# Revisiting and Generalizing Barkhausen's Equality

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**Abstract** A study of the conditions for sustaining signals in a loop shows that loop equations are essentially fixed-point equations over a space of functions, with the loop performing a mapping on that space of functions. When the space of functions is specified, one can derive particular conditions for the loop has a solution. Barkhausen conditions fall in this category. Loops composed of two subsystems are in the first place analyzed. The purpose of the chapter is to put into a general perspective the problems of loops, showings the general conditions that must be satisfied. The analysis aims to clarify several perspectives on and the framework of loops operation.

## 1 Introduction

This chapter is motivated in the first place by the intuitive nature and generality of the concept of loop, and by the numerous applications of this concept. In the second place, the motivation relates to the richness of results that may be derived from the basic condition of signals sustained by a loop, which is essentially an identity.

Probably the most favored method of approaching today loop-like problems is to write, for the subsystems in the loop, their differential equations, taking into account that the output of a system is the input to the next one. Then, the corresponding system of differential equations is solved, deriving the loop solutions.

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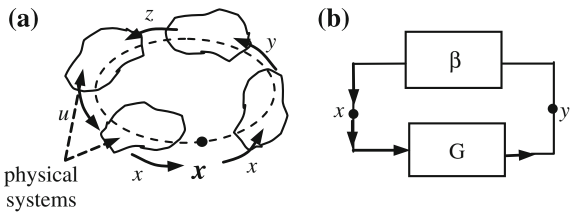


**Fig. 1** **a** Loop with a serried set of physical systems; each system output is the input to the subsequent one. **b** A two-system standard loop

We are interested in this chapter in another point of view. Namely, for a given loop composed of several subsystems, supposing that the loop has a non-trivial solution, we ask what conditions the subsystems do necessarily obey. While this problem is more general than that partly addressed by Barkhausen, it seems to be in line with Barkhausen's one.

Interestingly, it seems that sometimes the problem of existence of solution of a loop is confused with that of the stability of the solution; the confusion translates into the name often given to Barkhausen criterion indicating a stability criterion, not an existence one.

The concept of loop emerges in the physical and structural representation of sets of coupled equations whose variables are produced by individualized inter-dependent systems (Fig. 1a). Loops are system configurations used in numerous engineering fields. Control loops also appear in all natural sciences as a result of the coupling of equations in subsystems and are recently used to explain cellular and molecular "clocks". In economy and social sciences, loops are used to represent regulatory systems.

In many engineering applications, the equations representing systems take a simple multiplicative form, $y = G_1 \cdot x$, $z = G_2 \cdot y$, … For this type of systems, Barkhausen established a condition on the systems in a loop for the loop "accepts" a specified signal on it. The condition is a necessary one, not caring about the solution of the loop, i.e., on the signals in the loop. Barkhausen's conditions had reverberations in circuit theory in electronics and in control for several decades. They preserve some interest in relation with large systems, biologic models, and similar topics where loops occur. A longstanding academic tradition includes Barkhausen's condition in the introductory lectures on oscillators, but the interest in these conditions waned in scientific research during the recent decades.

Recently, Barkhausen's condition has attracted revived interest, along with some criticism [1–6]. He and coauthors wrote [1] "For too long a time, the Barkhausen criterion has been widely used as a condition of oscillation for the harmonic feedback oscillator. It is intuitive and simple to use. However it is wrong, and cannot give the real characteristics of an oscillator such as the startup condition and the oscillation frequency." In another direction, Singh [3, 4] shows that the supra-unitary Barkhausen condition, $G\beta > 1$ is not, contrary to the popular belief reflected in some textbooks, the condition for the starting up of oscillations; instead, Singh proves on a system example that the condition $G\beta < 1$ must hold. In a later paper, he proved for harmonic oscillators that the setting up of oscillations in a loop can occur in some specific cases if $G\beta < 1$, while in some other cases it occurs when $G\beta > 1$. Singh discuss the issue in the framework of Nyquist stability condition.

While Barkhausen's condition(s) of oscillation is a strong instrument, these conditions are offering only a partial answer to the problem of sustained signals in a loop. They do not give enough information on the signals in nonlinear loops, on the stability of the solutions, and on the starting up of such signals. For a discussion on the original Barkhausen identity, see the Appendix.

Simple feedback loops essentially represent an identity equation in the form $x \equiv x$, where $\equiv$ denotes the identity relation. The identity refers to any point in the loop, where literally the left part of the identity refers to the loop physical path from that point leftward, while the right part "sees" the loop rightward (Fig. 1a).

Sometimes, Barkhausen equality (frequently written as $G\beta = 1$or $A\beta = 1$) is confused with a criterion of stability of oscillations, or with a criterion for starting oscillations (see [6]), which Barkhausen criterion is not. Even worse, in some technical documentation, Barkhausen equality is replaced by an inequality. For example, in an older application note [7], it is written "For oscillation to occur, the Barkhausen criteria must be met: (1) The loop gain must be greater than one. …." Also, the above condition is not valid except for a subset of loops, including linear loops. Two more general criteria for loop solution existence are discussed, namely the loop identity condition (LIC) and the loop differential condition (LDC).

The chapter structure is as follows. In the next section, we present the general settings and introduce several notations and term definitions. In Sect. 2, the general setting is presented. Section 3 is devoted to loops depending on the derivatives of time-dependent functions. Conclusions and a short annex on the work of Barkhausen end the chapter.

## 2 General Setting and Non-Differential Loops

In this section, we set the loop problem framework. Two concepts are necessary for the discussion. The first is the concept of loop. The second is the notion of loop solution. In the first part of this section we will be less formal and precise, to connect to the general intuitive understanding of the topic in applications.

The loops are formally described by a system of (coupled) equations in the form $\{y = f_1(x);\ z = f_2(y);\ \ldots;\ x = f_n(u)\}$, or, more generally, $\{y = f_1(x, \dot{x}, \ddot{x}, \cdots), \cdots\}$, when the systems operate in continuous time and depend on the derivatives of the input. It is essential to recall that in this chapter $x, y, z, \ldots$ denote functions, not necessarily real valued, from a given space of functions, while $f_1$, …, the subsystems are seen as mappings from a space of functions to itself. This point of view is justified by the fact that we are interested in signals supported by the loops, not by constant values (equilibrium points) of the loops.

Throughout this chapter, one assumes that all the functions of the subsystems and the signals are derivable in the domain of interest, when time is continuous. The states of the system are not considered.

## 2.1 Terminology

A solution of the loop is defined as follows. First, reduce the loop to a single function, the function overall performed by all the systems in the loop, by combining all systems in one, $f_{loop}$. Assume that $f_{loop}$ has a single variable, $x$. Then, a $v$ variable dependent solution of the loop is a $v$-dependent function that is a fixed point of $f_{loop}$,

$$x(v) = f_{loop}(x(v)). \tag{1}$$

A time-dependent solution of the loop is a time-dependent function, named also signal (in the time domain), $x(t)$, which is a fixed point of $f_{loop}$. Assuming that the loop function does not depend on the derivatives of $x(t)$, then, a solution of the loop satisfies

$$x(t) = f_{loop}(x(t)). \tag{1$'$}$$

Equation (1) should be valid whatever is the variable of $x$, that is, whatever is the representation of $x$. For example, if $x$ is a Fourier representation of a time-domain signal, then the loop is represented by a mapping $F_{loop}$ such that $x(\omega) = F_{loop}(x(\omega))$, where $x(\omega)$ is the complex spectrum. Consider that the loop is composed of two systems performing multiplicatively, $y(x) = x(\omega)G(\omega)$, $x(y) = y(\omega)H(\omega)$, with $F_{loop}(x) = x(\omega)G(\omega)H(\omega)$, that is, $F_{loop}$ is linear in the input $x$, while $H, G$ are independent of $x$. Then, from $x(\omega) = F_{loop}(x(\omega))$ it results that, for all $\omega$where $x(\omega) \neq 0$, $G(\omega)H(\omega) = 1$. One may view the subsystems in the loop having the functions $\frac{dy(x)}{dx} = G, \frac{dx(y)}{dy} = H$, with $HG = 1$ ($y_x^{'}x_y^{'} = 1$.)

One can say about (1) that a loop is a mapping from the set of $v$-dependent functions to itself, and a solution of the loop is a fixed point of the mapping. A solution will be named trivial when $x(v) = constant$. Following Barkhausen and the literature, we are interested in the loops that can be written, for practical reasons, as $f_{loop} = f \circ g$, and in the conditions that relate the two mappings, $f$ and $g$, under specified general conditions, for the loop to have non-trivial solutions. For practical reasons, having physical loops in mind, a loop that allows the trivial solution $x(t) \equiv 0$ is named passive loop, or equilibrium-able loop. For example, a loop consisting of amplifiers and filters is an equilibrium-able loop when the amplifiers have no offset.

While the loop condition $x = L(x)$ is nothing but a fixed point definition for the "signal" in the loop, it resides on the physical trivial identity $x(t) = x(t)$. We know the general conditions for a fixed-point problem to have solutions, according to theorems in the class of theorems developed along Brouwer, Kakutani, and Schauder theorems. However, in this chapter we are interested in less general problems, related to loops composed of one or two subsystems. The focus is on the conditions satisfied by the subsystems, as implied by the hypothesis of non-trivial solutions of the loop general equations.

Throughout this chapter, we will say that a function $x(v)$ is accepted (supported) by the loop when it is a solution of the loop equation, that is, the fixed point condition (1). We will consider loops with at most two serried systems. In general, we will assume that the systems are intrinsically time-invariant.

In the case of loops consisting of two systems, assume $x = g(f(x)) = g(y)$ and $y = f(g(y)) = f(x)$. This formally identical (interchangeable) role of $f$ and $g$explains many of the symmetric results derived in this chapter.

The number of elements (subsystems) in the loop is irrelevant, because the string of elements in the loop can be seen as a single element. Also, the loop identity must hold on any loop in a graph, in a multi-loop configuration system where appropriate (Kirchhoff-like) conditions are added. The simplest case of loops we deal with is that of linear systems in the loop, $y = kx$.

The systems in the loop may produce an output depending on one or several variables, including the input, the derivatives of the input, and one or several parameters, $y = y(x, \dot{x}, \ddot{x}, \lambda_1, \cdots, \lambda_p)$. We need to consider these cases one by one.

Notice that loop Eq. (1) does not refer to steady state or transitory regimes specifically—it is just a translation for a loop of an identity condition. As such, it can be applied to steady state oscillations in nonlinear systems.

## 2.2 Basic Conditions

Consider a loop with the overall function denoted by $L$ and dependent only on the input, not on the input derivatives with respect to time, $L = L(x(v))$, where $v$ represents some variable in the representation space of $x$. For example, $v$ can stand for $\omega$ in the Fourier representation, or it may be some other suitable variable in another representation. Assume that both $L$ and $x$ have derivatives. By differentiation of the fixed point condition, we obtain that, at any fixed point $x^*$ of the loop,

$$\frac{\partial L}{\partial x}(x^*) \cdot \frac{dx^*}{dv} = \frac{dx^*}{dv}. \tag{2}$$

When $\frac{dx^*}{dv} \neq 0$, the above implies that

$$\frac{\partial L}{\partial x}(x^*) = 1 \tag{3}$$

for any function $x^*(v)$ that is a loop solution; in technical terms, this means that the "amplification" (slope) of the loop must be equal to 1 at $x^*$, for $x^*$ is a solution. A stability condition must be added, for the solution to be stable.

The condition (1), $x = L(x)$ will be named *loop identity condition* (LIC), while the equality (3) $1 = \partial L(x)/\partial x|_{x^*}$ will be named the *loop differential condition* (LDC).

Subsequently, we provide the formal general setting of the loop problem.

Consider a space of functions over the set $U$, with values in $\boldsymbol{R}$ or $\boldsymbol{C}$. Denote these functions as $x(v), y(v), \ldots, v \in U$. The set of these functions is denoted by $\Xi$. We assume that $U$, $\Xi$ are endowed with metrics and that derivation is defined for mappings from $\Xi$ to itself. Consider three operators, $L, G, H : \Xi \rightarrow \Xi$, all of them derivable at the fixed points of $L$. Assume that $L$ has at least one fixed point, $x^*(v)$. Also assume that on $\Xi$, $\frac{dx}{dv} = 0$ only when $x(v) \neq c \in \mathbf{R}(\boldsymbol{C})\forall v$. Then,

*Definition*. An operator $L$ that satisfies the condition $L = HG$ is named loop. A fixed point of the loop is named loop solution.

A loop solution is non-trivial when $x(v) \neq c \in \mathbf{R}(\boldsymbol{C})\forall v$. In the next Property, one assumes that derivatives are defined in some way (e.g., Fréchet differentiable, or in the sense of Alexandroff, or as in Sobolev spaces); for various definitions of derivatives, see [8–11]. One also assumes that the typical properties of the derivatives, as the derivative of a composed function and the chain rule hold. In the next property, $x$ is a function, for example a time-domain function, or the Fourier spectrum of a time domain function; $x^*$ is a solution of the loop.

**Property 1** *If a loop with two systems has a non-trivial solution $x^*$, then*

(1) $HoG(x^*) = \mathrm{Id}(x^*)$, where Id is the identity application on $\Xi$.
(2) When $x$ is real valued, $\frac{dL}{dx}(x^*) = 1$, where 1 denotes the unity in $\mathbf{R}$.
(3) $\frac{dG}{dy}(y)\frac{dH}{dx}(x^*) = 1$.
(4) If the loop is twice derivable, then $\frac{d^2L}{dx^2}(x^*) = 0$.
(5) If $G$ is invertible, then, $H(x^*) = G^{-1}(x^*)$.
(6) If $x$ is a derivable function in two variables, $x(v, w)$, then the loops solution satisfies the conditions $\frac{dH}{dy}\frac{dG}{dx}\frac{\partial x}{\partial v} = \frac{\partial x}{\partial v}$ and $\frac{dH}{dy}\frac{dG}{dx}\frac{\partial x}{\partial w} = \frac{\partial x}{\partial w}$ at the point (function) $x = x^*$.
(7) If the variable can be written as a linear decomposition $x(v) = \sum_k a_k x_k(v)$, $x$ real valued, with all functions $x_k$ derivable with respect to all $x_j$, $a_k \in \mathbf{R}$, then the solution of the loop satisfies the set of conditions $\frac{dH}{dy}\frac{dG}{dx}\frac{\partial x}{\partial x_j} = a_j + \sum_{k \neq j} a_k \frac{\partial x_k}{\partial x_j}$.

The above property directly results from the definitions. Conditions (1) and (2) resembles the typical form of Barkhausen condition written in the frequency domain, $H(\omega)G(\omega) = 1$; in fact, (1) is the general form of that condition.

We next give a negative result. Consider that "signal" means a real-valued, time-dependent function, and $x(t)$ is a solution of a loop, then for all values of $x(t) \in \mathbf{R}$, the loop identity $x(t) = L(x(t))$ holds. Suppose that $L(x(t))$ is a function only in $x$, not in its derivatives. But this implies that, in the range of $x(t)$, $L$ coincides with the identity operator $I(c) = c$, $c \in \mathbf{R}$, which satisfies (3). Then,

**Property 2** *A loop that depends exclusively on the value of the real-valued function $x(t)$, not on its derivatives, either can support only trivial solutions, $x(t) = constant$ ($\frac{dx}{dt} = 0$), or is an identity loop.*

*Example 1* Consider a loop with the function $L(x) = x^2$. It supports only two constant signals, $x = 0$ and $x = 1$.

*Example 2* Assume that a transform, for example Fourier transform, is applicable to (1), and that (1) becomes

$$X(v) = F(v)X(v),$$

where $X(v)$ is the transform of $x$ and $F(v)$ is the transform of $L$. When in (2) the loop corresponds to two serried systems with functions $G$ and $H$, from the above we obtain $F(v) = G(v)H(v) = 1$, one of the first Barkhausen condition for linear systems. This example refers to linear loops, which allow us to apply the Fourier transform.

The LIC criterion also implies that: If a steady-state signal appears in the loop, and the loop has a delay, $\tau$, independent on the value of the signal and on its derivatives, then the solution satisfies the condition $x(t) = x(t + \tau)$, meaning that the loop can support only periodical signals, moreover, the delay must be a multiple of the signal period. When the delay is dependent on the signal and its derivatives, we obtain the condition

$$x(t) = x(t + \tau(x, \dot{x}, \ddot{x}, \ldots)) \tag{4}$$

which is a supplementary condition (differential equation) to be satisfied by the fixed points of the loop.

Assuming that the loop is composed of two systems, characterized by two function $y(v) = f(x(v))$, $x(v) = g(y(v))$, then $x = (g \circ f)(x) = g(f(x))$. Further assuming that all functions are differentiable, we obtain,

$$\frac{dx}{dv} = \frac{dg}{dy}\frac{df}{dx}\frac{dx}{dv}.$$

Whenever $dx/dv \neq 0$, it follows that

$$\frac{dg}{dy}\frac{df}{dx} = 1. \tag{5}$$

This condition represents, for loops with two serried systems with differentiable functions $f, g$, both depending only in the input variable, not on its derivatives, the condition that the functions are one the inverse of the other. This could be named a *loop differential identity condition* (LDIC).

When the two functions are linear in the respective variables, the first Barkhausen condition is obtained, $A\beta = 1$, where $A = \partial f/\partial x$, $\beta = \partial g/\partial x$. We assumed above that $x$ is real and $f, g$ are real-valued functions of real variables. Whenever the function $g$ is invertible on the range of $f(x)$, the loop has a solution if the system satisfies the condition $f = g^{-1}$, which is an obvious generalization of the Barkhausen equality $A = 1/\beta$.

The differential equation $\frac{\partial g}{\partial y} \cdot \frac{\partial f}{\partial x} = 1$ does not need to be satisfied when the solution of the loop is trivial, that is when $x$ is a constant value with respect to the variable $v$; in that case, because in that case $dx/dv = 0$ maintains the differential equality above always true, independent of $g$ and $f$.

Recall that a solution of the loop fixed point equation may be stable or unstable according to the absolute value of the derivative of $g(f(.))$ in a vicinity of the solution, with a smaller than 1 absolute value of the derivative ensuring the stability. In the next section, we deal with general conditions for time variable solutions of the loop.

Assume that we look for solutions of a loop that are functions of time (such functions are named here "signals"). The next remark provides a sufficient condition for solutions of this type of loop.

*Remark 1* Denote by $f, g$ the two functions and assume $g$ is invertible for some range of $y$. Then, if the function $g$ is invertible, the loop identity $x = (g \circ f)(x)$ is satisfied by $f(x) = g^{-1}(y)$ for all $y$ in the given range.

A loop with two systems, both with functions depending only on the values of the signals (not of their derivatives) has a solution if (i) one of the two systems composing the loop has an invertible function on a domain of values of the function of the other system and (ii) the inverse function is equal to the function of the first system.

The above remark provides the principle for a method to build loops having a given solution, $x^*(t)$. Choose a function $g$ which is invertible on the range of $x^*(t)$. Then complete the loop with a function $f = g^{-1}$ on $\{g(x^*(t)) | t \in \mathbf{R}\}$.

## 3 Systems Depending on the Derivatives of the Signal

Consider the loop $L = L(x(v))$ and the loop condition $x(v) = L\big(x(t), x^{(1)}(t), \cdots, x^{(n)}(t)\big)$ for a time-dependent function $x(t)$. An equivalent, $n$-th order differential equation is readily obtained as

$$L\Big(x(t), x^{(1)}(t), \cdots, x^{(n)}(t)\Big) - x(t) = 0.$$

Assuming that all involved functions are derivable, taking once again the derivative as in (2), one obtains the equation representing the loop as

$$\frac{dx}{dt} = \frac{d}{dt}(L(x(t))) = \frac{dL}{dx^{(n)}} \frac{dx^{(n)}}{dt} + \cdots + \frac{dL}{dx} \frac{dx}{dt}.$$

Conversely, any differential equation $E\left(x^{(n)}, \cdots, x\right) = 0$, involving the derivatives of a function $x(t)$ can be seen as a loop; for that, take $L(x(t)) = E + x(t)$. The interesting situation is when the loop is not trivial, in the sense that it includes two subsystems, $L = H(G(x))$. Then, the equation writes $E = H\left(G\left(x, x^{(1)}(t), \cdots\right)\right) - x = 0$.

*Example 1* Consider the second order differential equation $\ddot{x} + \gamma\dot{x} + \lambda x = 0$. We wish to model it using a loop conveniently designed. Let $y = f(\dot{x}, x) = a\dot{x} + bx$, $g(y) = \alpha\dot{y} + \beta y$. The loop equation becomes then $g(y) = \alpha(a\dot{x} + bx)' + \beta(a\dot{x} + bx) = x$, or $\alpha a\ddot{x} + (\alpha b + \beta a)\dot{x} + \beta bx = x$, with $\alpha a = 1$, $\alpha b + \beta a = \gamma$, and $\beta b - 1 = \lambda$.

Consider a loop described by the function depending on the input derivatives, $x = L(x, \dot{x}, \ddot{x})$, assuming that $x(t)$ is derivable. We assume here that the loop systems have as variables the input signals and their first derivative, with $G(x, \dot{x})$ and $H(y, \dot{y})$ functions in the input variable and its derivative, where $y = G(x, \dot{x})$. It is easy to see that the loop depends on the second derivative of $x$, as in the Example 1.

The use of two systems is equivalent to using the d'Alembert decomposition of second order differential equations into first-order equations (see [8], p. 5). The loop identity (Barkhausen equality) has the form $x = \beta(G(x, \dot{x}), \dot{G}(x, \dot{x}))$.

Differentiating the equality

$$\frac{dx}{dt} = \frac{\partial\beta}{\partial u} \cdot \left(\frac{\partial G}{\partial x}\frac{dx}{dt} + \frac{\partial G}{\partial \dot{x}}\frac{d\dot{x}}{dt}\right) + \frac{\partial\beta}{\partial v} \cdot \left(\frac{\partial \dot{G}}{\partial x}\frac{dx}{dt} + \frac{\partial \dot{G}}{\partial \dot{x}}\frac{d\dot{x}}{dt}\right) \tag{6}$$

we obtain, assuming that all functions are derivable in $x$, and $x$ has its $n+1$ derivative,

$$\frac{dx}{dt} = \left(\frac{\partial\beta}{\partial u}\frac{\partial G}{\partial x} + \frac{\partial\beta}{\partial v}\frac{\partial \dot{G}}{\partial x}\right)\frac{dx}{dt} + \left(\frac{\partial\beta}{\partial u}\frac{\partial G}{\partial \dot{x}} + \frac{\partial\beta}{\partial v} \cdot \frac{\partial \dot{G}}{\partial \dot{x}}\right)\frac{d\dot{x}}{dt}. \tag{7}$$

This is a $(2+r)$ order differential equation $x''\left(\frac{\partial H}{\partial y}\frac{\partial G}{\partial x} + \frac{\partial H}{\partial y'}\frac{\partial G'}{\partial x'}\right) + x'\left(\frac{\partial H}{\partial y}\frac{\partial G}{\partial x} + \frac{\partial H}{\partial y'}\frac{\partial G'}{\partial x} - 1\right) = 0$, where $r$ is the highest order of the derivative of $x$ in the parentheses.

Suppose that we require a solution $x(t) = \cos(\omega t)$. Then, $x'$ and $x''$ are independent and the above equality can hold only if

$$\frac{\partial H}{\partial y}\frac{\partial G}{\partial x} + \frac{\partial H}{\partial y'}\frac{\partial G'}{\partial x} = 1, \tag{8}$$

$$\frac{\partial H}{\partial y}\frac{\partial G}{\partial x} + \frac{\partial H}{\partial y'}\frac{\partial G'}{\partial x'} = 0. \tag{9}$$

The above are generalizations of the Barkhausen conditions—compare the first with $\frac{\partial H}{\partial y}\frac{\partial G}{\partial x} = 1$. We conclude:

**Property 3** *When $\dot{x}$ and $\ddot{x}$ are independent in a second or larger order differential loop, (8) and (9) hold.*

The conditions (8) and (9) extend Barkhausen equality for differential loops of order at least 2.

*Example 2* Consider the second order differential equation $\ddot{x} + \gamma\dot{x} + \lambda x = 0$. It can be modeled using a loop conveniently designed. Let $y = G(x, \dot{x}) = a\dot{x} + by$, $x = H(y, \dot{y}) = \alpha\dot{y} + \beta y$. The loop equation becomes then $g(y) = \alpha(a\dot{x} + bx)' + \beta(a\dot{x} + bx) = x$, or $\alpha a\ddot{x} + (\alpha b + \beta a)\dot{x} + \beta bx = x$.

Replacing $a = \partial G/\partial\dot{x}$, $b = \partial G/\partial x$, $\alpha = \partial H/\partial\dot{y}$, $\beta = \partial H/\partial y$, and assuming that $\dot{x}$ and $\ddot{x}$ are independent, the condition (9) is satisfied when $\alpha b + \beta a = 0$. The last condition is known to mean that the loop is without attenuation—allowing the loop to sustain the oscillation.

## 4 Conclusions

The broad applicability of loops in technical systems ranging from oscillators to control applications requires a solid understanding of loops in the broader context of mappings on sets of functions and of fixed point theorems for such mappings. According to the application domain, one is interested to have trivial solutions of the loop, that is, non-oscillations, as in control, or non-trivial solutions, electronic as in oscillators. Barkhausen, and this chapter, adopted the latter point of view, emphasizing the “oscillation conditions.”

The requirement for the loop to consist of at least two subsystems (two mappings and the related mapping composition) brings specificities to the properties of the solutions. Whenever the mappings are derivable (in a weak, general sense), assuming the derivatives preserve some properties like chain property (which is true for the weak derivative in some spaces), the contour of stronger properties of the solutions appear, as shown in the properties in this chapter. Some of these properties can be seen as general conditions for a non-trivial fixed point exists, extending the type of conditions currently called with Barkhausen’s name. A few examples completed the general perspective proposed in this chapter.

While there is no true novelty in the results presented in this chapter, because all are only facets and generalizations of well-known facts like the condition of fixed point, the differential conditions for accepted signals on loops clarify some cases of loops of practical importance. We analyzed differential loops and determined conditions of fixed point of such loops. The cases of differential loops described by equations as $x = L(x, \dot{x}, \ddot{x})$ show the entire power of the loop concept and of fixed point conditions. We believe that the terms “inverse function” and "fixed point" should not lack from any textbook dealing with loops and oscillations.

**N.B.** Several serious errors appearing in the brief paper (H.N. Teodorescu, Revisiting Barkhausen Conditions, ECAI 2011 Conf., Pitesti, (Rev. Univ. Pitesti, Series Electronics, Computers, and Artificial Intelligence, vol. 4, nr. 1, 2011, pp. 7–10) have been corrected in this chapter.

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## Appendix

**Historical note.** Barkhausen has proposed what is better named but less known as Barkhausen *equation*, written by him as SDRi = 1, which translates to $G\beta = 1$ or $HG = 1$ in the notations used in this chapter. Barkhausen was also interested in the starting of oscillations (we name the topic today "stability of oscillations"), as proved by the manuscript pictured in [Eugen-Georg Woschni, The Life-Work of Heinrich Barkhausen http://www2.mst.ei.tum.de/ahmt/publ/symp/2004/2004_075.pdf]. The oscillation problem has been the subject of Brakhausen's doctoral thesis, Das Problem Der Schwingungserzeugung: MitBesondererBerucksichtigungSchnellerElektrischerSchwingungen, published in 1907. According to the above quoted article on his life, Barkhausen equation first appeared in a book he published in 1917. More on Barkhausen's life at http://de.wikipedia.org/wiki/Heinrich_Barkhausen. Publications by Barkhausen are listed in the Katalog der DeutschenNationalbibliothek, at https://portal.d-nb.de/opac.htm?query=Woe%3D118657240&method=simpleSearch.

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