Foreword

The present edition of the book differs substantially from the previous one. Over the period of time since the publication of the previous edition the author has accumulated quite a lot of ideas concerning possible improvements to some chapters of the book. In addition, some new opportunities were found for an accessible exposition of new topics that had not appeared in textbooks before but which are of certain interest for applications and reflect current trends in the development of modern probability theory. All this led to the need for one more revision of the book. As a result, many methodological changes were made and a lot of new material was added, which makes the book more logically coherent and complete. We will list here only the main changes in the order of their appearance in the text.

• Section 4.4 "Expectations of Sums of a Random Number of Random Variables" was significantly revised. New sufficient conditions for Wald's identity were added. An example is given showing that, when summands are non-identically distributed, Wald's identity can fail to hold even in the case when its right-hand side is well-defined. Later on, Theorem 11.3.2 shows that, for identically distributed summands, Wald's identity is always valid whenever its right-hand side is well-defined.

• In Sect. 6.1 a criterion of uniform integrability of random variables is constructed, which simplifies the use of this notion. For example, the criterion directly implies uniform integrability of weighted sums of uniformly integrable random variables.

• Section 7.2, which is devoted to inversion formulas, was substantially expanded and now includes assertions useful for proving integro-local theorems in Sect. 8.7.

• In Chap. 8, integro-local limit theorems for sums of identically distributed random variables were added (Sects. 8.7 and 8.8). These theorems, being substantially more precise assertions than the integral limit theorems, do not require additional conditions and play an important role in investigating large deviation probabilities in Chap. 9.

• A new chapter was written on probabilities of large deviations of sums of random variables (Chap. 9). The chapter provides a systematic and rather complete exposition of the large deviation theory both in the case where the Cramér condition (rapid decay of distributions at infinity) is satisfied and where it is not. Both integral and integro-local theorems are obtained. The large deviation principle is established.

• Assertions concerning the case of non-identically distributed random variables were added in Chap. 10 on "Renewal Processes". Among them are renewal theorems as well as the law of large numbers and the central limit theorem for renewal processes. A new section was written to present the theory of generalised renewal processes.

• An extension of the Kolmogorov strong law of large numbers to the case of non-identically distributed random variables having the first moment only was added to Chap. 11. A new subsection on the "Strong law of large numbers for generalised renewal processes" was written.

• Chapter 12 on "Random walks and factorisation identities" was substantially revised. A number of new sections were added: on finding factorisation components in explicit form, on the asymptotic properties of the distribution of the suprema of cumulated sums and generalised renewal processes, and on the distribution of the first passage time.

• In Chap. 13, devoted to Markov chains, a section on "The law of large numbers and central limit theorem for sums of random variables defined on a Markov chain" was added.

• Three new appendices (6, 7 and 8) were written. They present important auxiliary material on the following topics: "The basic properties of regularly varying functions and subexponential distributions", "Proofs of theorems on convergence to stable laws", and "Upper and lower bounds for the distributions of sums and maxima of sums of independent random variables".

As has already been noted, these are just the most significant changes; there are also many others. A lot of typos and other inaccuracies were fixed. The process of creating new typos and misprints in the course of one's work on a book is random and can be well described mathematically by the Poisson process (for the definition of Poisson processes, see Chaps 10 and 19). An important characteristic of the quality of a book is the intensity of this process. Unfortunately, I am afraid that in the two previous editions (1999 and 2003) this intensity perhaps exceeded a certain acceptable level. Not renouncing his own responsibility, the author still admits that this may be due, to some extent, to the fact that the publication of these editions took place at the time of a certain decline of the publishing industry in Russia related to the general state of the economy at that time (in the 1972, 1976 and 1986 editions there were much fewer such defects).

Before starting to work on the new edition, I asked my colleagues from our laboratory at the Sobolev Institute of Mathematics and from the Chair of Probability Theory and Mathematical Statistics at Novosibirsk State University to prepare lists of any typos and other inaccuracies they had spotted in the book, as well as suggested improvements of exposition. I am very grateful to everyone who provided me with such information. I would like to express special thanks to I.S. Borisov, V.I. Lotov, A.A. Mogul'sky and S.G. Foss, who also offered a number of methodological improvements.

I am also deeply grateful to T.V. Belyaeva for her invaluable assistance in typesetting the book with its numerous changes. Without that help, the work on the new edition would have been much more difficult.

A.A. Borovkov

Foreword to the Third and Fourth Editions

This book has been written on the basis of the Russian version (1986) published by "Nauka" Publishers in Moscow. A number of sections have been substantially revised and several new chapters have been introduced. The author has striven to provide a complete and logical exposition and simpler and more illustrative proofs. The 1986 text was preceded by two earlier editions (1972 and 1976). The first one appeared as an extended version of lecture notes of the course the author taught at the Department of Mechanics and Mathematics of Novosibirsk State University. Each new edition responded to comments by the readers and was completed with new sections which made the exposition more unified and complete.

The readers are assumed to be familiar with a traditional calculus course. They would also benefit from knowing elements of measure theory and, in particular, the notion of integral with respect to a measure on an arbitrary space and its basic properties. However, provided they are prepared to use a less general version of some of the assertions, this lack of additional knowledge will not hinder the reader from successfully mastering the material. It is also possible for the reader to avoid such complications completely by reading the respective Appendices (located at the end of the book) which contain all the necessary results.

The first ten chapters of the book are devoted to the basics of probability theory (including the main limit theorems for cumulative sums of random variables), and it is best to read them in succession. The remaining chapters deal with more specific parts of the theory of probability and could be divided into two blocks: random processes in discrete time (or random sequences, Chaps. 12 and 14–16) and random processes in continuous time (Chaps. 17–21).

There are also chapters which remain outside the mainstream of the text as indicated above. These include Chap. 11 "Factorisation Identities". The chapter not only contains a series of very useful probabilistic results, but also displays interesting relationships between problems on random walks in the presence of boundaries and boundary problems of complex analysis. Chapter 13 "Information and Entropy" and Chap. 19 "Functional Limit Theorems" also deviate from the mainstream. The former deals with problems closely related to probability theory but very rarely treated in texts on the discipline. The latter presents limit theorems for the convergence of processes generated by cumulative sums of random variables to the Wiener and Poisson processes; as a consequence, the law of the iterated logarithm is established in that chapter.

The book has incorporated a number of methodological improvements. Some parts of it are devoted to subjects to be covered in a textbook for the first time (for example, Chap. 16 on stochastic recursive sequences playing an important role in applications).

The book can serve as a basis for third year courses for students with a reasonable mathematical background, and also for postgraduates. A one-semester (or two-trimester) course on probability theory might consist (there could be many variants) of the following parts: Chaps. 1–2, Sects. 3.1–3.4, 4.1–4.6 (partially), 5.2 and 5.4 (partially), 6.1–6.3 (partially), 7.1, 7.2, 7.4–7.6, 8.1–8.2 and 8.4 (partially), 10.1, 10.3, and the main results of Chap. 12.

For a more detailed exposition of some aspects of Probability Theory and the Theory of Random Processes, see for example [2, 10, 12–14, 26, 31].

While working on the different versions of the book, I received advice and help from many of my colleagues and friends. I am grateful to Yu.V. Prokhorov, V.V. Petrov and B.A. Rogozin for their numerous useful comments which helped to improve the first variant of the book. I am deeply indebted to A.N. Kolmogorov whose remarks and valuable recommendations, especially of methodological character, contributed to improvements in the second version of the book. In regard to the second and third versions, I am again thankful to V.V Petrov who gave me his comments, and to P. Franken, with whom I had a lot of useful discussions while the book was translated into German.

In conclusion I want to express my sincere gratitude to V.V. Yurinskii, A.I. Sakhanenko, K.A. Borovkov, and other colleagues of mine who also gave me their comments on the manuscript. I would also like to express my gratitude to all those who contributed, in one way or another, to the preparation and improvement of the book.

A.A. Borovkov

http://www.springer.com/978-1-4471-5200-2

Probability Theory
Borovkov, A.A.
2013, XXVIII, 733 p. 22 illus., Softcover
ISBN: 978-1-4471-5200-2