Chapter 2
Non-parametric Tuning of PID Controllers

As pointed out in the Introduction, there are two approaches to tuning controllers: parametric and non-parametric. Non-parametric methods of tuning based on the continuous cycling principle are considered in this chapter. We start with the Ziegler–Nichols closed-loop tuning method and progress to the available methods of tuning, which involve the possibility of excitation of test oscillations at frequencies corresponding to phase lags of the process other than $-180°$. The necessity of such functionality is supported by examples provided.

2.1 PID Control

Proportional-integral-derivative (PID) control is the main control of the process industry. Research shows that the share of PID controllers in a typical overall plant control system is about 97 % (see [75]). The author's personal observations arising from his work in oil, gas and power plants unequivocally confirm this to be the case. Moreover, if controllers such as ratio controllers (because a ratio controller is not a feedback controller) and ON-OFF controllers are disregarded in this count then the presented figure may be even higher. We also mention that nearly all controllers for "elementary" processes (simple loops like flow, level, pressure and temperature not having much interaction with other loops) are in fact PID controllers. So even if there is a model-predictive controller in the system, for example, this controller produces set points to flow controllers, and flow controllers are implemented through PID algorithms. Therefore, PID control is utilised within the model-predictive control framework, too.

There are a number of features which count towards the advantages of PID control and many reasons why it is so popular. First of all it is a very simple algorithm to implement in the modern programmable logic controllers (PLC) and the distributed control systems (DCS). In fact, many PLCs have a built-in PID control algorithm, so that programming becomes simple. The same feature is seen in the DCS even to a greater degree. Another advantage of PID controllers supplied with a modern DCS

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is the variety of features empowering the PID control. Modern DCS have a rich variety of different PID controllers, which are provided with the features of switchable modes of operation, various equations that allow for applying the PID components to either error or process variable, nonlinear gains, the possibility of backtracking and back-initialisation, and several other. Some PID controllers have auto-tuners or they can easily interact with external loop tuning software. The performance of PID control can also be enhanced by introducing lead-lag and other compensators in the loop. Such systems as 2-input-2-output systems can normally be handled well by PID controllers with additional feed-forward signals, too.

There is vast body of literature on PID control. The fundamentals of PID control can be found in almost every book on process control. Some examples of these are (in chronological order) by Shinskey [76], Ogunnaike and Ray [68], Luyben and Luyben [54], Marlin [57], Corripio [32], Bequette [13], Seborg et al. [75], Corriou [31], Ellis [37], Altmann [4]. Detailed presentation of PID control is given in the books by Åström and Hägglund [6] and [7], Tan et al. [79], Visioli [82] and Johnson and Moradi [44].

PID control is a three-component control, the components being the proportional, the integral and the derivative. In the so-called expanded, or noninteracting form, the equations of the PID controller can be written in the Laplace domain as follows:

$$W_c(s) = K_c + \frac{K_i}{s} + K_d s. \tag{2.1}$$

In the expanded equation of the PID control, all the components are mutually independent and a change of any gain would change this component alone. The proportional, the integral, and the derivative gains K_c, K_i, and K_d, respectively, are referred to as tuning parameters. However, this equation is rarely used in practice and most controllers utilise the so-called parallel form of the PID equation (these are not the terms strictly used in academia and industry; sometimes equation (2.1) is referred to as ideal, for example). The parallel equation is given as follows:

$$W_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right). \tag{2.2}$$

In the parallel equation the tuning parameters are the proportional gain K_c, the integral time constant T_i and the derivative time constant T_d. There are a number of other forms of PID equations (see [75]). However, in this book only the above equations are used, and in most cases just the parallel equation (2.2). The obvious relationship between the two presented equations is as follows: the proportional gain is the same in both, the integral gain K_i is K_c/T_i and the derivative gain K_d is $K_c K_d$.

The task of tuning is to determine for a given process the values of K_c, T_i and T_d that provide optimal in a certain sense or acceptable performance of the control system (loop). The quality of tuning or loop performance can be determined based on the reactions of the closed loop to external control signals or disturbances, which may be intentionally created for the purpose of such a test or exist in the process (observations of the flow loop reactions to fluctuations of source pressure that naturally exist in the process, for example). There are a number of established and new methods of tuning, some of that were mentioned above. Below we consider non-parametric methods of tuning which fall into the category of methods based on the *continuous cycling*. The term "continuous cycling" is used to define the self-excited nonvanishing oscillations generated in the loop, which includes the process, aimed at measuring the parameters of these oscillations and producing the controller tuning parameters on the basis of the measurements obtained.

We will consider a few methods based on continuous cycling, presenting them in chronological order (at least the first three of them) and showing the development of the ideas as they are given sequentially in this overview.

2.2 Ziegler–Nichols Closed-Loop Test and Tuning

The open-loop and closed-loop tests proposed by J. Ziegler and N. Nichols [88] were the first methods in which a systematic and theoretically justified approach to controller tuning was introduced. Both methods—or at least some elements of both—are still used in practice and utilised in other methods of controller tuning. While the open-loop method is mainly related to parametric methods of loop tuning, the closed-loop method is purely a non-parametric method and related to other *continuous cycling* methods described in this book.

The methodology of closed-loop tuning proposed by Ziegler and Nichols is as follows. At the start the loop should be brought to a steady state. This can be done either manually through adjustments of valve position or by means of a PID controller through the use of the integral action. The controller might not be properly tuned but must deliver stability to the loop. It can have conservative gain values, which would provide a stable loop. At the second step, the integral and derivative components of the PID controller must be disabled and the loop should stay in the steady state. After that the proportional gain is incremented by steps and the behaviour of the loop is observed (see Figs. 2.1 and 2.2). Some sources recommend incrementing the gain by a value equal to half its current value. This, indeed, may speed up the process of tuning. Yet smaller increments must be applied once the tuning process approaches a sustained oscillation. What is observed is the occurrence of self-sustained oscillations. Because the loop is in a steady state, small increments of the set point up and down may be applied with attempts to excite an oscillation. If the oscillation vanishes (interval A of Fig. 2.2) then further increments of the proportional gain should be implemented. As shown for interval B, with an increase of the proportional gain, oscillations become less damped. And finally at some value of the gain they become divergent—as shown for interval C. At this point the gain value should be decreased to a safe level to exclude the possibility of significant process upset by an unstable loop. And, the gain value must be adjusted to obtain a nonvanishing self-sustained oscillation—as shown for interval D.

The Ziegler–Nichols test measures frequency and amplitude of oscillations, which are usually referred to as ultimate frequency Ω_u (or alternatively the ultimate period $T_u = \frac{2\pi}{\Omega_u}$) and ultimate gain K_u, which is the value of the proportional gain

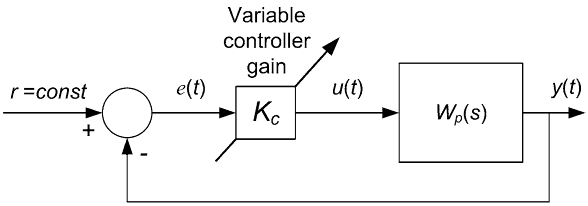

Fig. 2.1 Ziegler–Nichols closed-loop test

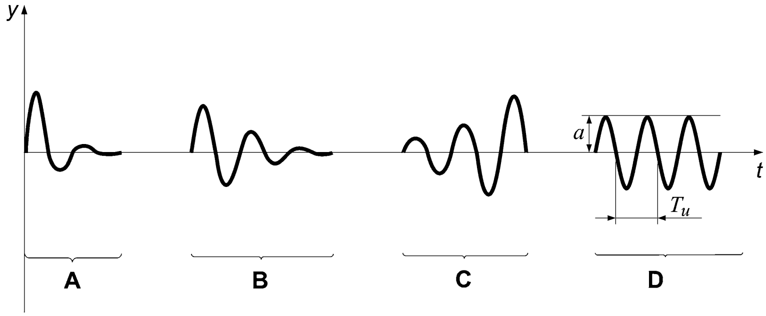

Fig. 2.2 Ziegler–Nichols closed-loop tuning. Steady self-excited oscillations are obtained in case D. Ultimate period $T_u = 2\pi/\Omega_u$ and ultimate gain are measured for this case

Table 2.1 Coefficients of tuning rules for Ziegler–Nichols closed-loop test

Controller	c_1	c_2	c_3
P	0.50		
PI	0.45	0.83	
PID	0.60	0.50	0.12

that provides nonvanishing self excited oscillations in the closed-loop system (option D in Figs. 2.2 and 2.3). In fact, oscillations in the loop are generated at the phase cross-over frequency ω_π of the process transfer function, which is the frequency at which the phase characteristic of the process is equal to $-\pi$ rad (or $-180°$).

The controller tuning parameters can be easily computed using the following formulas [88].

$$K_c = c_1 K_u, \qquad T_i = c_2 \frac{2\pi}{\Omega_u}, \qquad T_d = c_3 \frac{2\pi}{\Omega_u},$$

where c_1, c_2 and c_3 are coefficients defining the tuning rules. The coefficient values for the three types of controllers are presented in Table 2.1.

Tuning rules were developed to provide one quarter amplitude decay. This, however, is not always achieved in practice; Ziegler and Nichols determined the coefficient values using a specific process model. Moreover, even if this target requirement is met the result usually provides an aggressive tuning with an oscillatory response. However, the method provides a good starting point for subsequent fine tuning.

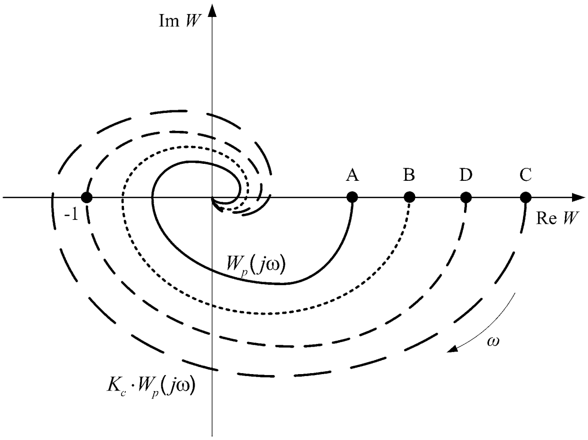

Fig. 2.3 Nyquist plots of Ziegler–Nichols closed-loop test

In terms of control theory, the closed-loop Ziegler–Nichols test is commonly interpreted by applying concepts linear systems analysis. This is a fairly simple and precise interpretation for most systems. There are some aspects of this test though that reveal effects which cannot be explained by linear theory. A respective example is considered in Chap. 4 of the book. The stepwise increase of the proportional gain can be seen as an expansion of the Nyquist plot of the process (Fig. 2.3). Each of the plots $K_c W_p(j\omega)$ in Fig. 2.3 can be associated with transients shown in Fig. 2.2. The correspondence is marked by the letters A, B, C and D. The Nyquist plots A and B do not encircle the point $(-1, j0)$ and correspond to vanishing oscillations in the system response to disturbance, while plot C encircles the point $(-1, j0)$ and corresponds to the diverging oscillations of the system output. And only Nyquist plot D goes through the point $(-1, j0)$ and, as a result, self-sustained nonvanishing oscillations exist in the system.

From Table 2.1, one can see that if, for example, a proportional controller is going to be used then the controller gain should be half the ultimate gain. And, the gain margin in this case is always guaranteed to be 2. This is a remarkable feature of the Ziegler–Nichols test. The desired gain margin of the proportional controller is guaranteed even without identification or any knowledge of the process parameters. This happens because measurement and tuning are realised in the same parametric space. Performance of the loop cannot be ensured, but it can be related to the gain margin, and the fact that stability is guaranteed is, of course, an advantageous and remarkable feature of the test. However, proportional control alone is seldom used in process control. PI and PID controllers are much more widely employed. And the noted property of guaranteed stability margins, unfortunately, cannot be applied to loops containing PI or PID controllers. This happens because of the shift of the ultimate frequency point of the Nyquist plot of the process from the real axis. This shift is due to the introduction of a PI or PID controller, which has a phase shift at frequency Ω_u. In the loop containing the PI/PID controller, the frequency ω_π does not coincide with the frequency Ω_u. Strictly speaking, we cannot say anything about the stability of such a system. Yet, in practice application of the Ziegler–

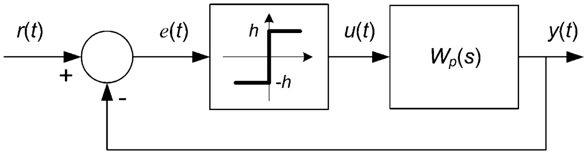

Fig. 2.4 Åström–Hägglund relay feedback test

Nichols closed-loop test and tuning rules provides a good enough result. However, the test may give a little more oscillatory response than desired. This is a drawback of the method, and so numerous research efforts have been aimed at eliminating it or mitigating its effects.

Another drawback or at least an inconvenience of the method is its iterative nature: it requires a sequence of incremental gain values and associated test responses. The iterative character of the method makes its implementation in automatic tuners inconvenient. This problem was successfully solved by the tuning method considered next.

2.3 Åström–Hägglund Relay Feedback Test

In 1984 K. Åström T. and Hägglund proposed a test that was a substantial improvement of the Ziegler–Nichols test in terms of convenience of implementation in automatic loop tuners. They proposed replacement of the variable proportional gain with a nonlinear function and the use of the relay nonlinearity as this function (see Fig. 2.4). The patent application was first filed in Sweden in 1981 and later patented in a number of countries (in the United States, for example [40]). The patent specification included only the tuning rules of the Ziegler–Nichols closed-loop tuning method. However, later [5] other tuning rules were developed. The proposed test is now commonly known as the *relay feedback test* (RFT).

The idea of the test is based on the observation that the relay feedback system (Fig. 2.4) generates oscillations of the same frequency as the oscillations in the Ziegler–Nichols test. Yet, the test realisation is noniterative and particularly suitable for computer controlled systems (PLC or DCS). The equality of these two frequencies is, however, only approximate; the describing function (DF) method[1] normally applied in this analysis does not allow one to see the difference. But exact methods [5, 15, 80] allow one to find the difference. It is usually insignificant and in consideration of the Åström–Hägglund method we shall assume that these frequencies are equal.

In an analysis of the relay feedback system by the DF method, the nonlinear function can be replaced with an equivalent gain (complex or real), that is the describing function itself. This gain describes the propagation of the fundamental frequency component (first harmonic) in the Fourier series expansion of the periodic error

[1] It is also sometimes referred to as the sinusoidal input describing function; we shall omit sinusoidal in the subsequent text.

signal through the nonlinearity. For the DF method to be applied, the so-called *filtering hypothesis* (which states that the linear part of the system (process) must be a low-pass filter) must be valid. The describing function is not a constant value but a function of the amplitude and sometimes of the frequency of the input signal to the nonlinearity (error signal). For the ideal relay nonlinearity, the describing function $N(a)$ is given by the expression $N(a) = \frac{4h}{\pi a}$, where a is the amplitude of the oscillations of the error signal, and h is the amplitude of the relay. Once the relay is replaced with the DF the frequency of the self-excited oscillations can be found from the harmonic balance equation

$$N(a_0)W_p(j\Omega_0) = -1,$$

which can be interpreted as the Nyquist stability criterion applied to the system in which the relay is replaced with the DF. Due to inherent instability[2] the relay feedback system excites nonvanishing oscillations of the frequency Ω_0 and the amplitude a_0, which are measured in the test. Frequency Ω_0 is the ultimate frequency, and the ultimate gain is computed as the value of the DF: $K_u = N(a_0) = \frac{4h}{\pi a_0}$. Because the RFT was proposed as a further development of the Ziegler–Nichols method, initially its tuning rules were those of Ziegler and Nichols (see Table 2.1).

Later, Åström and Hägglund proposed other tuning rules, in [5], which is remarkable as the paper where an attempt to design tuning rules that would provide required specifications on gain or phase margin was made. However, this attempt was not fully successful because the tuning rules were designed in a way that would ensure the phase lag of the PID controller at the frequency of the test oscillations would be zero (the phase lead due to the derivative term is equalised by the same value of the phase lag introduced by the integral term):

$$\frac{1}{jT_i\Omega_0} + jT_d\Omega_0 = 0,$$

which leads to the constraint

$$T_iT_d = \frac{1}{\Omega_0^2}.$$

The drawback of this approach is obvious. The designed tuning rules could hardly provide optimal or near-optimal tuning; even the PI controller falls out of the class of controllers covered by this approach. Nevertheless, it was a remarkable vision of the real requirement and needs of control practice. Furthermore, the whole RFT was an ingenious design one that gave a boost to industry as it opened the way to the automation of controller tuning.

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[2]From a theoretical point of view, zero can be a stable equilibrium point; for example it is an asymptotically stable equilibrium point in the relay system comprising an ideal relay and second-order linear dynamics [15].

2.4 Generating Test Oscillations in the Third Quadrant

Despite the innovative ideas given in [5], a popular notion has been that the most important point on system's frequency response is where the phase characteristic of the process equals to $-180°$ (frequency ω_π). This view appears in many publications. We shall refer to this point as the *phase cross-over frequency*. However, this is the most important point only in a system with a proportional controller, when introduction of the controller does not change the value of ω_π. This prerequisite is often neglected and the principle applied to all types of PID control. We consider the following motivating example and analyse how the introduction of the controller may affect the results of identification and tuning.

Example 2.1 Let us assume that a process is given by the following transfer function (which was used in a number of publications as a test process):

$$W_p(s) = \exp^{-2s} \frac{1}{(2s+1)^5}. \tag{2.3}$$

Find the first order plus dead time (FOPDT) approximating model $\hat{W}_p(s)$ to process (2.3) based on matching the values of the frequency response at frequency ω_π for the original and approximating models:

$$\hat{W}_p(s) = \frac{K_p e^{-s}}{T_p s + 1}, \tag{2.4}$$

where K_p is the process static gain, T_p is the time constant and τ is the dead time. Let us apply methods [88] to the tuning of process (2.3) and note that both (2.3) and (2.4) should produce the same ultimate gain and ultimate frequency in the Ziegler–Nichols closed-loop test [88] or the same values of the amplitude and the ultimate frequency in the RFT [5]. (Note: Strictly speaking, the values of the ultimate frequency in tests [88] and [5] are slightly different, as the frequency of the oscillations generated in the RFT does not correspond exactly to the phase characteristic of the process $-180°$; this fact follows from relay systems theory [15, 16, 80]; however, we shall use the describing function method and disregard inaccuracies.) Obviously, this problem has an infinite number of solutions, as (2.4) has three unknown parameters and only two measurements are obtained from the test. Assume the value of the process static gain is known: $K_p = 1$, and determine T_p and τ. These parameters can be found from equation

$$\hat{W}_p(j\omega_\pi) = W_p(j\omega_\pi),$$

where ω_π is the phase cross-over frequency for both transfer functions. Therefore, $\arg W_p(j\omega_\pi) = -\pi$. The value of ω_π is 0.283, which gives $W_p(j\omega_\pi) = (-0.498, j0)$, and the FOPDT approximation is, therefore, as follows (found via solution of the system of two algebraic equations):

$$\hat{W}_p(s) = \frac{e^{-7.393s}}{6.153s + 1}. \tag{2.5}$$

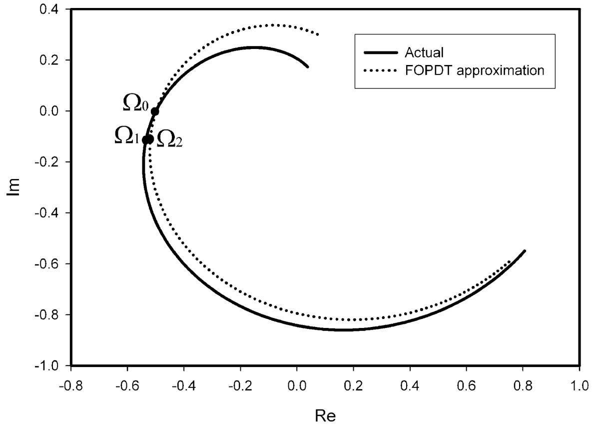


Fig. 2.5 Nyquist plots for process (2.3) and FOPDT approximation (2.5)

The Nyquist plots of process (2.3) and its approximation (2.5) are depicted in Fig. 2.5 (the meaning of frequencies Ω_1 and Ω_2 is explained below). The point of intersection of the two plots (denoted as Ω_0) is also the point of intersection with the real axis. Also $\Omega_0 = \omega_\pi$ for both process dynamics (2.3) and (2.5), and therefore $\hat{W}_p(j\Omega_0) = W_p(j\Omega_0)$. If the designed controller is of proportional type then the gain margins for processes (2.3) and (2.5) are the same. However, if the controller is of PI type then the stability margins for (2.3) and (2.4) are different. We illustrate this below. Design the PI controller given by the following transfer function:

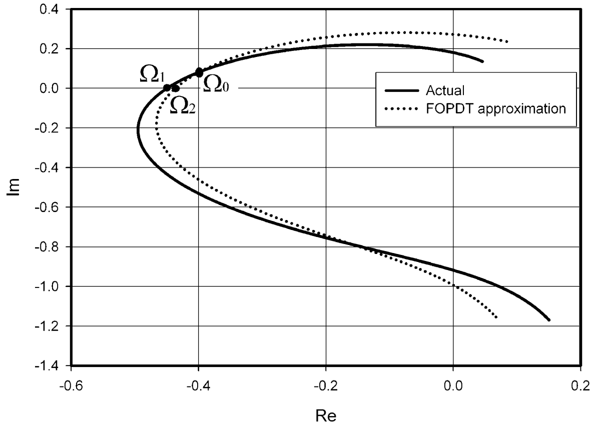
$$W_c(s) = K_c\left(1 + \frac{1}{T_c s}\right), \tag{2.6}$$

where K_c is the proportional gain and T_c is the integral time constant of the controller, using the Ziegler–Nichols tuning rules [88]. This results in the following transfer function of the controller:

$$W_c(s) = 0.803\left(1 + \frac{1}{17.76s}\right). \tag{2.7}$$

The Nyquist plots of the open-loop systems containing process (2.3) or its approximation (2.5) and controller (2.7) are depicted in Fig. 2.6. It follows from the frequency-domain theory of linear systems and the tuning rules used that the mapping of point Ω_0 in Fig. 2.5 into point Ω_0 in Fig. 2.6 is done via clockwise rotation of vector $\hat{W}_p(j\Omega_0)$ by the angle $\psi = \arctan(1/(0.8 \cdot 2\pi)) = 11.25°$ and multiplication of its length by certain value, so that its length becomes equal to 0.408. This is possible due to the serial connection of the controller and the process and the possibility of treating their frequency response (at Ω_0) as vectors. However, for the open-loop system containing the PI controller, the points of intersection of the Nyquist plots of the system and of the real axis are different for the system with process (2.3) and with process approximation (2.5). They are shown as points Ω_1 and Ω_2 in Fig. 2.6. The points of frequencies Ω_1 and Ω_2 on the Nyquist plots of the original process and its approximation, respectively, are also shown in Fig. 2.5. Therefore, the stability margins of the systems containing a PI controller are no longer the same. They

Fig. 2.6 Nyquist plots for open-loop system with PI controller and process



are revealed as different points of intersection of the plots and of the real axis in Fig. 2.6. In fact, the position of vector $\hat{W}_{ol}(j\Omega_0) = \hat{W}_c(j\Omega_0)\hat{W}_p(j\Omega_0)$ is fixed, but this vector does not reflect the stability of the system. As one can see in Fig. 2.6, the gain margin of the system containing the FOPDT approximation of the process is higher than the one of the system with the original process.

The considered example illustrates a fundamental problem of all methods of identification-tuning based on the measurements of process response at the critical point (Ω_0). This problem is the shift of the critical point due to the introduction of the controller. The question that follows from the above analysis is this: *Can the test point be selected in a different way, so that the introduction of the controller would be accounted for in the test itself*? And if this is possible, then *what kind of test should it be to ensure the measurements in the desired test point*?

We address the first question now. Assume that we can design a certain test so that we can generate the test frequency at the desired phase lag of the process $\arg W_p(j\Omega_0) = \varphi$, where φ is a given quantity, and measure $W_p(j\Omega_0)$ in this point. Consider the following example.

Example 2.2 Let the plant be the same as in Example 2.1. Assume that the introduction of the controller will give a mapping similar to the mapping described above—the vector of the frequency response of the open-loop system at the point Ω_0 will be a result of clockwise rotation of the vector $\hat{W}_p(j\Omega_0)$ by a known angle and multiplication by a certain known factor: $\hat{W}_{ol}(j\Omega_0) = \hat{W}_c(j\Omega_0)\hat{W}_p(j\Omega_0)$. Assume also that the controller will be the same as in Example 2.1 (for illustrative purpose, since the tuning rules are not formulated yet). Therefore, let us find the values of T_p and τ for transfer function (2.4) (we still assume $K_p = 1$) that ensure that the equality $\hat{W}_p(j\Omega_0) = W_p(j\Omega_0)$ holds, where $\arg W_p(j\Omega_0) = -180° + 11.25° = -168.75°$ (the angle is selected considering the subsequent clockwise rotation by $11.25°$).

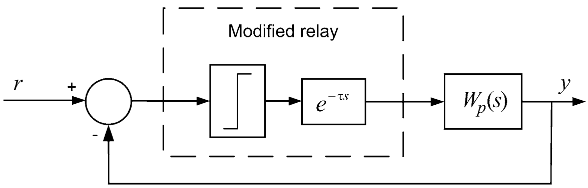


Fig. 2.7 Use of delay for identification in the third quadrant

Therefore, $\Omega_0 = 0.263$ and $W_p(j\Omega_0) = (-0.532, -j0.103)$. The corresponding FOPDT approximation of the process is

$$\hat{W}_p(s) = \frac{e^{-7.293s}}{5.897s+1}. \tag{2.8}$$

Application of controller (2.7) shifts the point Ω_0 of intersection of $W_p(j\Omega_0)$ and $\hat{W}_p(j\Omega_0)$ to the real axis. This point is still the point of intersection of the two Nyquist plots. Therefore, the gain margin of both systems, with the original process and with the approximated process, are the same. Consider now the problem of the *design of a test that can provide matching the points of the actual and approximating processes* at the point corresponding to a specified phase lag.

2.5 Tests that Ensure Frequency of Oscillations at Arbitrary Process Phase Lags

2.5.1 Test Using Additional Time Delay

There are a number of algorithms that can be used to generate oscillations in a closed-loop test over the process at the frequency corresponding to the third quadrant of the process frequency response (Nyquist plot).

Tan et al. [78] suggested that an additional time delay should be included after the relay nonlinearity to excite oscillations at the frequency lower than the phase cross-over frequency (see also [79]). This delay can then be adjusted by iteration to excite oscillations at the specified phase lag of the process. The proposed method can be illustrated by the diagram as in Fig. 2.7.

The proposed algorithm involves finding two points on the Nyquist plot of the process: at the ultimate frequency (phase cross-over frequency) and at the frequency that delivers the specified phase lag of the process $-\pi + \phi_m$, where $\phi_m \in [0, \pi/2]$.

The algorithm includes the following iterations.

- Carry out the conventional RFT and find the ultimate gain K_u and the ultimate frequency Ω_u.
- With these values available, calculate the initial guess for the frequency $\hat{\omega}_\phi$ for the specified phase $-\pi + \phi_m$ as follows: $\hat{\omega}_\phi = ((\pi - \phi_m)/\pi)/\Omega_u$ and initialisation of the time delay $\tau = \phi_m/\hat{\omega}_\phi$.
- Continue the RFT with delay updating the delay value at each iteration as $\tau = \phi_m/\omega_{osc}$, where ω_{osc} is the frequency of the oscillations in the test.

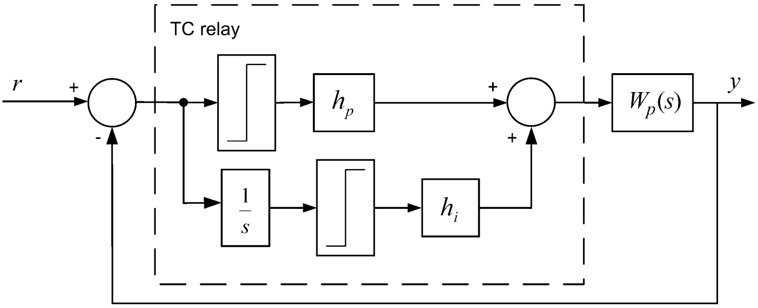


Fig. 2.8 Use of two-channel relay for identification in the third quadrant

The authors showed through simulations that the iterative algorithm converges to the frequency corresponding to the specified phase lag $-\pi + \phi_m$. The algorithm allows one to carry out identification for an arbitrary frequency point of the third quadrant of the complex plane but this is achieved at the expense of introducing iterations in the test, which significantly lengthen the test time and may cause unnecessary disturbance to the process.

2.5.2 Test Using Additional Integrator Term

Friman and Waller [38] proposed the use of the "two-channel relay" for identification of the process at a point of the third quadrant of the complex plane. It was proposed that an additional integrator having the error signal as the input and an additional relay, which output is added to the output of the main relay as depicted in Fig. 2.8, be used for exciting test oscillations.

An advantage of this test is that it does not require iteration, and the required phase lag can be obtained by the proper selection of the amplitudes of the two relays, as shown below.

The describing function of the *two-channel relay* can be computed as a sum of the two describing functions:

$$N(a) = \frac{4h_p}{\pi a} + \frac{1}{j\omega}\frac{4h_i}{\pi a_i},$$

where h_p and h_i are the amplitudes of the relays in the proportional link and the integral link, respectively, a is the amplitude of the oscillations (of the error signal), a_i is the amplitude of the oscillations after the integrator and ω is the frequency of the oscillations.

It is easy to see that $a_i = a/\omega$ and, therefore, the describing function of the algorithm can be rewritten as follows:

$$N(a) = \frac{4h_p}{\pi a} - j\frac{4h_i}{\pi a}.$$

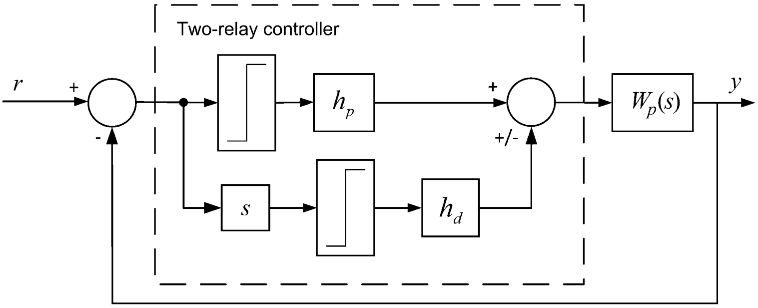


Fig. 2.9 Use of two-relay control for identification at the specified phase lag

Therefore, the magnitude of the describing function is

$$\left|N(a)\right| = \frac{4\sqrt{h_p^2 + h_i^2}}{\pi a}$$

and the phase response provided by the two-channel relay is

$$\arg N(a) = -\arctan \frac{h_i}{h_p}.$$

Overall the test is convenient and provides the required functionality but sometimes exhibits a long convergence time; in processes having nonlinearities (such as valve stiction) may result in low resolution in some points if the required $|h_p - h_i|$ difference is too small or too big.

2.5.3 Test Using Additional Derivative Term

A test that utilises the derivative of the error signal was proposed by Castellanos et al. [29]. The algorithm can be illustrated by Fig. 2.9. The algorithm originated from the analysis of the properties of the so-called *twisting* second-order sliding mode control algorithm. The original twisting control algorithm was modified to suit the purpose of identification in both ranges of the process phase lag: $[-\frac{\pi}{4}, -\frac{\pi}{2}]$ and $[-\frac{\pi}{2}, -\frac{3\pi}{2}]$:

$$N(a) = N_p(a) + j\omega N_d(a_d) = \frac{4h_p}{\pi a} + j\omega\frac{4h_d}{\pi a_d} = \frac{4}{\pi a}(h_p + jh_d), \tag{2.9}$$

where h_p and h_d are the amplitudes of the relays in the proportional link and the derivative link, respectively, a is the amplitude of the oscillations (of the error signal), a_d is the amplitude of the oscillations after the differentiator and ω is the frequency of the oscillations.

The magnitude of the describing function is

$$\left|N(a)\right| = \frac{4\sqrt{h_p^2 + h_d^2}}{\pi a}$$

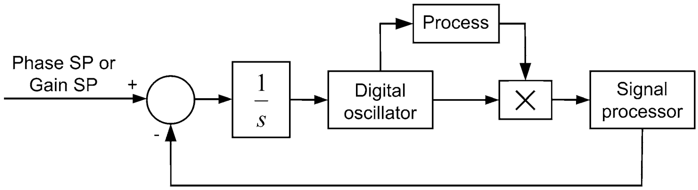


Fig. 2.10 Phase-lock loop non-parametric identification

and the phase response is

$$\arg N(a) = \arctan \frac{h_d}{h_p}.$$

Because $\arg N(a) > 0$ for $h_d > 0$, the two-relay algorithm provides identification in the range of process phase lags of $[-\frac{\pi}{2}, -\frac{3\pi}{2}]$. To change the range of the identification of the phase lags to $[-\frac{\pi}{4}, -\frac{\pi}{2}]$ the sign of h_d must be changed to negative.

The algorithm was initially designed to provide identification of electromechanical systems at frequencies higher than the phase cross-over frequency. Its use in process applications may feature high sensitivity to noise and, therefore, additional noise-reduction measures would be needed.

Exact analysis of oscillations in the system with the two-relay algorithm is presented in [18]; analysis of orbital stability of the periodic solution is given in [2] through the describing function method and in [1] through the construction of the linearised Poincaré map.

2.5.4 Test Using Phase-Lock Loop

A method aimed at the elimination of such known drawbacks of the relay feedback test as inaccuracy due to the use of the approximate describing function method and identification in only one point corresponding to the phase cross-over frequency was proposed by Crowe and Johnson [33]. The method is based on the phase-lock principle involving the organisation of a loop that includes a process. Oscillations are excited in this loop at the frequency that would ensure the desired phase lag. The block diagram of the system is given in Fig. 2.10.

The proposed identifier includes the following components (as per [34]):

- A feedback loop using a phase or gain reference.
- A digital controlled oscillator providing a sinusoidal signal.
- A multiplier.
- A digital signal processing unit, which computes the actual phase lag or gain of the process.
- A digital integrator, which is supposed to eliminate nonzero error and provide convergence to the steady state in the loop.

While providing such advantages as high accuracy of identification and the possibility of identification at a few frequency points, which motivated this method's development, this approach also features some drawbacks such as higher complexity, longer convergence time required for the phase-lock loop to come to a steady state and disconnection of the process from the control loop for the duration of the test. The results of the comparison of the phase-lock loop identification against the conventional RFT method, as well as details of implementation and accuracy analysis, are presented in [34], [35] and [36].

2.6 Conclusions

A brief overview of non-parametric methods was presented in this chapter, starting with the Ziegler–Nichols closed-loop test. Evolution of the ideas used in non-parametric tuning is shown through the review of methods. It is shown that the use of test oscillations of the frequency that corresponds to the $-180°$ phase response of the process does not allow one to design PI or PID controller tuning loops that would ensure specified gain or phase margins. This happens due to the shift of the test point from the real axis when the controller is introduced in the loop. It is also shown that generation of test oscillations in the third quadrant of the complex plane would be beneficial. A number of tests providing test oscillations in the third quadrant are reviewed. Another method of PID controller tuning, in which coordinated test and tuning are combined, is presented in Chap. 3.

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