

Contents

1	Introduction and Overview	1
1.1	Linear Computational Geometry	1
1.2	Non-linear Computational Geometry	4
1.3	Applications	5
	Appendix	6
 Part I Linear Computational Geometry		
2	Geometric Fundamentals	9
2.1	Projective Spaces	9
2.2	Projective Transformations	12
2.3	Convexity	13
2.4	Exercises	16
2.5	Remarks	17
3	Polytopes and Polyhedra	19
3.1	Definitions and Fundamental Properties	19
3.2	The Face Lattice of a Polytope	25
3.3	Polarity and Duality	28
3.4	Polyhedra	31
3.5	The Combinatorics of Polytopes	34
3.6	Inspection Using <code>polymake</code>	40
3.7	Exercises	44
3.8	Remarks	45
4	Linear Programming	47
4.1	The Task	47
4.2	Duality	49
4.3	The Simplex Algorithm	53
4.4	Determining a Start Vertex	60
4.5	Inspection Using <code>polymake</code>	61
4.6	Exercises	63
4.7	Remarks	64

5	Computation of Convex Hulls	65
5.1	Preliminary Considerations	65
5.2	The Double Description Method	66
5.3	Convex Hulls in the Plane	72
5.4	Inspection Using <code>polymake</code>	76
5.5	Exercises	77
5.6	Remarks	78
6	Voronoi Diagrams	81
6.1	Voronoi Regions	81
6.2	Polyhedral Complexes	83
6.3	Voronoi Diagrams and Convex Hulls	84
6.4	The Beach Line Algorithm	88
6.5	Determining the Nearest Neighbor	96
6.6	Exercises	97
6.7	Remarks	98
7	Delone Triangulations	99
7.1	Duality of Voronoi Diagrams	99
7.2	The Delone Subdivision	102
7.3	Computation of Volumes	104
7.4	Optimality of Delone Triangulations	105
7.5	Planar Delone Triangulations	109
7.6	Inspection Using <code>polymake</code>	114
7.7	Exercises	116
7.8	Remarks	116
Part II Non-linear Computational Geometry		
8	Algebraic and Geometric Foundations	119
8.1	Motivation	119
8.2	Univariate Polynomials	122
8.3	Resultants	123
8.4	Plane Affine Algebraic Curves	125
8.5	Projective Curves	127
8.6	Bézout's Theorem	129
8.7	Algebraic Curves Using <code>Maple</code>	133
8.8	Exercises	135
8.9	Remarks	136
9	Gröbner Bases and Buchberger's Algorithm	137
9.1	Ideals and the Univariate Case	137
9.2	Monomial Orders	141
9.3	Gröbner Bases and the Hilbert Basis Theorem	145
9.4	Buchberger's Algorithm	149
9.5	Binomial Ideals	152
9.6	Proving a Simple Geometric Fact Using Gröbner Bases	153

9.7 Exercises	155
9.8 Remarks	155
10 Solving Systems of Polynomial Equations Using Gröbner Bases . . .	157
10.1 Gröbner Bases Using Maple and Singular	157
10.2 Elimination of Unknowns	158
10.3 Continuation of Partial Solutions	162
10.4 The Nullstellensatz	164
10.5 Solving Systems of Polynomial Equations	167
10.6 Gröbner Bases and Integer Linear Programs	171
10.7 Exercises	177
10.8 Remarks	177
Part III Applications	
11 Reconstruction of Curves	181
11.1 Preliminary Considerations	181
11.2 Medial Axis and Local Feature Size	182
11.3 Samples and Polygonal Reconstruction	185
11.4 The Algorithm NN-Crust	187
11.5 Curve Reconstruction with polymake	190
11.6 Exercises	190
11.7 Remarks	192
12 Plücker Coordinates and Lines in Space	193
12.1 Plücker Coordinates	193
12.2 Exterior Multiplication and Exterior Algebra	194
12.3 Duality	199
12.4 Computations with Plücker Coordinates	203
12.5 Lines in \mathbb{R}^3	204
12.6 Exercises	206
12.7 Remarks	206
13 Applications of Non-linear Computational Geometry	209
13.1 Voronoi Diagrams for Line Segments in the Plane	209
13.2 Kinematic Problems and Motion Planning	212
13.3 The Global Positioning System GPS	219
13.4 Exercises	221
13.5 Remarks	222
Appendix A Algebraic Structures	223
A.1 Groups, Rings, Fields	223
A.2 Polynomial Rings	224
Appendix B Separation Theorems	227
Appendix C Algorithms and Complexity	231
C.1 Complexity of Algorithms	231
C.2 The Complexity Classes P and NP	233

Appendix D	Software	237
D.1	polymake	237
D.2	Maple	237
D.3	Singular	238
D.4	CGAL	238
D.5	Sage	238
Appendix E	Notation	241
References		243
Index		247

Polyhedral and Algebraic Methods in Computational
Geometry

Joswig, M.; Theobald, T.

2013, X, 250 p. 67 illus., 17 illus. in color., Softcover

ISBN: 978-1-4471-4816-6