

## Chapter 2

# Technology in Growth Models

For about 100 years, the discipline of economic growth and its models use the idea of technology (technical progress) in a rather objective fashion, quantifying its contribution to value production. This discipline is a crucial area where it is possible to realize how the concept of technology has been understood so far. The main objective of this chapter is to describe several growth models such that the role of technology is described as captured by their authors and eventually quantified. It will be shown how ambiguously and inaccurately the concept of technology is used.

The issue is how does technology impact on a transformation process and thus on economic systems? There is a consensus that it does impact and many authors modeled economic systems in order to show how, but it is not straightforward to understand their conclusions, because the concept of technology itself has not been made clear. An equivalent issue is how can technology contribute to growth? Growth models also try to reproduce in a schematic fashion how that contribution works, although somehow unsuccessfully.

We will start with some basic ideas about models, namely economic models, and an explanation of different types. A model describes a system whether it is the nature, a society or a product of the human mind, though in a limited fashion. An economic model replicates parts of an economic system to simulate its operation, gathering a number of input parameters and the system's output, and linking them through a specific rule. This rule is described as a particular set of relationships, which characterize the model and, hopefully the system. Typically, there are two categories: Static and dynamic models. The former reflects a finite number of closed and necessary relationships between the model's parameters, in the form of mathematical identities. It shows how system parts interact, enlightens the main concepts involved, and clarifies the respective semantics. The latter is an open relationship between system parameters, written in the form of mathematical equations representing a combination of functions, possibly non-linear and time dependent, for which is necessary to find solutions and then to interpret the results appropriately.

## 2.1 Comparative-Static Models

It can be said that accounting systems are also economic models, as they describe the economic activity using units of value, whether they refer to firms, sectors, or national economies. These systems, for both micro and macroeconomics, provide nowadays the structural semantics of this field of knowledge, along with its epistemological component. Unfortunately, this is a main first argument, technology is a concept that is not present within those systems, such that its contribution to production has not been valued so far. Moreover, the accounting systems, being international standards, bring the highest possible objectivity to the economic process, since they ensure the verification of economic facts regardless of place, time, and the observer. Also, they are the only available bridge to interpret how economic value translates into social value. The models based on the (double entry) accounting systems, which are approximately standard all around the world, are static and conservative, what means that the sum of the parts makes the whole, just as energy in Physics or as within Mathematics. These conservative features create the best possible environment to delimit unambiguously the production factors, which somehow relate to technology and to knowledge.

In order to evaluate the evolution of economic systems, thus growth conditions, it is not enough to work with static patterns like accounting identities. It is also required to establish hypotheses between the system parameters along the time. These hypotheses are, after all, conjectures about how the model dynamically relates to its reality context. Examples of conjectures would be the increase in human capital reflecting the increase in fixed capital, or the increase in household consumption being activated by the increase of their income, or even the total factor productivity (TFP) gains representing the earnings that are not measured directly on the contributions of labor and capital inputs. Of course, these hypotheses should be previously verified in the social and economic contexts.

Although the static models provide a static balance, a certain degree of its time dependence can be directly verified. In fact, for a sequence of time periods, it is possible to infer differential models, which, besides relating static parameters representing production, establish relations between their time variations. This generalization of static models allows both the application of a method known as comparative statics and the design of models described by differential identities.

## 2.2 Dynamic Models

On the other hand, a dynamic model based on a number of time-dependent characteristics, i.e. a time-dependent rule that relates its inputs with the output, can be built. Dynamic economic models are typically referred to as growth models. It will be seen that the neo-classic growth models attempted to include technology in the rule, making an explicit difference with capital, though mixing it with knowledge and technical progress, whatever they mean.

Before resuming to growth models, I will briefly describe what is meant by economic growth and what the fundamental identities in macroeconomics are. The problem of economic growth is a subset of the wider issue of development. The idea of economic growth is basically associated to the increasing accessibility of every individual to goods and services, which will, on the one hand, compensate for the natural fading of his/hers physical and intellectual condition and, on the other hand, provide supplementary capabilities. Therefore, growth relates directly to the increasing yield of an economy and with an appropriate distribution of total income among different identities of a society [1]. Income is at the origin of expenditure, which may have two different forms: Consumption and investment (savings). Consumption benefits the individual in the short run but diverts value from investment. Investment feeds production and may increase output and then income in the medium and long term. As such, the conditions for increasing income are directly related to optimal budgeting between consumption and investment, as seen from the expense view, and with the proper use of resources, from the production view. It is the appropriate balance between consumption and investment that controls the maintenance of a certain growth rate. To find this balance is the main enquire. So, technology, even if only considered as a mix of resources, is surely in the center of the growth problem.

How have the growth ideas and models developed? In Antiquity, the extent of fertile land, the number of subjects and slaves and the size of the armies provided the first measures of wealth. With Mercantilism, wealth was measured at a higher level of abstraction, by the accumulation of currency and precious goods. To achieve that accumulation, the doctrine was to get an export surplus over imports, using customs barriers and promoting exports. Those times emphasized the importance of the monetary capital available, adding this factor to the need of a large labor force.

The models of economic growth appeared with the physiocracy. The physiocrats, named for the supremacy they attributed to the natural order and the Earth elements in the eighteenth century France, reacted against the mercantilist influence on national economic strategies. According to Quesnay's [2] growth scheme, growth was proportional to the cultivated land, from where a free gift was provided, i.e. a value added by the land itself, value that made the difference between the process's starting value, and the ending output value. One can complete this scheme by introducing the capital investment, as Quesnay regarded it, and its influence on the process productivity. The importance of labor and of machinery as capital was clearly acknowledged.

Adam Smith [3] developed, further, the growth scheme of the physiocrats emphasizing the importance of manufacturing to wealth creation, modeling, and describing it in great detail, highlighting many of its peculiarities, naming many of its parameters, and describing its cost structure. Economic growth, according to this author, depended basically on two parameters: Capital accumulation and division of labor, the latter being the responsible for greater productivity. His conception of economic growth, which he did not model schematically, made it clear that growth would depend on the increase of labor productivity being greater than

the increase in wages. According to this idea, as explained by Deane [4, point 3], growth could be achieved step-by-step by increases of technical progress: For a certain level of technical progress, economic growth would tend to minimize profits and wages; but, with additional technical progress, thus increasing productivity, profits and wages would rise again toward a new equilibrium. Still, what technical progress was about has not been objectively explained, and the word technology was still absent in the lexicon of those times.

The works published by Harrod [5] and Domar [6], following Keynes' ideas of growth, modeled for the first time the dynamics of economic growth. Even if independently, they followed studies of Lundberg [7] and, according to Imparato [8], of Cassel [9] and Kalecki [10]. Later, they would be continued and developed extensively by the Cambridge School economists, notably Nicholas Kaldor and Joan Violet Robinson, school known as post Keynesian and somewhat neo-Ricardian. Those models are based on three fundamental principles regarding the aggregated parameters of an economy: A balance between supply and demand; production to equal income and income to equal expense; and the central ideas of Keynes that savings equals investment and that this acts as a lever for growth of the expenditure and of the product.

This is the beginning of macroeconomics, the establishment of its fundamental relations and the start of the quest for the determinants of economic growth. The Keynesian school supported the theoretical development until the proposals of Solow [11, 12] and Swan [13], who started a new era that would dominate for 20 or 30 years and has not yet been drained. Later, the endogenous growth theory became popular, as well as the movements that placed knowledge and innovation at the heart of growth.

## 2.3 Technology and the Solow Model

Solow and Swan presented the first most successful long-run growth model. Swan acknowledged the importance of progress and not just technical progress, drawing attention to the relevance of the administration and other institutions, perhaps with roles as important as technical change. Solow explicitly included technological change [11] or technical change [12] as a basic parameter in his Cobb-Douglas aggregate production function. How does technical change relate with technology? It is not clear, because at the time neither it was well defined. Furthermore, over the course of the next 50 years, it was not objectively established the significance of this technical change factor. To illustrate this, I quote, among many other important authors, Jones [14] who refers to this factor using the following words: *Stock of ideas; technology; technology variable; productivity term; stock of knowledge; people with ideas, labor augmenting technology*, etc. The ambiguity is evident but the current ideas of technology and knowledge are surely contained by it. Solow himself in his 1957 paper explained his technical change parameter as representing any kind of shift in the production function, such as slowdowns,

speedups or improvements, or the education of the labor force. It is also important to note that Solow recognized both capital and labor force as explicit factors in his production function, what means that they were conceptually well identified, and that other factors like labor force education and, in general technology were not.

I will explain what the significance of the parameter technical change might be, showing that it is a different thing depending on where it is placed within the Cobb-Douglas's production function. I shall also show how this ambiguity contributed to the difficulty of interpreting some empirical results.

Solow's model builds on two main conditions, both known at the time and already used by other economists, concluding that the economy tends to a steady-state equilibrium when there is no technical change. With modifications in economic conditions, other than capital and labor, the parameter representing technical level will change and new steady-state equilibrium will be reached.

The first condition can be written as a capital accumulation equation, using the idea of comparative statics. It states that the change of the capital value  $\Delta C$ , from one period of time to the next, equals investment  $I$  minus the depreciation of capital during that period of time. Considering differentials to time  $t$ , investment  $I$  as a part of the product  $Y$ , such that  $I = s \cdot Y$ , and capital depreciation as  $\delta \cdot C$ , the capital accumulation equation is written as in Eq. 2.1, where  $\dot{C} = \frac{\Delta C}{\Delta t}$ ,  $s$  is the saving coefficient (dimensionless),  $\delta$  is the capital depreciation coefficient (dimensionless), and  $Y$  is the product of the economy. Typically,  $Y$ ,  $C$ , and  $\Delta C$  are in units of value, for example €.

$$\dot{C} = s \cdot Y - \delta \cdot C \quad (2.1)$$

This equation reads: In one period of time  $\Delta t$ , the change in capital value  $\Delta C$  equals the savings part of the economy's product minus the amount of capital depreciated value.

The second condition is a given production function, which describes the relation between the production system inputs with its output  $Y$ . This relation corresponds to a conjecture about how the product (output) depends on inputs, i.e. a relationship expressed as a mathematical function in which inputs are independent variables referred here as production factors. Considering as production factors the value of capital  $C$  and the number of workers  $N$ , the output  $Y$ , which is the value of the product, is written as a mathematical function  $F(C, N)$ . This function, as Solow [12] puts it, represents technological possibilities.

$$Y = F(C, N) \quad (2.2)$$

Works of Johan Gustaf Knut Wicksell, Philip Wickstedd, Charles Wiggins Cobb, and Paul Howard Douglas proposed the function  $F$  now widely known as the Cobb-Douglas function (2.3).

$$F(C, N) = Y = C^\alpha \cdot N^{1-\alpha} \quad (2.3)$$

As said above when charactering a model, this function is proposed as the rule that reflects the system operation. What this function says is that, in order to obtain the output  $Y$ , capital and labor must operate together in a way that can be represented

by a multiplication of their values, each considered not as the whole but only one part of the respective whole:  $C^\alpha$  as a part of  $C$ , and  $N^{1-\alpha}$  as a part of  $N$ , where  $\alpha$  is a dimensionless parameter. This parameter  $\alpha$ , typically with a value less than one, balances the contributions to the output of both capital and labor force, meaning that not all capital relates with all workers, only one part of  $C$  is worked by one part of  $N$ . The rule of this model is this multiplication between the two production factors and the parameter  $\alpha$ . It is important to note that a different value of  $\alpha$  makes a different function and so a different way the model works. Two models with the same function type but with a different  $\alpha$  are not easily comparable, as we will see.

This multiplication means that the output is built by adding  $N^{1-\alpha}$  terms, each equal to  $C^\alpha$ . To make it more clear, suppose  $C = 64$  €,  $N = 16$  workers and  $\alpha = 0.5$ . The output would be built by adding four times ( $=16^{0.5}$ ) the value of 8 ( $=64^{0.5}$ ), yielding 32. The units of the result are [ $\text{€}^{0.5} \cdot \text{workers}^{0.5}$ ]. This production function is dimensionally incorrect as the first member has the dimension unit of value [ $uv$ ], for example €, and thus the second member should have the same dimensions, which it does not. There is, here, a missing parameter  $A$ , with appropriate dimensions, which would make the equation correct.

Swan used it in this incorrect way and many authors continue to do so, what represents a serious theoretical matter and a more or less important pragmatic issue as it affects the interpretation of results in an unknown way. Other dimensions have been used for this type of production function, as stated in many economic text books. For example, if dealing with the electricity sector,  $N$  may be hours worked by labor,  $C$  hours worked by the turbines and  $Y$  in units of energy like MWh. For instance, Kaldor [15] proposed steel contents, in tonnes, within the capital assets. It would be wrong again.  $Y$  could be tons of cotton,  $C$  the value of capital and  $N$  the number of workers. It is still wrong. Most authors apparently forget to make this production function dimensionally correct. Hunten [16] makes an excellent short biography of the difficulties found by the empirical literature on TFP and the Solow residual, though without identifying this dimensions error. He quotes Robert Solow in 1987: “We can see the computer age everywhere but in the productivity statistics”. The result of this dimension’s inaccuracy is a story of apparent successes and many inconsistencies within empirical econometric analysis. Next, I will try to clarify this dimensions’ problem, as it deals directly with the measurement of what many authors understand as technical progress and technological change.

Solow [12] starts explaining his theory of the convergence for steady-state equilibrium with a production function not fully defined but equivalent to Eq. 2.3, in the form of Eq. 2.4.  $Y_N$  and  $C_N$  are product and capital per worker.

$$Y_N = C_N^\alpha \quad (2.4)$$

The combination of Eqs. 2.1 and 2.4 leads to his famous steady-state equilibrium. Eq. 2.4 is dimensionally incorrect. Still, for comparisons of different solutions, how right or wrong could this be, in view of this problem of the missing parameter  $A$ ? It is right if that parameter  $A$  does not change its value over time when  $Y$ ,  $C$ , and  $N$  change. It is wrong if it changes, and the most I can say is that it will be less right if it changes a little and more wrong if it changes a lot.

In Solow's paper section IV, example 3, the following production function is also proposed:  $Y = \alpha^2 C + N + 2 \propto \sqrt{CN}$ , where this problem of dimensions is very obvious, as  $Y$  and  $N$  do not have the same dimensions (the labels were changed to the current notation).

Then Solow introduced another formulation in section VI, and later in Solow [12], by entering the parameter  $A$  representing something related to knowledge and technology, which he said that represented technological change. This could be the missing parameter that would put the equations right. However, the ambiguity with which this parameter was introduced did not allow for a clarification, actually it presented extra problems.

The Cobb-Douglas production function was written and used by Solow and many authors placing the technological change parameter  $A$  in three different positions: *Augmenting labor*, *augmenting capital*, or *neutral*. The three respective forms are written as follows:

$$Y = C^\alpha \cdot (A \cdot N)^{1-\alpha} \quad (2.5)$$

$$Y = (A \cdot C)^\alpha \cdot N^{1-\alpha} \quad (2.6)$$

$$Y = A \cdot C^\alpha \cdot N^{1-\alpha} \quad (2.7)$$

A dimensional analysis immediately shows that, in each case,  $A$  has different dimensions and so must signify different causes, what should have different effects.

For *labor augmenting* (2.5), the dimension of  $A$  is units of value per number of workers  $[uv/nw]$ .

For *capital augmenting* (2.6), the dimension of  $A$  is  $[uv/nw]^{(1-\alpha)/\alpha}$ .

For *neutral* (2.7), the dimension of  $A$  is  $[uv/nw]^{(1-\alpha)}$ .

It must be concluded that this technological change parameter is only exogenous in the *labor augmenting* case (2.5), where its dimensions do not depend on  $\alpha$ . Note, as explained above, that exogenous parameters are those that do not depend on the model rule. On the other two cases,  $A$  dependence on  $\alpha$  means that its economic significance is depending on the model characteristics such that, for different values of  $\alpha$ ,  $A$  means a different thing. Moreover, it should not be referenced with the same symbol. In order to investigate further their meanings, we will use  $A_1$ ,  $A_2$ , and  $A_3$ , respectively.

This analysis suggests that the three technological change factors may be a type of labor productivity, measured as units of value per worker  $[uv/nw]$  to the power 1,  $(1 - \alpha)/\alpha$ , or  $(1 - \alpha)$ , respectively. But they could also be a kind of capital productivity, measured as units of value per worker  $[uv/nw]$  to the power 1,  $(1 - \alpha)/\alpha$ , or  $(1 - \alpha)$ , respectively.

### 2.3.1 Technological Change as a Type of Labor Productivity

Let us first consider them as charged labor productivities, meaning labor productivities affected by a charge or a weight. What type of charge is this? And on what



is that charge depending? To answer these questions, we will first take the classic and consensual definition of labor productivity  $LP = Y/N$ , which unquestionably has dimensions  $[uv/nw]$ .

In the first case (2.5),  $A = A_1$  has the dimensions  $[uv/nw]$ , the same as  $LP$ 's. This means that it is a labor productivity (charged) multiplied by a dimensionless factor  $f_1$ . In other words,  $A_1 = LP \cdot f_1$ .

To calculate  $f_1$  we write  $LP = Y/N$  and substitute  $Y$  by Eqs. 2.5, yielding 2.8. For  $A_2$  and  $A_3$ , and  $f_2$  and  $f_3$  factors, the equivalent results are shown in Eqs. 2.9 and 2.10.

$$f_1 = C_N^{-\alpha} \cdot A_1^{\alpha} \quad (2.8)$$

$$f_2 = A_2^{1-\alpha} \cdot C_N^{-\alpha} \quad (2.9)$$

$$f_3 = (C_N)^{-\alpha} \quad (2.10)$$

This shows that the factor  $f$  is a multifaceted entity with a meaning not easy to understand. Accordingly,  $A$  is not really a labor productivity, meaning that it does not relate output value to labor activity in a straightforward fashion.

### 2.3.2 Technological Change as a Type of Capital Productivity

Let us now inquire the parameters  $A$  as charged capital productivities and find out on what are they depending. Again, we will first take the classic and consensual definition of capital productivity  $CP = Y/C$ , which unquestionably is dimensionless. We will follow the same method as above.

In the first case (2.5),  $A = A_1$  has the dimensions  $[uv/nw]$ . This means that it is a capital productivity multiplied by a factor  $f_1$ , which dimensions are  $[uv/nw]$ . In other words, it would be  $A_1 = CP \cdot f_1$ .

To calculate  $f_1$  we write  $CP = Y/C$  and substitute  $Y$  by Eq. 2.5, yielding  $f_1 = A_1^{\alpha} \cdot C_N^{1-\alpha}$ . For  $A_2$  and  $A_3$ , and  $f_2$  and  $f_3$  factors, the equivalent results are:  $f_2 = A_2^{1-\alpha} \cdot C_N^{1-\alpha}$ ; and  $f_3 = A_3^{1-\alpha} \cdot C_N^{1-\alpha}$ . Again, this shows that the parameter  $A$  is not capital productivity, not even charged capital productivity.

From this brief yet objective analysis the conclusion is that: (1) The parameter  $A$  most probably reflects other factors than  $C$  and  $N$ ; (2) however, it is not independent of  $C$  and  $N$ .

This has important implications on the calculations of the TFP, what will be analyzed in the next section. For now, we may conclude that the neo-classic parameter  $A$ , referred to as technical progress, technology, or as everything else contributing to output besides capital and labor, is not independent of  $C$  and  $N$  and does not inform on the role of technology. One aspect was, however, made clear: Neo-classic and Keynesian authors clearly established a conceptual difference between capital and technology, defining and quantifying capital but leaving technology as an indefinite, and confusing idea.



## 2.4 Technology and Total Factor Productivity

The objective of computing TFP is to assess the importance and contributions to the output of everything else than labor force  $N$  and capital  $C$ . That residual part would include workers' knowledge and new ideas, technological assets that could not be considered as capital (disembodied technology), management structure, organizational forms, impacts from markets, financial costs, profits, etc. What neo-classic authors called technology was diffusedly within that lot, though the most significant part. Taking the production function (2.7), the parameter  $A$  may play that role. In fact, even if with variations, the literature of growth accounting, productivity growth, or equivalent nominations takes this production function with *neutral* parameter  $A$  as the basis for calculating this productivity. Assigning to  $A$  the idea of TFP, it reflects the concern of identifying a parameter that relates the output  $Y$  with the whole of inputs, and so revealing the influence of everything else besides known and objective inputs, like  $C$  and  $N$ .

How can TFP be computed? Taking three time series of  $Y$ ,  $C$ , and  $N$  for  $n$  years, it is possible to construct two time series of  $Y_N$  and  $C_N$ , and scatter  $n$  points, one for each year, on a two-dimensions plane  $(Y_N, C_N)$ . Note that (2.7) may be written as (2.11). Using the method of ordinary least squares or least squares fitting, it is possible to find the function  $Y_N(C_N)$  that estimates and describes how  $Y_N$  depends on  $C_N$ . Prescribing a power function as the estimator (as the production function suggests), the regression algorithm will return the best fit function, and with it the values of  $A$  and of  $\alpha$ . The value of  $A$  will be the value of TFP.

$$Y_N = A \cdot C_N^\alpha \quad (2.11)$$

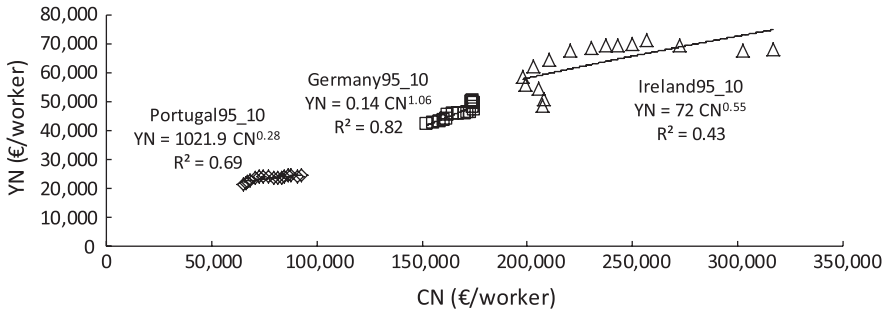
Alternatively, taking logarithms of (2.11), the Eq. 2.12 can be written. In this case, the estimator function would be linear and the regression algorithm will return the values of  $\ln A$  and  $\alpha$ , the former being the intercept, the point where the linear function crosses the  $\ln Y_N$  axis ( $\ln C_N = 0$ ), and the latter being the inverse tangent function of the angle between the linear function and the  $\ln C_N$  axis.

$$\ln Y_N = \ln A + \alpha \cdot \ln C_N \quad (2.12)$$

This is perfectly equivalent to the first method. In some literature, TFP is said to correspond to  $\ln A$  instead of  $A$ . In one or in the other way, for the time period corresponding to the time series, the values of TFP are computed as described. A conclusion relevant to our analysis is that  $A$  and  $\alpha$  are a pair of solutions, meaning that their values depend on each other and on the input distributions.

Alternatively, it is possible to force exogenously a specific value of  $\alpha$  and compute the linear regression, what would return a different value of  $\ln A$ . Or, force a value of  $\ln A$  and have a return of a different value of  $\alpha$ . In both cases, there will be higher minimum square errors (MSE), such that the production function would not represent the real distribution equally well.

An example will help to understand this problem of comparing values of  $A$  in different periods of time and between different countries. Taking time series for



**Fig. 2.1** Description of Portugal, Germany, and Ireland economies according to production function (2.11), 1995–2010, constant prices of 2005, data from AMECO

**Table 2.1** Values for A and  $\alpha$ —Portugal, Germany and Ireland economies according to production function (2.11), 1995–2010, constant prices of 2005, data from AMECO

	$A [(\text{€}/N)^{(1-\alpha)}]$	$\alpha$
Portugal	1021.9	0.28
Germany	0.14	1.06
Ireland	72.0	0.55

the Portuguese, German, and Irish total economies, from 1995 to 2010, of  $Y_N$  and  $C_N$ , where Y is the GVA at constant prices (2005) for total branches,<sup>1</sup> C is the net capital stock at constant prices for total economy,<sup>2</sup> and N is the total labor force, from the European annual macroeconomic database AMECO,<sup>3</sup> the results for the three countries, according to the production function (2.11) are depicted in Fig. 2.1 and listed in Table 2.1.

The values of A for each country in the same period of time, which could be interpreted as the TFP, are extremely different and, as we have seen in Sect. 2.3, their units are different hence not comparable. It is obvious from this example that A is a different thing in each of the countries studied here.

If instead of comparing different economies, along the same period of time, we would compare different periods of time for the same economy, the results would be equally illusive. Taking the same three economies and calculating in the same way, the values of A and  $\alpha$  for the periods 1995–1999, 2000–2004, and 2005–2009, the results are shown in Table 2.2. Again, they are very different and not comparable.

It is very clear that these results do not show what it was expected. In no way either it is possible to interpret the parameter A as a productivity that could be comparable between different economies or different periods of time, or interpret it as reflecting anything like technology changes or technical progress. So why is

<sup>1</sup> Definition (ESA 95 [17]): 8.11–8.12, 9.23, 10.27–10.30.  
<sup>2</sup> Definition (ESA 95 [17]): 6.02 f.  
<sup>3</sup> [http://ec.europa.eu/economy\\_finance/db\\_indicators/ameco/index\\_en.htm](http://ec.europa.eu/economy_finance/db_indicators/ameco/index_en.htm) (consulted March 2012).

**Table 2.2** Values for A and  $\alpha$ —Portugal, Germany and Ireland economies according to production function (2.11), constant prices of 2005, data from AMECO

	1995–1999		2000–2004		2005–2009	
	A $[(\text{€}/N)^{(1-\alpha)}]$	$\alpha$	A $[(\text{€}/N)^{(1-\alpha)}]$	$\alpha$	A $[(\text{€}/N)^{(1-\alpha)}]$	$\alpha$
Portugal	0.169	1.06	1.564E + 05	−0.17	1,826.214	0.22
Germany	0.004	1.3	3,361.021	0.21	0.000003	1.95
Ireland	1.696E + 20	−2.9	2.51	12.3	0.7	13.2

this methodology being used by many authors and organizations? Possibly for two reasons:

First, because the values of A are not usually calculated, but in its place the yearly differences of A are computed, such that it is not the value of TFP, but the TFP growth rate that is calculated. This shades this problem of the dimensions of A being dependent on the values of  $\alpha$ .

Second, because many authors, as Solow [12] himself did, consider the parameter  $\alpha$  as input to the system.

We shall start by analyzing the first. In fact, taking derivatives to time of (2.11) and then dividing by  $Y_N$ , the equation can be written as in (2.13), where the cap symbol over a parameter means growth rate. The equation reads as follows: The output per worker growth rate equals the input capital per worker growth rate, multiplied by the factor  $\alpha$ , plus the TFP growth rate  $\hat{a}$ . As each member of this equation and the two terms of the second member have dimensions of  $[t^{-1}]$ , the problem of the dimensions of A dependence on  $\alpha$  becomes opaque, though it is still there.

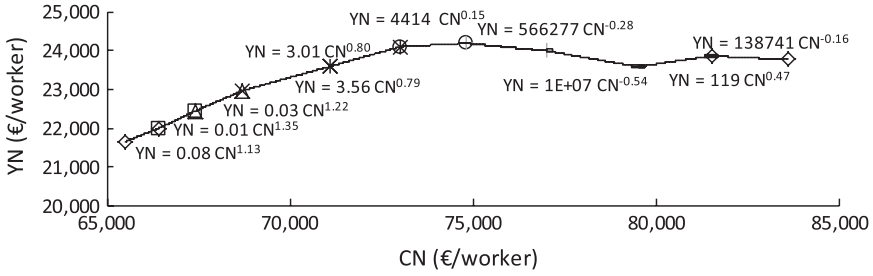
$$\widehat{y}_N = \alpha \cdot \widehat{c}_N + \widehat{a} \quad (2.13)$$

Let us analyze one example. The following TFP growth rate calculation is equivalent to what was presented in Table 2.2, though using periods of 1 year instead of 5 years. Using Portugal data as before and Eq. 2.11, Fig. 2.2 shows the results of the production function for ten periods of one year, from 1995–1996 to 2004–2005. From each production function of each year, values of A and  $\alpha$  are extracted and shown in Table 2.3, columns 2 and 3.

The calculations of the TFP growth rate, displayed in Table 2.3, show once again that the yearly TFP growth rates do not seem comparable, and the reason is what was pointed out above: As the dimensions of A depend on  $\alpha$ , and  $\alpha$  is different from period to period, the values of A in each period represent different things.

Now analysing the second. Solow valued it as the ratio of the return to capital to total return, and for output he used the gross domestic product (GDP). A similar approach is followed today by the Organisation for Economic Co-operation and Development (OECD). This approach makes the problem much smoother in what concerns the large variations of either A or  $\hat{a}$  that we have seen in the cases above, but does not change the nature of the problem.

Solow [11] considered  $\alpha = 0.35$ , value that changed every year proportionally to the ratio (labor force)/(labor force employed), from 1909 to 1949. The empirical



**Fig. 2.2** Description of the Portugal economy according to production function (2.11), constant prices of 2005, data from AMECO. Results for ten periods of one year are shown, starting on 1995–1996, on the left, up to 2004–2005, on the right part of the figure. In each period, the production function shown is estimated with only two points, where from the values of  $A$  and  $\alpha$  are extracted to columns 2 and 3 of Table 2.3

**Table 2.3** Second and third columns show values for  $A$  and  $\alpha$  for Portugal economy according to production function 2.11, constant prices of 2005, data from AMECO. Values are taken from equations on Fig. 2.2. Fourth and fifth columns show  $\hat{A}$  and  $\alpha$ , the latter as the average for two consequential periods

	$A$	$\alpha$	$\Delta A/A = \hat{A}$	$\alpha$
1995–1996	0.08	1.13		
1996–1997	0.01	1.35	−0.88	1.24
1997–1998	0.03	1.22	2.00	1.29
1998–1999	3.56	0.79	117.67	1.01
1999–2000	3.01	0.8	−0.15	0.80
2000–2001	4,414	0.15	1465.45	0.48
2001–2002	566,277	−0.28	127.29	−0.07
2002–2003	1.E + 07	−0.54	16.66	−0.41
2003–2004	119	0.47	−1.00	−0.04
2004–2005	138,741	−0.16	1,164.89	0.16

values of  $\alpha$  evolved between 0.312 and 0.397. Using an equation equivalent to (2.13) and time series for  $Y_N$  and  $C_N$ , he could calculate  $\hat{Y}_N, \hat{C}_N$  and thus  $\hat{A}$  as the so-called yearly rate of technological change. Considering  $\alpha$  as an input, and knowing that its value for a specific country or sector does not change much along the time, makes the dimensions of  $A$  changing less and so making yearly comparisons probably more acceptable. How acceptable? That depends very much on the values of  $A$  and  $\alpha$ , as their relation is not linear but a power function. It cannot be said, a priori and not easy to assess a posteriori. For example, it will not be possible to compare TFP growth rates between sectors and countries with very different mean values of  $\alpha$ .

Moreover, an almost constant  $\alpha$ , which geometrically means an almost constant angle between the linear function and the horizontal axis, may have a destroying effect on the regression technique, such that the chosen linear function will no longer represent that specific distribution (even if with only two points).

OECD [18] uses as inputs yearly values of  $\alpha$  for evaluating what they refer to as multifactor productivity (MFP)<sup>4</sup> (based on value added). This organization uses an equation, somehow equivalent to (2.12) and representing yearly changes as in (2.13), which can be written as (2.14). The subscripts  $t$  and  $t - 1$  mean year  $t$  and year  $t - 1$ .

$$\ln \frac{y_t}{y_{t-1}} = \ln \frac{A_t}{A_{t-1}} + \alpha \cdot \ln \frac{CS_t}{CS_{t-1}} + (1 - \alpha) \cdot \ln \frac{N_t}{N_{t-1}} \quad (2.14)$$

This equation could present the same problems as before; however, this methodology uses, instead of capital  $C$ , what they call capital services  $CS$ , which are measured in working hours [19]. Concisely, these services, multiplied by prices that are paid for 1 hour use of each type of service, correspond to a capital use value, or remuneration for capital services, in units of value. The latter, divided by the total costs of inputs, is the value of  $\alpha$ ; and total costs of inputs are considered to be the sum of the remuneration for labor  $w \cdot N$  and the remuneration for capital services.  $N$  is the number of hours worked and  $w$  is a wage per hour worked (prices and wages are considered in such a way that the final equation is written in constant prices). As such, the basic production function that OECD uses is (2.15), and a dimensions analysis shows that  $A$  (what they call MFP based on value added) is measured in  $[uv/hw]$  and so becoming independent of  $\alpha$ . This methodology overcomes the main problem pointed out before.

$$Y = A \cdot CS^\alpha \cdot N^{1-\alpha} \quad (2.15)$$

What now does it measure this MFP? To answer this question we can consider  $Y$ , which is the GVA, as the sum of its two most important terms, according to the fundamental accounting identity from the production view (2.16).

$$Y = w \cdot N + m \cdot C \quad (2.16)$$

I call  $m$  the management factor [20], which, proportionally to capital  $C$ , represents everything else besides the contribution of labor (compensation of employees) to GVA. It covers capital depreciation and revaluations (or consumption of fixed capital), taxes, and other financial costs and operating surplus. So, any  $\Delta A$  (or the MFP growth rate) accounts for changes in wages, capital service prices, financial costs, and operating surplus. Where is technology or technical change here? It is diluted in all these terms. So, once again, the so-called methods computing MFP do not inform on the influence of technology, whatever that could be.

## 2.5 Technology and A.C Models

There is a special class of models that are based on production functions of the type shown in (2.17), where the product  $Y$ , the model's output, is proportional to capital  $C$ , an input to the model and the sole explicit production factor, being the proportionality given by the parameter  $A$ .

$$Y = A \cdot C \quad (2.17)$$

<sup>4</sup> Definition in <http://stats.oecd.org/glossary/detail.asp?ID=1698> (Accessed March 2012).

The parameter  $A$  has had interpretations as in the neo-classic model. Thus,  $A$  reflects the knowledge and technology within the system, one or the other depending on the author, and  $C$  represents capital with the variant that it can include human capital as well. As pointed out before, these ambiguities continue to challenge an objective interpretation of results.

This type of models is characterized by increasing returns to scale, for example, if  $C$  and  $A$  double,  $Y$  more than doubles, which was not the case in the Solow model, where the economy had decreasing returns to scale in its dependence on capital. The interpretation of  $A$  is indistinct but its dimensions are clear. If  $C$  is capital, measured in units of value [uv], as well as the product  $Y$ ,  $A$  comes dimensionless. Its meaning is a clear capital productivity,  $A = Y/C$ , everything in the system that contributes to produce  $Y$  out of the use of capital  $C$ , in other words, labor and their skills, technology (whatever it means), technical progress in organizational structures, and other contributors to output like financial costs and profits.

This model is handled with the consensual capital accumulation Eq. 2.1, which says that the capital increase in each period of time equals the investment  $I$ , or savings  $S = I = s \cdot Y$ , less depreciation of the existing capital  $\delta \cdot C$ . Dividing (2.1) by  $C$ , considering (2.17) and writing the corresponding expression in terms of growth rates originates (2.18).

$$\hat{y} = s \cdot A - (\delta - \hat{a}) \quad (2.18)$$

The product's growth rate  $\hat{y}$  comes proportional to the saving coefficient  $s$ . An economy with constant technical progress  $A$ , what can be represented by  $\hat{a} = 0$ , and a constant depreciation coefficient  $\delta = \text{const.}$ , grows only according to savings increases. Similar conclusion had been taken from Harrod/Domar model's results, where the equivalent of  $A$  was named by Domar [6] as *potential social average investment productivity*, which, in turn, was a concept close to the famous Keynesian multiplier [21], Chaps. 3, 8, and 10).

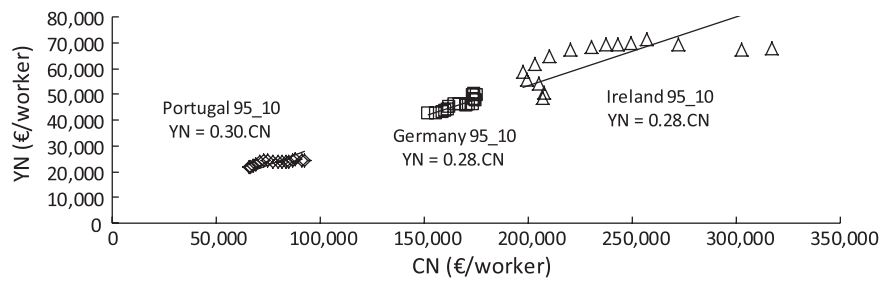
Let us see how  $A$  behaves in such a model. Writing (2.17) per number of workers does not change the meaning and the values of  $A$ , and we can write (2.19):

$$Y_N = A \cdot C_N \quad (2.19)$$

Taking the same economies along the same period of time as in sections above, the results for  $A$  are depicted in Fig. 2.3 and Table 2.4 (linear regressions were forced to an intercept equal to zero).

The results show average values for the capital productivity about the same for the three economies, during this period of time. As there are no differences in dimensions of  $A$ , these values can be compared and the comparison is significant.

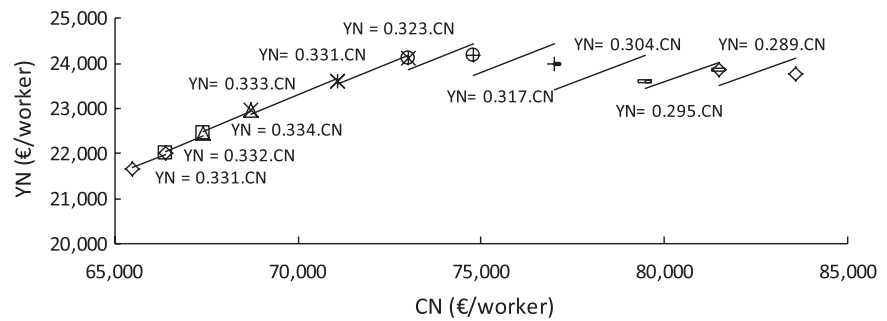
In the same way, the capital productivity growth rate can be computed using periods of 1 year and evaluating  $\hat{a} = \Delta A/A$ . Using Portugal data as before and Eq. 2.19, Fig. 2.4 shows the results of the linear production function for ten periods of one year, from 1995–1996 to 2004–2005 (linear regression were forced to a zero intercept). From the production function of each period, the value of  $A$  is extracted and shown in Table 2.5.



**Fig. 2.3** Description of Portugal, Germany, and Ireland economies according to production function (2.19), 1995–2010, constant prices of 2005, data from AMECO

**Table 2.4** Values for A—Portugal, Germany and Ireland economies according to production function (2.19), 1995–2010, constant prices of 2005, data from AMECO

	A
Portugal	0.30
Germany	0.28
Ireland	0.28



**Fig. 2.4** Description of the Portugal economy according to production function (2.19), constant prices of 2005, data from AMECO. Results for ten periods of one year are shown, starting on 1995–1996, on the left, up to 2004–2005, on the right part of the figure. In each period, the linear production function shown is estimated with only two points, from where the values of A are extracted to column 2 of Table 2.5

These numbers for the capital productivity illustrate how the A.C dynamic model may describe an economy, rigorously though in a very limited fashion. The parameter A, used in these models, has a clear significance, is dimensionless, and reveals correctly and unambiguously how capital relates to output. However, from the point of view we are pursuing, which is the influence of technology on growth, it is still very ambiguous.



**Table 2.5** Second and third columns show values for  $A$  and  $\hat{a} = \Delta A/A$  for Portugal economy according to production function (2.19), constant prices of 2005, data from AMECO. Values are taken from equations on Fig. 2.4

	$A$	$\Delta A/A$
1995–1996	0.331	
1996–1997	0.332	0.003
1997–1998	0.334	0.006
1998–1999	0.333	–0.003
1999–2000	0.331	–0.01
2000–2001	0.323	–0.02
2001–2002	0.317	–0.02
2002–2003	0.304	–0.04
2003–2004	0.295	–0.03
2004–2005	0.289	–0.02

## 2.6 Endogenous Technology Models

In previous neo-classic models, technology, technical progress, and knowledge are accounted for within one or two parameters ( $A$  and  $\alpha$  for the Solow model and  $A$  for the A.C model). In the Solow model, with its Cobb-Douglas production function,  $\alpha$  is an input as well as labor and capital, and  $A$  is said to be exogenous, meaning that its value cannot be computed only from inputs. The same applies to A.C models. As most authors considered  $A$  as somehow representing technology, those are known as exogenous technology models.

Romer [22] introduced a new model in which the variable that reflects knowledge and technology is a function of some of the model's inputs. This and other similar models are known by endogenous growth or endogenous technology models. He devised a model with three activity sectors: First, a sector where ideas for new goods are produced, characterized by an R&D activity, with a population of researchers  $N_A$ ; second, a sector that produces capital equipment  $C$  that results from the ideas developed in sector 1, and that does not have labor force; third, a sector that produces final consumption goods, with a labor force  $N_C$  and using the capital produced in sector 2. All goods from the first sector are used in the second and all goods from the second are used in the third. This last sector sells its products to the market. The output product of this economy is produced exclusively by the third sector.

In the final consumption goods sector, the production function used is neoclassical, with constant returns to scale. If  $C$  is the capital used by this last sector and  $Y$  is the economy output, the model's production function is (2.20), in its simplest form:

$$Y = C^\alpha \cdot (A \cdot N_C)^{1-\alpha} \quad (2.20)$$

$A$  is now an index that reflects the stock of knowledge or ideas, ideas that are generated by  $N_A$  researchers within sector 1. Researchers discover new

ideas in every period of time, such that  $A$  increases with time, with the number of researchers  $N_A$  and with  $H$ , an equally important parameter reflecting human capital, such as education years of every worker. On the other hand, each idea corresponds to a unit of capital  $x$ , starting with  $A$  ideas and growing every year. Thus, capital  $C$  becomes proportional to  $A \cdot x$  and growing as  $A$ .

Overall, the model works as a one sector neo-classical model with technological change but with an endogenous explanation of the source of technological change. This is triggered by R&D activity, a non-rival good, which immediate costs are well overcome by overall social benefits.

As in the neo-classic models, the endogenous technology models take that residual idea of everything else besides capital and labor, and name it, definitely, as technology. This technological portion is expressed as a combination of human capital, taken as a measure of workers' knowledge, with labor activity, measured as a number of workers, and also with capital formation, representing embodied knowledge. So, the idea of technology is still used overlapping the ideas of knowledge and capital but there is already a timely sequence showing how knowledge originates first more productive work, and later how this generates new forms of capital.

## 2.7 A New Linear Model

As shown above, along the twentieth century, the idea of technology became central to the process of value adding; consequently, it was progressively considered in production functions describing production processes. Labor and capital are two inputs that have been quantified explicitly, weighted differently in the production function and represented as one multiplied by the other. Technology, because it could not be defined objectively, has not been an explicit input. In fact, how could it be assigned a quantity or a value to a still ambiguous idea? The idea of technology was within a mix of everything else besides labor and capital contributing to output. That mix is said to include workers knowledge and skills, organization structure, and technical solutions. In reality, it is more than that because financial costs and operational profits also contribute positively to output (GVA).

Suppose, as a working hypothesis, that we could define technology such that its concept would point to an autonomous entity, and thus allowing a quantification and valuation of what I will call technological assets  $TA$ . Also, as a corollary, that we could objectively discriminate technological assets from capital assets  $CA$ , and finally that we would understand the differences and the borders between human knowledge, work, and technology. If we could do that, we should be able to consider, independently, three inputs to a production process: Labor, technology, and capital. In [Chap. 3](#), it will be described how these new operational concepts are constructed such that this working hypothesis is justified. For now, I propose to follow a sequence of economic events, increasingly more complete, sequences that describe production systems and their models, which will enlighten the process of

value adding when using human knowledge, work, technology, and capital contributions as inputs. This will introduce a step-by-step new production model with a linear production function, which I believe is simple, intuitive, and thorough.

### ***2.7.1 A Simple Production Model***

The simplest economic model involves only one primitive family and the surrounding Nature. The family has their own knowledge, which, in absence of any tools, comprehends only the ways and strategies to remain alive and to prosper. Naturally, families weaken without consumption. That fading away is both physical degradation and information decay. In the limit, if the family would not eat and internalize information would degrade their organic constituents, decompose, and disperse to nature. On the other hand, if the family works in accordance with her knowledge, will obtain from Nature what is needed to compensate for the natural degradation, thus keeping its ability to live and thrive.

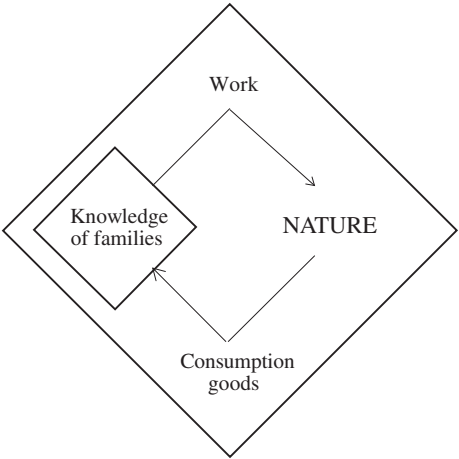
I will consider stocks and flows to explain how models work. Moreover, as I will be talking about entities with apparently different natures, it will be used a single quality for all, which is economic value. As such, there will be stocks and flows of value.

In this first model, there is a flow of knowledge value from the family to Nature, carried by action and work of the family members. The result of that work is whatever is harvested from Nature, and the product of that process, the goods produced, goes back to be consumed by the family. This is an economic cycle. If the work value equals the goods value, the family will be in equilibrium. If it is larger, the family will lower its capacity to survive; and if it is smaller the family will have the chance to grow. In this model, the only entity that can accumulate value is the family and only in the form of more knowledge. There is value accumulation when the flow in the family's direction exceeds the one in the direction of Nature. This is outlined in Fig. 2.5.

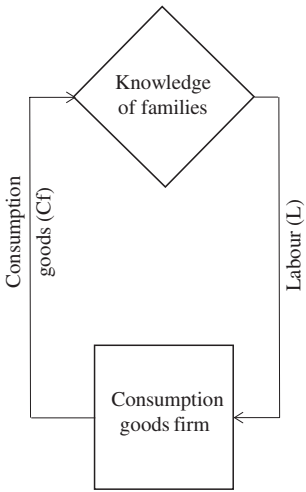
If the stock of knowledge grows, it is foreseeable that, in the same period of time and in comparable circumstances, the goods produced increase beyond the basic consumption daily needs, what will lead to an increasing stock of goods. If work consists of hunting and gathering food, the values are equal when the amount of work produces the goods needed to keep the community at the same level of knowledge, i.e. feeding and dressing enough to compensate for the natural wear and tear.

This is the reference model. However, we can now consider that there may be a surplus of food and furs. Any surplus of food and clothing is a surplus of value created, which, not being consumed, can be kept allowing future exchanges for other products of other families' surpluses. Thus, under these new circumstances, there is accumulation of value in more forms than just human knowledge. The quantity and the value of knowledge existing in the human mind cannot be directly counted, but human communities realize that who uses more efficient techniques

**Fig. 2.5** Economic model for families and Nature. *Arrows indicate value flow directions*



**Fig. 2.6** Economic model for families and consumption goods firms. Nature remains as a background. *Arrows indicate value flow directions*

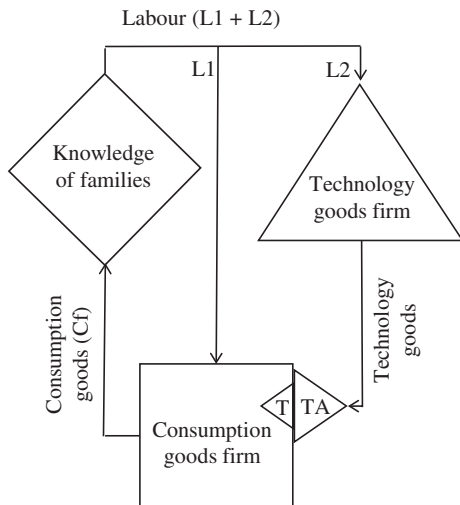


contributes to a higher value work, precisely because, in the same period of time and in comparable circumstances, can generate extra surpluses.

A similar model considers families and Nature and a new entity, which I call a consumer goods firm. This firm is a group of people with an explicit social objective: To produce specific consumer goods. In a primitive economy, such a firm can be a group of hunters, shepherds, or farmers. For simplicity, several of those groups, or firms, are here reduced to a single firm, and it is assumed that they do not have any technical tools besides their own knowledge. The production process remains as it was explained above and the model is described in Fig. 2.6.

Families contribute to firms with the work value  $L$  and the firms return to families the value of consumer goods  $C_f$ . In static equilibrium, the value flows are

**Fig. 2.7** Economic model describing value flows among families, firms producing consumption goods, and firms producing technological goods. TA is the technology goods value that is now available for consumer goods firms. T refers to the part of TA that is used by the consumer goods firm in one economic cycle



equal and the added value can be measured in either flow. The firm, by hypothesis, cannot accumulate value. Once again, the only entities that can do so are the individual families. Nature gives away, at no cost (no value flows), all the raw materials. The value added  $Y$  equals the consumption  $C_f$  and equals the work value  $L$ .

$$Y = C_f = L \quad (2.21)$$

### 2.7.2 A More Complete Model

If the goods produced are not fully consumed there will be savings. They can be used to “finance” extra work besides hunting and gathering, for example, tools manufacturing. This situation would lead to a new entity, a tools producing firm. Those new groups could be dedicated, for example, to the manufacture of tools for treating animal skins or the preparation of spears for hunting. These tools are conceived out of families’ knowledge and work experience and embodied in material forms. We will call these groups as technology firms, because they develop and produce technological assets with value TA.

A new model, described in Fig. 2.7, shows how value flows among these three entities: Families, consumption goods firms, and technology goods firms. It is assumed that both families and consumer goods firms can accumulate value, in the form of knowledge and savings and as technology assets, respectively. Again, furnished by Nature, raw materials are a *don gratuit*.<sup>5</sup>

The families work is now divided by consumer goods and technology goods firms. The labor value flows from families to these firms ( $L = L1 + L2$ ). The

<sup>5</sup> As it was said in 1758 by Quesnay [2] in his *Tableau Economique*.

technology firms deliver their technological assets value  $TA$  to consumption goods firms. In the technology firms, the labor value input is  $L_2$ , which equals the output value  $TA$ , hence  $L_2 = TA$ .

Consumption goods firms deliver their output to families, which value is  $C_f$ . However, unlike in the previous case, the consumer goods value  $C_f$  is greater than the labor value  $L_1$ .  $C_f$  equals to the sum of labor value  $L_1$  plus a value  $T$ , i.e.  $C_f = L_1 + T$ . The value  $T$  may be less or equal to  $TA$  and therefore less or equal to  $L_2$ . The remaining part of technology assets value  $(TA-T)$ , if exists, is accumulated in the consumption goods firm.

The value added by this economy  $Y$  is the value of the goods produced  $Y = C_f + TA$ . However, as  $TA = L_2$ ;  $C_f = L_1 + T$ ; and  $L = L_1 + L_2$ ; therefore, the value added by this economy is also the sum of the value added by work  $L$  and the value added by the use of technology  $T$ .

$$Y = C_f + TA = L + T \quad (2.22)$$

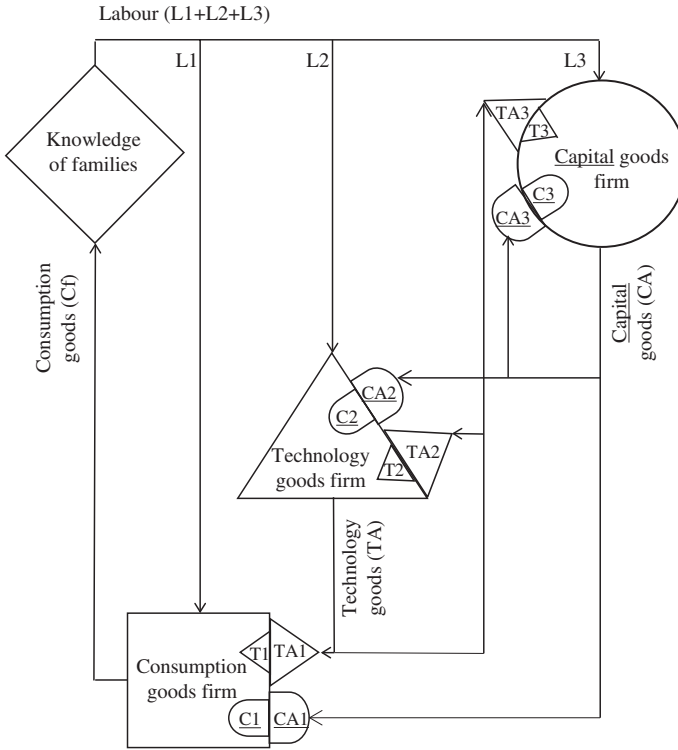
The families' balance sheet is in equilibrium if the labor value  $(L_1 + L_2)$  equals to consumer value  $C_f$ , what will happen when  $T = TA$ , in other words, when the technological goods value  $TA$  is 100 % used by the consumption goods firm. If not, the families accumulate value in the form of technological assets. In fact, they will have a positive balance sheet when  $TA > T$ , hence we could say that the economy grows. The accumulated value would be  $TA-T$ , which is stocked in the firm, firm that is the property of families.

Besides the accumulation possibility in the firm's assets, there may be consumption goods accumulation of in the households. In that case, there will be the chance to dispose of extra work for other goals. The families already have one part of the group hunting and gathering, another part manufacturing tools and other technological forms. They may want to form a third group to arrange for new and better living and manufacturing conditions. This could be for example preparing land, building barns, or arranging for barter. I will refer to this third type of activity as producing capital goods.<sup>6</sup> This new situation is described in Fig. 2.8.

Capital goods companies have at their disposal the  $TA_3$  technology assets value and the  $CA_3$  capital assets value, of which they use for their production the parts  $T_3$  and  $C_3$ . With their labor  $L_3$  and with the parts  $T_3$  and  $C_3$  they produce capital goods with value  $CA = CA_1 + CA_2 + CA_3$ . Technology goods companies have at their disposal the technology assets value  $TA_2$  and the capital assets value  $CA_2$ , of which they use the parts  $T_2$  and  $C_2$  for their production. With their labor  $L_2$  and the parts  $T_2$  and  $C_2$ , they produce technology goods of value  $TA = TA_1 + TA_2 + TA_3$ . Consumer goods companies have at their disposal the technology assets value  $TA_1$  and the capital assets value  $CA_1$ , of which they use

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<sup>6</sup> This new idea of capital is somehow different from the current idea of capital, as it differentiates from technology. As such, the current idea of capital  $C$  includes the new ideas of technology (assets)  $TA$  and the capital assets  $CA$ : such that  $C = TA + CA$ . The new (capital) is written underlined.



**Fig. 2.8** Economic model describing value flows among families, firms producing consumption goods, firms producing technological goods, and firms producing capital goods. Technology goods TA are now available for consumer goods firms, for technology goods firms and for capital goods firms. Capital goods CA are available for consumer goods firms, for technology goods firms and for capital goods firms. T refers to the part of TA that is used by the firm in one economic cycle. C refers to the part of CA that is used by the firm in one economic cycle

for their production the parts T1 and C1. With their labor L1 and the parts T1 and C1, they produce consumer goods of value  $C_f = L1 + T1 + \underline{C1}$ .

This model, even if explained with a primitive economy that was gradually completed, reflects any modern economy. The value added Y in a certain period of time is equal to the sum of the value of consumer goods  $C_f$  with the value of the technology goods TA and with the value of capital goods CA, produced in the same period of time.

$$Y = C_f + TA + \underline{CA} \quad (2.23)$$

As  $TA = L2 + T2 + \underline{C2}$ ;  $CA = L3 + T3 + \underline{C3}$ ; and  $C_f = L1 + T1 + \underline{C1}$ , whereas L groups the values of L1, L2, and L3; T groups the technology uses values T1, T2, and T3; and C groups the values of the uses of capital assets C1, C2, and C3.



Then it comes finally:

$$Y = L + T + \underline{C} \quad (2.24)$$

Based on the working hypothesis considered at the beginning of Sect. 2.6, it was shown that the value added in an economy can be calculated as the sum of the value contributions of the uses of three basic production factors: knowledge, technology (assets) TA, and capital (assets) CA. The three terms of the value added Y are respectively labor L, as the expression of knowledge value, the use value of technology T and the use value of capital C. Expression (2.24) shows a linear production function as the sum of three components, assuming that it is possible to objectively differentiate among knowledge, technology, and capital.

How to differentiate the three concepts and how to objectively quantify their independent contributions to value added is the goal of the next chapter.

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<http://www.springer.com/978-1-4471-5000-8>

The Contribution of Technology to Added Value

Fernandes, A.S.C.

2013, VIII, 104 p., Hardcover

ISBN: 978-1-4471-5000-8