{22} & \cdots & \bar{x}_{2M} \\ \vdots & \vdots & \cdots & \vdots \\ \bar{x}_{N1} & \bar{x}_{N2} & \cdots & \bar{x}_{NM} \end{bmatrix} \quad (2.40)$$

- Calculation of the weighted normalized decision matrix  $\hat{X}$ . The weighted normalized values  $\hat{x}_{ij}$  are calculated as,

$$\hat{x}_{ij} = \bar{x}_{ij} * w_j, \quad i = 1, 2, \dots, N \text{ and } j = 1, 2, \dots, M. \quad (2.41)$$

where,  $w_j$  is significance (weight) of the  $j$ th attribute. After this step the weighted normalized decision making matrix is formed as:

$$\hat{X} = \begin{bmatrix} \hat{x}_{11} & \hat{x}_{12} & \cdots & \hat{x}_{1M} \\ \hat{x}_{21} & \hat{x}_{22} & \cdots & \hat{x}_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{x}_{N1} & \hat{x}_{N2} & \cdots & \hat{x}_{NM} \end{bmatrix} \quad (2.42)$$

- Calculate sums  $P_i$  of attributes values for which larger values are more preferable i.e. beneficial attributes, for all the alternatives:

$$P_i = \sum_{j=1}^k \hat{x}_{ij} \quad (2.43)$$

Where,  $k$  is number of attributes which must be maximized i.e. beneficial attributes. (It is assumed that in the decision matrix the first columns are of beneficial attributes and columns for non-beneficial attributes are placed afterwards).

- Calculate sums  $R_i$  of attributes values in which smaller values are more preferable i.e. non-beneficial attributes, for all the alternatives:

$$R_i = \sum_{j=k+1}^M \hat{x}_{ij} \quad (2.44)$$

Hence  $(M-k)$  is the number of non-beneficial attributes which must be minimized.

- Determining the minimal value of  $R_i$ :

$$R_{\min} = \min_i R_i; \quad i = 1, 2, 3, \dots, N \quad (2.45)$$

- Calculation of the relative weight of each alternative  $Q_i$ :

$$Q_i = P_i + [(R_{\min} \sum_i^M R_i) / (R_i \sum_i^M (R_{\min}/R_i))] \quad (2.46)$$

Equation (2.46) can be written as follows:

$$Q_i = P_i + [(\sum_i^M R_i)/(R_i \sum_i^M (1/R_i))] \quad (2.47)$$

- Determination of the optimality criterion  $K$ :

$$K = \max_i Q_i \quad ; \quad i = 1, 2, 3, \dots, N \quad (2.48)$$

- Determination of the priority of the alternative: The greater significance (relative weight of alternative)  $Q_i$ , the higher is the priority (rank) of the alternative. In the case of  $Q_{\max}$ , the satisfaction degree is the highest.
- Calculation of the utility degree of each alternative:

$$N_i = Q_i/Q_{\max} * 100 \% \quad (2.49)$$

where,  $Q_i$  and  $Q_{\max}$  are the significance of alternatives obtained from Eq. (2.47).

The improved COPRAS method presented in this section uses a stepwise ranking and evaluating procedure of the alternatives considering fuzzy scale for qualitative attributes and calculating relative importance weights of attributes using AHP method for better consistency in judgments. The ranking is determined by examining utility degree calculated from Eq. (2.49). Complete ranking can be obtained by arranging the alternatives in the descending order of their utility degrees as higher values of utility degree are preferred over the lower ones. The proposed decision making framework using improved COPRAS method provides a complete ranking of the alternatives from the best to the worst one.

## 2.7 Improved Gray Relational Analysis Method

Gray relational analysis (GRA) is one of the derived evaluation methods based on the concept of gray relational space (GRS). The GRA method is widely applied in various areas, such as economics, marketing, and agriculture [34, 35].

The main procedure of GRA is firstly translating the performance of all alternatives into a comparability sequence. This step is called data pre-processing. According to these sequences, a reference sequence (ideal target sequence) is defined. Then, the gray relational coefficient between all comparability sequences and the reference sequence for different values of distinguishing coefficient ( $\xi$ ) are calculated. Finally, based on these gray relational coefficients, the gray relational grade between the reference sequence and every comparability sequences is calculated. If an alternative gets the highest gray relational grade with the reference sequence, it means that the comparability sequence is most similar to the reference sequence and that alternative would be the best choice [36].

The steps of improved GRA are described below [37].

### Step 1: Data pre-processing

For a multiple attribute decision making problem having  $N$  alternatives and  $M$  attributes, the general form of decision matrix is as shown in Table 1.1.

It may be mentioned here that the original GRA method can effectively deal mainly with quantitative attributes. However, there exists some difficulty in the case of qualitative attributes. In the case of a qualitative attribute (i.e. quantitative value is not available); a ranked value judgment on a fuzzy conversion scale can be adopted as explained in Sect. 2.4.

The term  $m_{ij}$  can be translated into the comparability sequence  $m_{ij} = (x_{i1}, x_{i2}, \dots, x_{ij}, \dots, x_{iM})$ , where  $x_{ij}$  is the normalized value of  $m_{ij}$  for attribute  $j$  ( $j = 1, 2, 3, \dots, M$ ) of alternative  $i$  ( $i = 1, 2, 3, \dots, N$ ). After normalization, decision matrix becomes the normalization matrix. However, the normalized values of  $m_{ij}$  are determined by the use of the Eqs. (2.50)–(2.52), which are for beneficial type, non-beneficial type, and target value type attributes, respectively. These are described as follows [36]:

1. If the expectancy is larger-the-better (i.e. beneficial attribute), then it can be expressed by

$$x_{ij} = \frac{m_{ij} - \min\{m_{ij}\}}{\max\{m_{ij}\} - \min\{m_{ij}\}} \text{ for } i = 1, 2, \dots, N \text{ and } j = 1, 2, \dots, M \quad (2.50)$$

2. If the expectancy is smaller-the-better (i.e. non-beneficial attribute), then it can be expressed by

$$x_{ij} = \frac{\max\{m_{ij}\} - m_{ij}}{\max\{m_{ij}\} - \min\{m_{ij}\}} \text{ for } i = 1, 2, \dots, N \text{ and } j = 1, 2, \dots, M \quad (2.51)$$

3. If the expectancy is nominal-the-best (i.e. closer to the desired value or target value), then it can be expressed by

$$x_{ij} = 1 - \frac{|m_{ij} - m_j^*|}{\max\{\max\{m_{ij}\} - m_j^*, m_j^* - \min\{m_{ij}\}\}} \quad (2.52)$$

for  $i = 1, 2, \dots, N$  and  $j = 1, 2, \dots, M$

where  $m_j^*$  is closer to the desired value of  $j$ th attribute.

### Step 2: Reference sequence

In comparability sequence all performance values are scaled to  $[0, 1]$ . For an attribute  $j$  of alternative  $i$ , if the value  $x_{ij}$  which has been processed by data pre-processing procedure is equal to 1 or nearer to 1 than the value for any other alternative, then the performance of alternative  $i$  is considered as best for the attribute  $j$ . The reference sequence  $X_0$  is defined as  $(x_{01}, x_{02}, \dots, x_{0j}, \dots, x_{0M}) = (1, 1, \dots, 1, \dots, 1)$ , where  $x_{0j}$  is the reference value for  $j$ th attribute and it aims to

find the alternative whose comparability sequence is the closest to the reference sequence.

#### Step 3: Gray relational coefficient

Gray relational coefficient is used for determining how close  $x_{ij}$  is to  $x_{0j}$ . The larger the gray relational coefficient, the closer  $x_{ij}$  and  $x_{0j}$  are. The gray relational coefficient can be calculated by Eq. (2.53) [36].

$$\gamma(x_{0j}, x_{ij}) = \frac{(\Delta_{\min} + \xi \cdot \Delta_{\max})}{(\Delta_{ij} + \xi \cdot \Delta_{\max})} \text{ for } i = 1, 2, \dots, N \text{ and } j = 1, 2, \dots, M \quad (2.53)$$

In Eq. (2.53),  $\gamma(x_{0j}, x_{ij})$  is the gray relational coefficient between  $x_{ij}$  and  $x_{0j}$  and

$$\begin{aligned} \Delta_{ij} &= |x_{0j} - x_{ij}|, \\ \Delta_{\min} &= \text{Min} \{ \Delta_{ij}, i = 1, 2, \dots, N; j = 1, 2, \dots, M \} \\ \Delta_{\max} &= \text{Max} \{ \Delta_{ij}, i = 1, 2, \dots, N; j = 1, 2, \dots, M \} \end{aligned}$$

$\xi$  = distinguishing coefficient,  $\xi \in (0, 1]$ .

Distinguishing coefficient ( $\xi$ ) is also known as the index for distinguishability. Smaller the  $\xi$  is, the higher is its distinguishability. It represents the equation's "contrast control". The purpose of  $\xi$  is to expand or compress the range of the gray relational coefficient. Different  $\xi$  may lead to different solution results. Decision makers should try several different  $\xi$  and analyze the impact on the GRA results [36]. In many situations,  $\xi$  takes the value of 0.5 because this value usually offers moderate distinguishing effects and good stability [38]. In this work, different  $\xi$  values are considered for the analysis.

#### Step 4: Gray relational grade

The measurement formula for quantification in GRS is called the gray relational grade. The gray relational grade (gray relational degree) indicates the degree of similarity between the comparability sequence and the reference sequence. It is a weighted sum of the gray relational coefficients and it can be calculated using Eq. (2.54) [36].

$$\Gamma(X_0, X_i) = \sum_{j=1}^M w_j \cdot \gamma(x_{0j}, x_{ij}) \text{ for } i = 1, 2, \dots, N \quad (2.54)$$

Where,  $\sum_{j=1}^M w_j = 1$

The original GRA method has not specified any systematic method of deciding the weights of relative importance of the attributes. Hence, in this method, an (AHP) procedure is suggested for deciding the weights of relative importance of the attributes.

In Eq. (2.54),  $\Gamma(X_0, X_i)$  is the gray relational grade between the comparability sequence  $X_i$  and reference sequence  $X_0$ . It represents the level of correlation between the reference sequence and the comparability sequence.  $w_j$  is the weight of attribute  $j$  and usually depends on decision makers' judgment or the structure of the proposed problem. The gray relational grade indicates the degree of similarity

between the comparability sequence and the reference sequence. If an alternative gets the highest gray relational grade with the reference sequence, it means that comparability sequence is most similar to the reference sequence and that alternative would be the best choice [39].

In this book, several values of  $\xi$  are considered to find the rankings of given alternatives. Each  $\xi$  gives its own ranking. To get the final GRA ranking “Mode principle” is applied, which considers the effect of all  $\xi$  values. The “Mode” is the value that occurs most often. In “Mode principle”, the alternative having mode number at rank 1 position is selected and given final GRA rank as 1. Similarly, the alternative having mode number at rank 2 position is selected and given final GRA rank as 2 and so on. Those alternatives, which have been already given final GRA rankings, are not considered further to find next final GRA ranking.

## 2.8 Improved Utility Additive Method

The purpose of this method is to assess the additive utility functions which aggregate multiple criteria in a composite criterion, using the information given by a subjective ranking on a set of stimuli or actions (weak order comparison judgments) and the multiple criteria evaluations of these actions. It is an ordinal regression method using LP to estimate the parameters of the utility function.

The model assessed by Utility Additive (UTA) is not a single utility function, but is a set of utility functions, all of them being models consistent with the decision maker's a priori preferences. In order to assess such a set of utility functions, an ordinal regression method is used. Using LP, it adjusts optimally additive nonlinear utility functions so that they fit data which consist of multiple criteria evaluations of some alternatives and a subjective ranking of these alternatives given by the decision maker.

The UTA method proposed by Jacquet-Lagrange and Siskos [40] aims at inferring one or more additive value functions from a given ranking on a reference set  $A_R$ . The method uses special LP techniques to assess these functions so that the ranking obtained through these functions on  $A_R$  is as consistent as possible with the given one. This LP model is solved and marginal utility values are obtained. Then the utility value of each alternative is calculated. Higher the utility value, better the alternative. The ranking based on utility value is the ranking without considering the weights of attributes, i.e. equal weightage is given to all attributes. To consider the weights of attributes, the alternatives are chosen based on weighted utility of alternatives. The procedure of UTA method is described below.

A set of alternatives, called ‘A’, is considered which is valued by a family of attributes  $g = (g_1, g_2, \dots, g_M)$ . The method uses a classical operational attitude of assessing a model of overall preference of an individual and leads to the aggregation of all attributes into a unique criterion called a utility function  $U(g)$  [40].

$$U(g) = U(g_1, g_2, \dots, g_M) \quad (2.55)$$



Let,  $P$  is the strict preference relation and  $I$  is the indifference relation. If  $g(a) = [g_1(a), g_2(a), \dots, g_M(a)]$  is the multiple attribute evaluation of an alternative 'a', then the following properties generally hold for the utility function  $U$ :

$$U[g(a)] > U[g(b)] \Leftrightarrow aPb \quad (2.56)$$

$$U[g(a)] = U[g(b)] \Leftrightarrow aIb \quad (2.57)$$

And the relation  $R = P \cup I$  is a weak order.

The criteria (i.e. attributes) aggregation model in UTA is assumed to be an un-weighted additive value function of the following form

$$U(g) = \sum_{j=1}^M u_j(g_j) \quad (2.58)$$

where  $u_j(g_j)$  is the marginal utility of the attribute  $g_j$  for the given alternative, which is entirely determined by the attribute  $g_j$ . Let  $g_j^*$  and  $g_{j*}$  be respectively the most and least preferred value (grade) of the attribute  $j$ .

The most common normalization constraints are the following:

$$\begin{cases} \sum_{j=1}^M u_j(g_j^*) = 1 \\ u_j(g_{j*}) = 0, \text{ for all } j = 1, 2, \dots, M \end{cases} \quad (2.59)$$

On the basis of the above additive model and taking into account the preference conditions, the value of each alternative  $a \in A_R$  may be written as

$$U'[g(a)] = \sum_{j=1}^M u_j[g_j(a)] + \sigma(a) \quad \forall a \in A_R \quad (2.60)$$

where  $\sigma(a)$  is a potential error relative to the utility

$$U[g(a)] = \sum_{j=1}^M u_j[g_j(a)] \quad (2.61)$$

In order to estimate the corresponding marginal value functions in a piecewise linear form, Jacquet-Lagrange and Siskos [40] proposed the use of linear interpolation. For each attribute, the interval  $[g_j^*, g_{j*}]$  is cut into  $(\alpha_j - 1)$  equal intervals, and thus the end points  $g_j^i$  are given by the formula:

$$g_j^i = g_{j*} + \frac{i-1}{\alpha_j-1} (g_j^* - g_{j*}) \quad \forall i = 1, 2, \dots, \alpha_j \quad (2.62)$$

The marginal value of an alternative 'a' is approximated by a linear interpolation, and thus, for  $g_j(a) \in [g_j^i - g_j^{i+1}]$ .

$$u_j[g_j(a)] = u_j(g_j^i) + \frac{g_j(a) - g_j^i}{g_j^{i+1} - g_j^i} [u_j(g_j^{i+1}) - u_j(g_j^i)] \quad (2.63)$$

The set of reference alternatives  $A_R = \{a_1, a_2, \dots, a_N\}$  is also rearranged in such a way that  $a_1$  is the head of the ranking (best) and  $a_N$  its tail (worst). Since the ranking has the form of a weak order  $R$ , for each pair of consecutive alternatives  $(a_k, a_{k+1})$  it holds either  $a_k \succ a_{k+1}$  (preference) or  $a_k \sim a_{k+1}$  (indifference). Thus, if

$$\Delta(a_k, a_{k+1}) = U'[g(a_k)] - U'[g(a_{k+1})] \quad (2.64)$$

Then one of the following holds:

$$\begin{cases} \Delta(a_k, a_{k+1}) \geq \delta & \text{if } a_k \succ a_{k+1} \\ \Delta(a_k, a_{k+1}) = 0 & \text{if } a_k \sim a_{k+1} \end{cases} \quad (2.65)$$

where  $\delta$  is a small positive number so as to discriminate significantly two successive equivalence classes of  $R$ .

The marginal value functions are finally estimated by means of the following LP, in which objective function is depending on the  $\sigma(a)$  and indicating the amount of total deviation.

LP model: Minimize( $F$ ) =  $\sum_{a \in A_R} \sigma(a)$

Subject to:

$$\left. \begin{aligned} &\text{LP model : Minimize } (F) = \sum_{a \in A_R} \sigma(a) \\ &\text{Subject to:} \\ &\left. \begin{aligned} &\Delta(a_k, a_{k+1}) \geq \delta \quad \text{if } a_k \succ a_{k+1} \\ &\Delta(a_k, a_{k+1}) = 0 \quad \text{if } a_k \sim a_{k+1} \end{aligned} \right\} \forall k \\ &u_j(g_j^{i+1}) - u_j(g_j^i) \geq 0 \quad \forall i \text{ and } j; \\ &\sum_{j=1}^M u_j(g_j^*) = 1; \quad u_j(g_{j*}) = 0 \\ &u_j(g_j^i) \geq 0, \quad \sigma(a) \geq 0 \quad \forall a \in A_R, \quad \forall i \text{ and } j \end{aligned} \right\} \quad (2.66)$$

This LP model is solved and marginal utility values are obtained. Then the utility value  $[U(a)]$  of each alternative is calculated. Higher the  $U(a)$  value, better the alternative. The ranking based on  $U(a)$  is the ranking without considering the weights of attributes. In this method, the improvement is made by incorporating the weights to get rank of alternatives. To consider the weights of attributes, the alternatives are chosen based on weighted utility of alternatives rather than the utility value which is given in Eq. 2.67. In this method, an AHP procedure is suggested for deciding the weights of relative importance of attributes.

$$\text{Weighted } U(a) = \sum_{j=1}^M w_j u_j [g_j(a)] \quad (2.67)$$

Steps for solving MADM problems using the improved UTA method are as follows:

1. Decision matrix: find the decision matrix. The attributes of the decision matrix can be objective or subjective in nature. The subjective values of attributes are to be converted into corresponding crisp value. In this method, to take care of subjectiveness of attributes a seven point fuzzy scale can be used which systematically converts the subjective values of attributes into corresponding crisp values as explained in [Sect. 2.4](#).
2. Reference sequence of alternatives: get the reference sequence ( $A_R$ ) of alternatives; it can be found based on  $\sum_{j=1}^M x_{ij}$ , where  $x_{ij}$  is the normalized value of  $j$ th attribute for  $i$ th alternative or it is decided by the decision maker(s).
3. Equations for utility values of the alternatives: divide the range for each attribute in equal interval of parts and form the utility value equations for each alternative.
4. Mathematical formulation: formulate the mathematical model of the problem as LP model.
5. Solution: solve the LP model to get the utility value of each alternative and then get the weighted utility value of each alternative by using Eq. (2.67). If different weights of attributes are given, then ranking of alternatives is based on the weighted utility value of alternatives.

The UTA method is improved in this book by proposing a seven point fuzzy scale for the systematic conversion qualitative attributes into corresponding quantitative values. Another improvement made is the incorporation of the weights of attributes and determining the rank of alternatives based on weighted utility values of alternatives.

## 2.9 VIKOR Method

The compromise ranking method, also known as VIKOR (Višekriterijumsko Kompromisno Rangiranje), was introduced as an applicable technique to implement within MADM. The foundation for compromise solution was established by Yu [41] and Zeleny [42] and later advocated by Opricovic and Tzeng [43–46]. The compromise solution is a feasible solution, which is closest to the ideal solution, and a compromise means an agreement established by mutual concessions.

The VIKOR method is improved in this book by introducing the analytical hierarchy process (AHP) procedure for deciding the weights of relative importance of the attributes. Another improvement made is the incorporation of a new seven point fuzzy scale for the systematic conversion of qualitative attributes into corresponding quantitative values.

The multiple attribute merit for compromise ranking was developed from the  $L_p$ -metric used in the compromise programming method [42].

$$L_{p,i} = \left\{ \sum_{j=1}^M (w_j [(m_{ij})_{\max} - m_{ij}] / [(m_{ij})_{\max} - (m_{ij})_{\min}]^p \right\}^{1/p} \quad (2.68)$$

$$1 \leq p \leq \infty; \quad i = 1, 2, \dots, N$$

where  $N$  is the number of alternatives and  $M$  is the number of attributes.  $m_{ij}$  value (for  $i = 1, 2, 3, \dots, N$ ;  $j = 1, 2, 3, \dots, M$ ) indicates the values of attributes for different alternatives.

Within the VIKOR method  $L_{1,i}$  and  $L_{\infty,i}$  are used to formulate the ranking measure  $E_i$  and  $F_i$  respectively. The main procedure of VIKOR method is described below:

Step 1: Decision matrix

The first step is to determine the objective and to identify the pertinent evaluation attributes and prepare a decision matrix. For a multiple attribute decision making problem having  $m$  alternatives and  $n$  attributes, the general form of decision matrix is as shown by Table 1.1.

Step 2: Calculate the values of  $E_i$  and  $F_i$

The values of measures  $E_i$  and  $F_i$  are calculated using the formulae given below.

$$E_i = \sum_{j=1}^M w_j [(m_{ij})_{\max} - m_{ij}] / [(m_{ij})_{\max} - (m_{ij})_{\min}]; \text{ for beneficial attribute} \quad (2.69)$$

$$E_i = \sum_{j=1}^M w_j [m_{ij} - (m_{ij})_{\min}] / [(m_{ij})_{\max} - (m_{ij})_{\min}]; \text{ for non-beneficial attribute} \quad (2.70)$$

$$F_i = \text{Max}^m \text{ of } \{w_j [(m_{ij})_{\max} - m_{ij}] / [(m_{ij})_{\max} - (m_{ij})_{\min}]\}; \text{ for beneficial attribute} \quad (2.71)$$

$$F_i = \text{Max}^m \text{ of } \{w_j [m_{ij} - (m_{ij})_{\min}] / [(m_{ij})_{\max} - (m_{ij})_{\min}]\}; \text{ for non-beneficial attribute} \quad (2.72)$$

Step-3: Calculate the values of  $P_i$

$$P_i = v \left[ \frac{E_i - E_{i,\min}}{E_{i,\max} - E_{i,\min}} \right] + (1 - v) \left[ \frac{F_i - F_{i,\min}}{F_{i,\max} - F_{i,\min}} \right] \quad (2.73)$$

where  $E_{i,\max}$  is the maximum value of  $E_i$  and  $E_{i,\min}$  is the minimum value of  $E_i$ .  $F_{i,\max}$  is the maximum value of  $F_i$  and  $F_{i,\min}$  is the minimum value of  $F_i$ .  $v$  is introduced as weight of the strategy of 'the majority of attributes'. Normally, the value of  $v$  is taken as 0.5. However, it can be taken any value from 0 to 1.

Step 4: Arrange the alternatives in the ascending order according to the values of  $P_i$ ,  $E_i$ , and  $F_i$

By arranging the alternatives in the ascending order of  $P_i$ ,  $E_i$  and  $F_i$  values, the three ranking lists can be obtained. Compromise ranking list for a given  $\nu$  is obtained by ranking with  $P_i$  measure. The best alternative ranked by  $P_i$  is the one with the minimum value of  $P_i$ .

Step 5: Compromise solution

For given attribute weights, proposed as a compromise solution, alternative  $A_k$ , which is the best ranked by the measure  $P$ , if the following two conditions are satisfied [47, 4]:

Condition 1: “Acceptable advantage”  $P(A_1) - P(A_k) \geq (1/(m - 1))$ .  $A_1$  is the second best alternative in the ranking list by  $P$ .

Condition 2: “Acceptable stability in decision making”: Alternative  $A_k$  must also be the best ranked by  $E$  or/and  $F$ . This compromise solution is stable within a decision making process, which could be: “voting by majority rule” (when  $\nu > 0.5$  is needed) or “by consensus” (when  $\nu \approx 0.5$ ) or “with veto” (when  $\nu > 0.5$ ).

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternatives  $A_k$  and  $A_1$  if only condition 2 is not satisfied.
- Alternatives  $A_k, A_1, \dots, A_p$  if condition 1 is not satisfied; and  $A_p$  is determined by the relation  $P(A_p) - P(A_1) \approx (1/(m - 1))$ .

VIKOR is a helpful tool in MADM, particularly in a situation where the decision maker is not able, or does not know to express preference at the beginning of system design. The obtained compromise solution could be accepted by the decision makers because it provides a maximum “group utility” (represented by  $E_{i,\min}$ ) of the “majority”, and a minimum of the individual regret (represented by  $F_{i,\min}$ ) of the “opponent”.

## 2.10 Improved Ordered Weighted Averaging Method

Ordered weighted averaging (OWA) method is based on OWA operator. Yager [48] introduced the OWA operator to provide a method for aggregating multiple inputs that lie between the maximum and minimum operators. The aggregation of OWA operator is generally composed of the following three steps [49]: (1) reorder the input arguments (attribute values) in descending order, (2) determine the weights associated with the OWA operator by using a proper method, and (3) utilize the OWA weights to aggregate the reordered arguments (attribute values).

In OWA method, the paired judgments on the alternatives are the decision maker’s holistic judgments. The paired judgments can be obtained on comparing the attributes data for a pair of alternatives. For the given set of paired judgments the problem is formulated as a LP model which gives the ordered weight of the attributes.

The combined goodness values of the alternatives are obtained using the ordered weights. Any alternative with the highest combined goodness measure will be considered the most preferred decision. The components of input vector are to be ordered (descending order) before multiplying them by the ordered weights. The OWA method is improved in this book to deal with the subjective as well as objective attributes.

Steps of OWA method are given below:

Step 1: Decision matrix and its normalization

This step includes getting the measures of attributes for each alternative and the normalization of the attribute data. The attributes may be objective or subjective. The subjective attributes are converted into corresponding crisp scores as explained in Sect. 2.4. The normalization is necessary to keep all the attributes on same scale. For an MADM problem, if there are  $N$  alternatives and  $M$  attributes, the  $i$ th alternative can be expressed as  $Y_i = (m_{i1}, m_{i2}, \dots, m_{ij}, \dots, m_{iM})$  in decision matrix form, where  $m_{ij}$  is the performance value of attribute  $j$  ( $j = 1, 2, \dots, M$ ) for alternative  $i$  ( $i = 1, 2, \dots, N$ ). The term  $Y_i$  can be translated into the comparability sequence  $X_i = (x_{i1}, x_{i2}, \dots, x_{ij}, \dots, x_{iM})$  using Eqs. (2.74) and (2.75), where  $x_{ij}$  is the normalized value of  $m_{ij}$  for attribute  $j$  ( $j = 1, 2, 3, \dots, M$ ) of alternative  $i$  ( $i = 1, 2, 3, \dots, N$ ).

Let  $x_{ij}$  is the normalized value of  $y_{ij}$  for attribute  $j$  of alternative  $i$ , then

$$x_{ij} = \frac{m_{ij}}{\max_j(m_{ij})}; \text{ if } j\text{th attribute is beneficial} \quad (2.74)$$

$$x_{ij} = \frac{\min_j(m_{ij})}{m_{ij}}; \text{ if } j\text{th attribute is non-beneficial} \quad (2.75)$$

Step 2: Paired judgments on the alternatives

Paired judgments on the alternatives are the decision maker's holistic judgments between the alternatives. The paired judgments can be obtained on comparing the attributes data for a pair of alternatives. If  $A = \{A_1, A_2, \dots, A_N\}$  is the set of alternatives and let  $\theta \subseteq A \times A$  denote the set of ordered pair  $(i, j)$ , where  $i$  is designated as a preferred alternative.

Step 3: Find OWA weights consistent with the ordered pairs

An OWA operator of dimension  $n$  is a mapping  $f: R^n \rightarrow R$  defined as,

$$f(x_1, \dots, x_M) = \sum_{j=1}^M w_j \cdot b_j = w_1 b_1 + w_2 b_2 + \dots + w_M b_M \quad (2.76)$$

where  $b_j$  is the  $j$ th largest element in the set of inputs  $\{x_1, x_2, \dots, x_M\}$ , and  $\{w_1, w_2, \dots, w_M\}$  are the ordered weights. It is assumed that  $w_j \leq 1$  for all  $j$  and  $\sum_{j=1}^M w_j = 1$ . Vector  $W = \{w_1, w_2, \dots, w_M\}$  is called order weights vector and  $f$  is the combined goodness measure of a decision alternative if the inputs are its evaluations with respect to  $M$  attributes. Any alternative with the highest  $f$  value will be considered the most preferred decision. The components of input vector are to be ordered (descending order) before multiplying them by the ordered weights [50, 51].

The solution would be consistent with the decision maker's holistic judgments between alternatives if  $f(A_i) - f(A_j) > 0$  for every priory ordered pair  $(i, j) \in \theta$ . Now, for all  $(i, j) \in \theta$ ,

$$\sum_{k=1}^M (b_{ik} - b_{jk})w_k > 0 \quad \text{for } w_k \in W \quad (2.77)$$

where  $b_{ik}$  and  $b_{jk}$  are the reordered values of the attributes of the alternative  $A_i$  and  $A_j$  respectively. Now the goal is to determine the optimum ordered weights for which the condition

$$\sum_{k=1}^M (b_{ik} - b_{jk})w_k \geq \varepsilon \quad (2.78)$$

for every ordered pair  $(i, j) \in \theta$  is violated as minimally as possible in which  $\varepsilon$  is a small arbitrary positive number to make the problem tractable by the LP. To attain the objective "as minimally as possible", the auxiliary variable  $\delta_{ij}$  is introduced which is given in Eq. (2.79).

$$\sum_{k=1}^M (b_{ik} - b_{jk})w_k + \delta_{ij} \geq \varepsilon \quad (2.79)$$

For every ordered pair  $(i, j) \in \theta$  and the sum of auxiliary variables is minimized [50].

The mathematical model is:

$$\left. \begin{aligned} &\text{Minimize } \sum_{(i,j) \in \theta} \delta_{ij} \\ &\sum_{k=1}^M (b_{ik} - b_{jk})w_k + \delta_{ij} \geq \varepsilon \quad \text{for all } (i, j) \in \theta \\ &\sum_{k=1}^M w_k = 1 \\ &\delta_{ij} \geq 0 \text{ and } w_k \geq 0 * \end{aligned} \right\} \quad (2.80)$$

Step 4: Find combined goodness measures of the alternatives and get the ranking.

Higher is the combined goodness measure  $f(A_i)$  value, higher is the preference given to that alternative. Rank of  $i$ th alternative is based on  $f(A_i)$  value.

The next chapter presents the applications of various improved MADM methods discussed in this chapter for different decision making situations of the manufacturing environment.

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