

Preface

Overview

The objective of this book is to give the reader a flavour of mathematics used in the computing field. The goal is to show how mathematics is applied in computing, rather than the study of mathematics for its own sake.

Organization and Features

The first chapter discusses the contributions made by early civilisations to computing. This includes work done by the Babylonians, Egyptians and Greeks. The Egyptians applied mathematics to solve practical problems such as the construction of pyramids. The Greeks made a major contribution to mathematics and geometry, and most students are familiar with the work of Euclid.

Chapter 2 provides an introduction to fundamental building blocks in mathematics including sets, relations and functions. A set is a collection of well-defined objects and it may be finite or infinite. A relation between two sets A and B indicates a relationship between members of the two sets, and is a subset of the Cartesian product of the two sets. A function is a special type of relation such that for each element in A there is at the most one element in the co-domain B . Functions may be partial or total and injective, surjective or bijective.

Chapter 3 provides an introduction to logic including propositional and predicate logic. The nature of mathematical proof is discussed.

Chapter 4 provides an introduction to the important field of software engineering. The birth of the discipline was at the Garmisch conference in Germany in the late 1960s. The extent to which mathematics should be employed in software engineering is discussed, and this remains a topic of active debate.

Chapter 5 discusses formal methods, which consist of a set of mathematical techniques to specify and derive a program from its specification. Formal methods may be employed to rigorously state the requirements of the proposed system; they may be employed to derive a program from its mathematical specification; and they

provide a rigorous proof that the implemented program satisfies its specification. They have been mainly applied to the safety critical field.

Chapter 6 presents the Z specification language, which is one of the most widely used formal methods. It was developed at Oxford University in the UK.

Chapter 7 presents the fundamentals of number theory, and discusses prime number theory and the greatest common divisor and least common multiple of two numbers.

Chapter 8 discusses cryptography, which is an important application of number theory. The codebreaking work done at Bletchley Park in England during the Second World War is discussed, and the fundamentals of cryptography, including private and public key cryptosystems, are discussed.

Chapter 9 presents coding theory and is concerned with error detection and error correction codes. The underlying mathematics is discussed, and this includes abstract mathematics such as group theory, rings, fields, and vector spaces.

Chapter 10 discusses language theory and includes a discussion on grammar, parse trees, and derivations from a grammar. The important area of programming language semantics is discussed, including an overview of axiomatic, denotational and operational semantics.

Chapter 11 discusses computability and decideability. The Church-Turing thesis states that anything that is computable is computable by a Turing machine. Church and Turing showed that mathematics is not decideable. In other words, there is no mechanical procedure (i.e., algorithm) to determine whether an arbitrary mathematical proposition is true or false, and so the only way is to determine the truth or falsity of a statement is try to solve the problem.

Chapter 12 discusses probability and statistics and includes a discussion on discrete and continuous random variables, probability distributions, sample spaces, sampling, the abuse of statistics, variance and standard deviation, and hypothesis testing. The application of probability to the software reliability field is discussed.

Chapter 13 discusses matrices including 2×2 and general $n \times m$ matrices. Various operations such as the addition and multiplication of matrices are considered, and the determinant and inverse of a matrix is discussed. The application of matrices to solve a set of linear equations using Gaussian elimination is considered.

Chapter 14 discusses complex numbers and quaternions. Complex numbers of the form $a + bi$ where a and b are real numbers, and $i^2 = -1$. Quaternions are a generalization of complex numbers to quadruples that satisfy the quaternion formula $i^2 = j^2 = k^2 = -1$.

Chapter 15 provides a very short introduction to calculus, and provides a high-level overview of limits, continuity, differentiation, integration, and numerical analysis. Fourier series, Laplace transforms and differential equations are briefly discussed.

Chapter 16 discusses graph theory where a graph $G = (V, E)$ consists of vertices and edges. It is a practical branch of mathematics that deals with the arrangements of vertices and the edges between them. It has been applied to practical problems such as the modeling of computer networks, determining the shortest driving route between two cities, and the traveling salesman problem.

Mathematics in Computing

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