Chapter 2
Elementary Cam Lift Curve Synthesis

Abstract This chapter describes the derivation and piece-wise integration of the first half of an analytically simple valve acceleration curve. Two simultaneous algebraic equations are obtained. The first equates an expression for the velocity on the nose of the cam to zero, and the second the sum of the increments of valve lift to the maximum specified lift. The two unknowns are the maximum positive acceleration, which is on the flank of the cam and the maximum negative acceleration, which is on the nose of the cam. The two equations can then be solved for these two unknown quantities. This example has been chosen for analytical simplicity, to demonstrate the method, but such an acceleration curve would not result is a good cam design with smooth valve acceleration, and should not be used in practice. A superior and useable, but analytically more complex acceleration curve is considered in the next chapter.

2.1 Introduction

Some of the concepts of cam lift curve synthesis were described in Chap. 1. Over the years many methods of obtaining the acceleration diagram have been used and the method described below and refined in Chap. 3 is only one of these. The example given here has been chosen for its analytical simplicity, but this type of acceleration curve should not be used in practice, as it is not a good one.

It has been said that misconceptions tend to harden into axioms, and the simple example given below is based on a method which was surprisingly still being used by at least one company for cam design in the early 1980s. The use of some cams designed with especially rapid changes in acceleration, or jerk, resulted in considerable valve spring surge and consequent spring failures, which then resulted in destruction of much of the engine. However, despite the difficulties in synthesising

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acceptably smooth cam lift curves before the advent of digital computers, they were produced and used, but this was very time consuming.

The use of cams exhibiting significant jerk with push rod mechanisms or other mechanisms subject to relatively flexible behaviour can result in a loss of cam–follower contact, which will impair reliability and power output. This sort of cam will also be far more likely to induce valve spring surge in otherwise stiffer mechanisms which apart from valve spring failure will again result in a loss of cam–follower contact at a lower engine speed. Unstable behaviour of the cam mechanism may lead to valve clash, damage to the piston, valve and valve seat, or loss of power due to poor gas flow, or a combination of these undesirable effects.

2.2 An Elementary Cam Lift Curve

The cam lift curve is obtained by designing an acceleration curve and then integrating to obtain a velocity curve and then a lift curve. It is convenient to divide the curve into two parts, the opening half and a closing half. These two parts meet on the nose, and at this point the curves need to be smoothly continuous. In fact we shall require them to be continuously differentiable.

For this initial example it is helpful to consider a curve with few components and consisting of analytically simple sections, with discontinuous rates of change of acceleration. A more realistic example is considered later, but this has more sections and is more complicated and the amount of algebra involved tends to mask the basic method.

When the engine is assembled it is necessary to specify a tappet clearance, s_0, to allow for thermal expansion of the valve stem when the engine is at working temperature. If there were no initial clearance then the valve would not be closed, when the tappet was touching the cam's base circle. It is necessary to take up this initial clearance as smoothly as possible, and one solution might be to have a ramp of length, T_0, linearly increasing lift to a height of s_0 at T_0.

Derivatives with respect to time are denoted in this chapter using Newton's dot notation. The slope of this ramp, $\dot{s}_0$, would then be, given by: $\dot{s}_0 = s_0/T_0$. Unfortunately, this results in an instantaneous change in the velocity, $\dot{s}_0$, which would require an infinite acceleration for an infinitesimally small time, as described in Sect. 1.2.1. This, so-called Delta function, would not be realised in practice but such a ramp design is best avoided, as the jerk needs to be as small as possible. For this initial example let the ramp have a constant acceleration, A_0, as shown in Fig. 2.1. The notional tappet clearance, s_0, is specified by the designer.

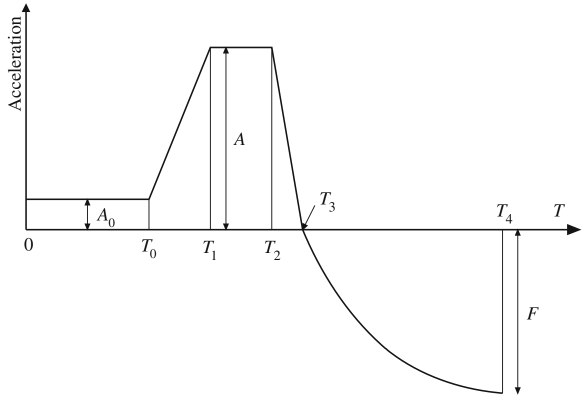

Fig. 2.1 An elementary acceleration diagram

2.2.1 Notation

A	Maximum acceleration
$\bar{A}$	Parameter defined in text
$\hat{A}$	Parameter defined in text
A_0	Initial ramp height
$\hat{A}_0$	Parameter defined in text
F	Maximum deceleration
$\hat{F}$	Parameter defined in text
L	Maximum lift
s	Lift
s_0	Lift at end of ramp, tappet clearance
t	Time
T	Time interval

2.2.2 An Elementary Acceleration Curve

After the initial ramp the next three sections are linear and the final section consists of a quarter sine wave. Although this would not be an acceptable design in practice, it will permit a minimum of mathematics and will therefore be easier to follow the method. By splitting the acceleration curve into sections and integrating twice we can obtain two equations in two unknowns, A and F. At T_4 the velocity is zero and the valve lift, L, is specified by the designer. The slope of the acceleration curve at T_4 is $\dddot{s}_4 = 0$.

The acceleration curve shown in Fig. 2.1 has discontinuities in slope at 0, T_0, T_1, T_2 and T_3, which cause large instantaneous changes in the rate of change of acceleration or jerk. This will lead to surging and premature failure of metallic coil valve-springs, and a tendency for instabilities in the motion of cam mechanisms with low elastic stiffness such as those involving push-rods. At very high camshaft speeds, even stiff mechanisms with pneumatic valve springs can have a tendency to behave in an unstable manner, if there are significantly large values of jerk. There is also insufficient flexibility to permit the designer to optimise his design, and the use of a section which is a quarter sine-wave will not allow the energy stored in the spring to be used efficiently to maintain contact between follower and cam; this limits the maximum engine speed that can safely be used before contact is lost between cam and tappet.

By considering each section in turn the equation for the acceleration is integrated twice and the constants of integration determined from the initial boundary conditions for each section.

Constant Acceleration Ramp. $0 \le t \le T_0,\ 0 \le T \le T_0$

With notation of Fig. 2.1:

Integrating Eq. (2.1) twice w.r.t. t:

$$\ddot{s} = A_0 \tag{2.1}$$

$$\dot{s} = A_0 t \tag{2.2}$$

$$s = \frac{A_0 t^2}{2} \tag{2.3}$$

At $t = T_0$:

$$\ddot{s} = \ddot{s}_0 = A_0 \tag{2.4}$$

$$\dot{s}_0 = A_0 T_0 \tag{2.5}$$

$$s = s_0 = \frac{A_0 T_0^2}{2} \tag{2.6}$$

Hence:

$$A_0 = \frac{2 s_0}{T_0^2} \tag{2.7}$$

As s_0 and T_0 are specified by the designer, A_0 can be determined.

Linearly Increasing Acceleration. $T_0 \le t \le T_1,\ T_0 \le T \le T_1$

$$\ddot{s} = A_0 + \frac{(A - A_0)t}{T_1} \tag{2.8}$$

Integrating Eq. (2.8) twice w.r.t. t:

$$\dot{s} = \dot{s}_0 + A_0 t + \frac{(A - A_0)t^2}{2T_1} \tag{2.9}$$

$$s = s_0 + \dot{s}_0 t + \frac{A_0 t^2}{2} + \frac{(A - A_0)t^3}{6T_1} \tag{2.10}$$

At $t = T_1$:

$$\ddot{s} = \ddot{s}_1 = A \tag{2.11}$$

$$\dot{s} = \dot{s}_1 = \dot{s}_0 + \frac{A_0 T_1}{2} + \frac{A T_1}{2} \tag{2.12}$$

$$s = s_1 = s_0 + \dot{s}_0 T_1 + \frac{A_0 T_1^2}{3} + \frac{A T_1^2}{6} \tag{2.13}$$

Constant Acceleration. $T_1 \leq t \leq T_2,\ T_1 \leq T \leq T_2$

$$\ddot{s} = A \tag{2.14}$$

Integrating Eq. (2.14) twice w.r.t. t:

$$\dot{s} = s_1 + At \tag{2.15}$$

$$s = s_1 + \dot{s}_1 t + \frac{At^2}{2} \tag{2.16}$$

At $t = T_2$:

$$\ddot{s} = \ddot{s}_2 = A \tag{2.17}$$

$$\dot{s} = \dot{s}_2 = \dot{s}_1 + AT_2 \tag{2.18}$$

$$s = s_2 = s_1 + \dot{s}_1 T_2 + \frac{AT_2^2}{2} \tag{2.19}$$

Linearly Decreasing Acceleration. $T_2 \leq t \leq T_3,\ T_2 \leq T \leq T_3$

$$\ddot{s} = A\left(1 - \frac{t}{T_3}\right) \tag{2.20}$$

Integrating Eq. (2.19) twice w.r.t. t:

$$\dot{s} = \dot{s}_2 + A\left(t - \frac{t^2}{2T_3}\right) \tag{2.21}$$

$$s = s_2 + \dot{s}t + A\left(\frac{t^2}{2} - \frac{t^3}{6}\right) \tag{2.22}$$

At $t = T_3$:

$$\ddot{s}_3 = 0 \tag{2.23}$$

$$\dot{s}_3 = \dot{s}_2 + \frac{AT_3}{2} \tag{2.24}$$

$$s_3 = s_2 + \dot{s}_2 T_3 + \frac{AT_3^2}{3} \tag{2.25}$$

Sinusoidal Deceleration. $T_3 \le t \le T_4,\ T_3 \le T \le T_4$

$$\ddot{s} = -F \sin\left(\frac{\pi t}{2T_4}\right) \tag{2.26}$$

Integrating Eq. (2.25) twice w.r.t.t:

$$\dot{s} = \dot{s}_3 - F\left(\frac{2T_4}{\pi}\right)\left[1 - \cos\left(\frac{\pi t}{2T_4}\right)\right] \tag{2.27}$$

$$s = s_3 + \dot{s}_3 t - F\left(\frac{2T_4}{\pi}\right)^2\left[\frac{\pi t}{2T_4} - \sin\left(\frac{\pi t}{2T_4}\right)\right] \tag{2.28}$$

At $t = T_4$:

$$\ddot{s}_4 = -F \tag{2.29}$$

The velocity on the nose is zero therefore:

$$\dot{s}_4 = \dot{s}_3 - \frac{2FT_4}{\pi} = 0 \tag{2.30}$$

The maximum lift is specified by the designer hence:

$$s_4 = s_3 + \dot{s}_3 T_4 - F\frac{4T_4^2}{\pi^2}\left(\frac{\pi}{2} - 1\right) = L \tag{2.31}$$

***Solution of Equations for* A *and* F.** The equations for $\dot{s}_4$ and s_4 have two unknowns, A and F. By substituting Eqs. (2.5), (2.12), and (2.18) into Eq. (2.30) and after some algebra, we can write:

$$\dot{s}_4 = A_0\left[T_0 + \frac{T_1}{2}\right] + A\left[\frac{T_1 + T_3}{2} + T_2\right] - \frac{2FT_4}{\pi} = 0 \tag{2.32}$$

Let:

$$\bar{A}_0 = A_0\left(T_0 + \frac{T_1}{2}\right) \tag{2.33}$$

and

$$\bar{A} = \left(\frac{T_1 + T_3}{2} + T_2\right) \tag{2.34}$$

Substituting Eqs. (2.6), (2.13) and (2.19) into (2.33) together with Eqs. (2.5), (2.12) and (2.18) into Eq. (2.32), and after further lengthy back substitutions and algebra, we can write:

$$\begin{aligned} s_4 = A_0&\left[\frac{T_0^2}{2} + T_0(T_1 + T_2 + T_3 + T_4) + \frac{T_1^2}{3} + \frac{T_1}{2}(T_2 + T_3 + T_4)\right] \\ &A\left[\frac{T_1^2}{6} + \frac{T_2^2}{2} + \frac{T_3^2}{3} + \frac{T_1}{2}(T_2 + T_3 + T_4) + T_2(T_3 + T_4) + \frac{T_3 T_4}{2}\right] \\ &- F\frac{4T_4^2}{\pi^2}\left(\frac{\pi}{2} - 1\right) = L \end{aligned} \tag{2.35}$$

Let:

$$\hat{A}_0 = A_0\left[\frac{T_0^2}{2} + T_0(T_1 + T_2 + T_3 + T_4) + \frac{T_1^2}{3} + \frac{T_1}{2}(T_2 + T_3 + T_4)\right] \tag{2.36}$$

and

$$\hat{A} = A\left[\frac{T_1^2}{6} + \frac{T_2^2}{2} + \frac{T_3^2}{3} + \frac{T_1}{2}(T_2 + T_3 + T_4) + T_2(T_3 + T_4) + \frac{T_3 T_4}{2}\right] \tag{2.37}$$

When simplifying lengthy algebraic equations, it is helpful to equate some expressions to new parameters. This makes the analysis simpler to follow and when writing computer code this makes for shorter expressions and reduces coding errors. By considering the individual increments of the equations the simplification process can be made more easily which results in less likelihood of terms being missed and errors made. The method given below may not be necessary for the present example, but is used in Chap. 3 where the acceleration diagram is more complex.

From Eq. (2.5):

$$dV_0^{A_0} = A_0 T_0 \tag{2.38}$$

From Eq. (2.6):

$$dS_0^{A_0} = \frac{A_0 T_0^2}{2} \tag{2.39}$$

From Eq. (2.12):

$$dV_1^{A_0} = \frac{A_0 T_1}{2} \tag{2.40}$$

$$dV_1^{A} = \frac{A T_1}{2} \tag{2.41}$$

$$dv_1 = \frac{dV_1^{A}}{A} \tag{2.42}$$

From Eq. (2.13):

$$dS_1^{A_0} = \frac{A_0 T_1^2}{3} \tag{2.43}$$

$$dS_1^{A} = \frac{A T_1^2}{6} \tag{2.44}$$

$$ds_1 = \frac{dS_1}{A} \tag{2.45}$$

From Eq. (2.18):

$$dV_2 = AT_2 \tag{2.46}$$

$$dv_2 = \frac{dV_2}{A} \tag{2.47}$$

From Eq. (2.19):

$$dS_2 = \frac{A T_2^2}{2} \tag{2.48}$$

$$ds_2 = \frac{dS_2}{A} \tag{2.49}$$

From Eq. (2.24):

$$dV_3 = \frac{A T_3}{2} \tag{2.50}$$

$$dv_3 = \frac{dV_3}{A} \tag{2.51}$$

From Eq. (2.25):

$$dS_3 = \frac{A T_3^2}{3} \tag{2.52}$$

$$ds_2 = \frac{dS_2}{A} \tag{2.53}$$

From Eq. (2.30):

$$dV_4 = \frac{-2FT_4}{\pi} \tag{2.54}$$

$$dv_4 = \frac{dV_4}{F} \tag{2.55}$$

From Eq. (2.31):

$$dS_4 = \frac{-4FT_4^2}{\pi^2}\left(\frac{\pi}{2} - 1\right) \tag{2.56}$$

$$ds_4 = \frac{dS_4}{F} \tag{2.57}$$

Let:

$$\Sigma V^{A_0} = V_0^{A_0} + V_1^{A_0} \tag{2.58}$$

Let:

$$\Sigma V^A = A(dv_1 + dv_2 + dv_3) \tag{2.59}$$

Let:

$$\Sigma S^{A_0} = dS_0^{A_0} + dS_1^{A_0} + dV_0^{A_0}(T_1 + T_2 + T_3 + T_4) + dV_1^{A_0}(T_2 + T_3 + T_4) \tag{2.60}$$

Let:

$$\Sigma S^A = A\left[ds_1^A + ds_2^A + ds_3^A + dv_1^A(T_2 + T_3 + T_4) + dv_2^A(T_3 + T_4) + dv_3^A T_4\right] \tag{2.61}$$

Equation (2.30) can be written as:

$$\dot{s}_4 = \Sigma V^{A_0} + \Sigma V^A - \frac{2FT_4}{\pi} \tag{2.62}$$

Equation (2.31) can be written as:

$$S_4 = \Sigma S^{A_0} + \Sigma S^A - \frac{4FT_4^2}{\pi^2}\left(\frac{\pi}{2} - 1\right) \tag{2.63}$$

Evaluation of Eqs. (2.59) and (2.60) confirm Eqs. (2.37) and (2.38):

$$\Sigma V^{A_0} = \bar{A}_0 = A_0\left(T_0 + \frac{T_1}{2}\right) \tag{2.64}$$

$$\Sigma V^A = \bar{A} = \left(\frac{T_1 + T_3}{2} + T_2\right) \tag{2.65}$$

Evaluation of Eqs. (2.66) and (2.67) confirm Eqs. (2.37) and (2.38):

$$\Sigma S^{A_0} = \hat{A}_0 = A_0\left[\frac{T_0^2}{2} + T_0(T_1 + T_2 + T_3 + T_4) + \frac{T_1^2}{3} + \frac{T_1}{2}(T_2 + T_3 + T_4)\right] \tag{2.66}$$

$$\Sigma S^{A} = \hat{A} = A\left[\frac{T_1^2}{6} + \frac{T_2^2}{2} + \frac{T_3^2}{3} + \frac{T_1}{2}(T_2 + T_3 + T_4) + T_2(T_3 + T_4) + \frac{T_3 T_4}{2}\right] \tag{2.67}$$

Substituting Eqs. (2.33) and (2.34) into Eq. (2.32):

$$\dot{s}_4 = \bar{A}_0 + A\bar{A} - \frac{2FT_4}{\pi} = 0 \tag{2.32}$$

$$F = \frac{\pi(\bar{A}_0 + A\bar{A})}{2T_4} \tag{2.68}$$

Substituting Eqs. (2.36) and (2.37) into Eq. (2.35):

$$s_4 = \hat{A}_0 + A\hat{A} - F\frac{4T_4^2}{\pi^2}\left(\frac{\pi}{2} - 1\right) = L \tag{2.69}$$

Hence:

$$A\hat{A} = L - \hat{A}_0 + \frac{\pi(\bar{A}_0 + A\bar{A})}{2T_4}\frac{4T_4^2}{\pi^2}\left(\frac{\pi}{2} - 1\right) \tag{2.70}$$

Let:

$$\hat{F} = \frac{2T_4}{\pi}\left(\frac{\pi}{2} - 1\right) \tag{2.71}$$

From Eqs. (2.70) and (2.71):

$$A = \frac{L - \hat{A}_0 + \hat{F}\bar{A}_0}{\hat{A} - \hat{F}\bar{A}} \tag{2.72}$$

$$F = \frac{\pi\left[\bar{A}_0 + \bar{A}\left(L - \hat{A} + \hat{F}\bar{A}_0\right)\right]}{2T_4\left(\hat{A} - \bar{A}\hat{F}\right)} \tag{2.73}$$

Having solved these equations for the parameters A and F, we can compute the lift velocity, and acceleration for each section of each curve. When initially checking a program the velocity on the nose should be identically zero and the computed maximum lift should agree with the specified value.

The parameters acceleration, velocity and lift can then be obtained for each section of the curve in turn using a new loop for each section. Other errors in the program may be found by checking that the values of the parameters give continuous curves at the joints between sections. Any discontinuities will indicate where to look for an error.

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