

Preface

Mathematical finance—the mathematical theory of financial markets—not only underpins modern financial practice, but is a thriving area of mathematical research. This book provides an introduction to the way in which real world financial markets are modelled, and how derivatives can be priced in a rational way. Derivatives are, as the name suggests, financial entities whose value is derived from the value of other more concrete assets such as stocks and commodities. *Options* and *forwards* are two of the better known types of derivative.

We consider only *discrete time models*; that is, models where time proceeds in steps (minutes, hours, days, etc.). The main reason for doing so is to keep the mathematics as simple as possible, while explaining the basic principles, without the need for the sophisticated mathematical tools required by the more complicated *continuous time* theory. All of the basic ideas can be seen in even the simplest of discrete time models, and there is a precise sense in which the continuous theory is simply the limit of the discrete theory. For simplicity we also restrict attention to models with only finitely many scenarios (or price histories), thus avoiding the need for general measure and integration theory.

There are many texts that deal with discrete time mathematical finance. This one provides a treatment that aims to be more rigorous and comprehensive than most, while remaining elementary. For example, since *fair pricing* of derivatives is a fundamental issue, we provide a rigorous definition (often lacking or a little fuzzy) and complete proofs concerning the identification of fair prices (in many texts the fair price is not actually shown to be fair). We also cover

comprehensively the fair pricing of derivatives (including American options) in incomplete markets, a topic that is often omitted in introductory courses.

The development proceeds slowly and carefully in the early chapters. The introductory chapter (Chapter 1) gives a little background on derivatives and may be skipped by the student wishing to get straight into the theory. It is surprising that many of the basic ideas can be explained in simple financial models with only two time steps—“today” and “tomorrow”—so we spend considerable time looking at such models (Chapters 2 and 3). Chapters 4 and 5 extend the theory to models with several time steps. The more advanced topics comprise (in Chapter 6) an introduction to the way in which a continuous model can be seen as the limit of discrete models and (in Chapter 7) a complete and rigorous treatment of American options in a discrete setting. Finally, in Chapter 8, we give a brief introduction to a number of topics that would form natural further study.

The mathematical prerequisites are no more than what should have been encountered in the first two years of an undergraduate mathematics programme: mainly finite dimensional linear algebra and elementary probability theory, combined with the ability to follow a line of mathematical reasoning—since we place strong emphasis on complete proofs. To that extent the treatment is more “pure” than “applied”—although the subject matter is by definition relevant to the real world. Related to this is the emphasis in the main on general theory rather than specifics, although plenty of worked examples and exercises are provided. Solutions to a selection of exercises are provided in the appendix. The mark [†] next to an exercise indicates that we have provided a partial solution (a hint or numerical answer without the working) and [‡] indicates an exercise (typically one that is less routine or more theoretical) for which we have provided a full solution. Instructors can obtain a complete set of solutions from the publisher.

A selection of the material (omitting Sections 3.1.4, 3.6, 5.6, 6.4 and 7.3 onwards) would be suitable for an undergraduate course. The material in the sections mentioned requires a little more mathematical maturity but is still elementary in the sense that it does not require any additional mathematical prerequisites. It would be suitable for inclusion in a rigorous course at MSc level where, in our experience, students are frequently graduates with a more limited mathematical background coming from disciplines such as economics or physics.

In real-world financial markets the buying and selling of financial instruments is often done by institutions such as companies, investment firms and governments. For convenience of language, we shall use the words “he” and “him” in this book in a non-gender-specific sense when referring to agents in financial models, with the understanding that these agents are as likely to be institutions as individuals.

The book grew out of the introductory module developed by the authors for the MSc in Mathematical Finance at the Universities of Hull and York, and a corresponding module at undergraduate level. It has been taught also by Kasia Grzesiak and Ekkehard Kopp, who thus contributed to its evolution, and to whom thanks are therefore due.

The website <http://www.springer.com/978-1-4471-4407-6> may be consulted for the latest corrections to the book. It also explains how instructors may access the complete set of solutions to the exercises.

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