

## Chapter 2

# Higher-Order Risk Attitudes

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**Abstract** Risk aversion has long played a key role in examining decision making under uncertainty. But we now know that prudence, temperance, and other higher-order risk attitudes also play vital roles in examining such decisions. In this chapter, we examine the theory of these higher-order risk attitudes and show how they entail a preference for combining “good” outcomes with “bad” outcomes. We also show their relevance for non-hedging types of risk-management strategies, such as precautionary saving. Although higher-order attitudes are not identical to preferences over moments of a statistical distribution, we show how they are consistent with such preferences. We also discuss how higher-order risk attitudes might be applied in insurance models.

**Keywords** Hedging • Expected utility • Precautionary motives • Prudence • Risk • Temperance

### 2.1 Introduction

Ever since Daniel Bernoulli (1738), risk aversion has played a key role in examining decision making under uncertainty. Within an expected-utility framework, this property corresponds to the simple feature that the utility function is concave. Although somewhat newer, the higher-order risk attitude of “prudence” and its relationship to precautionary savings also has become a common and accepted assumption. The term “prudence” was coined by Kimball (1990), although its importance in determining a precautionary savings demand was noted much earlier by Leland (1968) and Sandmo (1970). Indeed, Kimball’s (1990) analysis is compelling, in part, due to the way he extends the “logic” of risk aversion to a higher order. Since then, numerous empirical contributions have used prudence to test for a precautionary demand for saving.

Risk aversion is defined in several different ways. Some, assuming an expected-utility framework, might say that the von Neumann–Morgenstern utility function  $u$  is concave. Others might define risk aversion in a more general setting, equating it to an aversion to mean-preserving spreads, as defined

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by [Rothschild and Stiglitz \(1970\)](#). Such a definition allows the concept of risk aversion to be applied in a broader array of settings, not confined within expected utility. It also helps to obtain a deeper understanding of the concept, even within expected utility.

Ask someone to define what it means for the individual to be “prudent” and they might say that marginal utility is convex ( $u''' > 0$ ) as defined in [Kimball \(1990\)](#), but they also might define prudence via behavioral characteristics. For example, [Gollier \(2001, p. 236\)](#) defines an agent as prudent “if adding an uninsurable zero-mean risk to his future wealth raises his optimal saving.” Interestingly, prudence was defined by Kimball in order to address the issue of precautionary saving. But such characterizations necessarily introduce aspects of particular decision problems into definitions of risk attitudes. They also are typically derived within a specific type of valuation model, most commonly expected utility. In this chapter, we describe an alternative approach to defining higher-order risk attitudes, such as prudence. Since our definitions are perfectly congruous to those based within expected utility, it helps to give a deeper understanding of their application to risk-management decisions.

In an expected-utility framework, it is interesting to note that an assumption of a third derivative of utility being positive was often seen as “more severe” than assuming the generally accepted property of decreasing absolute risk aversion (DARA)—even though the latter assumption is stricter mathematically. Indeed, the early articles of [Leland \(1968\)](#) and [Sandmo \(1970\)](#) both point out how  $u''' > 0$  will lead to a precautionary demand for saving. But assumptions about derivatives seemed rather ad hoc and technical at that time. Both of these authors pointed out that DARA, whose intuition had already been discussed in the literature, is sufficient to obtain a precautionary demand for saving.

Although it predates [Kimball \(1990\)](#), the concept of “downside risk aversion” as defined by [Menezes et al. \(1980\)](#), which we now know is equivalent to prudence, helps in our understanding. A pure increase in “downside risk” does not change the mean or the variance of a risky wealth prospect, but it does decrease the skewness. More generally, prudence plays an important role in the tradeoff between risk and skewness for economic decisions made under uncertainty, as shown by [Chiu \(2005\)](#). Hence, prudence (downside risk aversion) can be quite important for empirical economists, wanting to measure such tradeoffs.

A lesser known higher-order risk attitude affecting behavior towards risk is *temperance*, a term also coined by [Kimball \(1992\)](#). [Gollier and Pratt \(1996\)](#) and [Eeckhoudt et al. \(1996\)](#) show how temperance plays an important role in decision making in the presence of an exogenous background risk. As was the case with prudence, first notions of temperance relied upon its application to certain decision problems, and they were also explained in terms of utility, more particularly as a negative fourth derivative of the utility function.

Although not a perfect analog, in the same way that risk aversion is not a perfect analog for aversion to a higher variance ([Rothschild and Stiglitz 1970](#)), a temperate individual generally dislikes kurtosis. In an expected-utility setting, [Eeckhoudt and Schlesinger \(2008\)](#) show that temperance is both necessary and sufficient for an increase in the downside risk of future labor income to always increase the level of precautionary saving.

More recently, prudence and temperance, as well as even higher-order risk attitudes, have been defined without using an expected-utility context. In particular, [Eeckhoudt and Schlesinger \(2006\)](#) define these higher-order risk attitudes as preferences over particular classes of lottery pairs. What makes these characterizations particularly appealing is their simplicity, as they are stated in terms of comparing simple 50–50 lottery pairs. The intuition behind such preference is described via a concept defined as “risk apportionment.”

In this chapter, we summarize many of the interesting results about these higher-order risk attitudes. The lottery preferences that are defined here are basic, and they do not require any particular model: neither expected utility nor a particular framework for non-expected utility. Since much of insurance theory is based on expected-utility models and since much of what we know about higher-order risk attitudes is easy to characterize in an expected-utility setting, this chapter is mainly (though

not exclusively) focused on expected utility. Since this area of research is relatively new, it is our hope that this chapter will stimulate new research—both theoretical and empirical/experimental—in this relatively nascent topic. We are especially interested in ways that our basic results extend to non-expected utility models and to behavioral models.

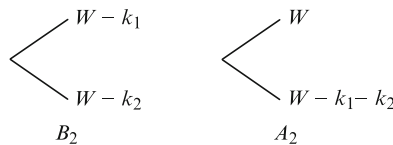
We first give a very brief overview of the [Eeckhoudt and Schlesinger \(2006\)](#) lottery preference approach, and we explain the rationale behind what we refer to as “risk apportionment.” Then, in Sect. 2.3, we show how these results have quite simple ties to expected-utility theory. In Sect. 2.4, we generalize the concept of risk apportionment, which can be described as preference for “disaggregating the harms,” to a preference for mixing “good outcomes” with “bad outcomes.” In Sect. 2.5, we examine how our results can be applied to the best known of higher-order risk effects, namely, to precautionary motives. Section 2.6 extends the analysis to cases where preferences are bivariate, such as preferences over both wealth and health status. Section 2.7 looks at the special case of univariate preferences, but where various risks are jointly applied in a multiplicative manner, such as when stochastic nominal wealth is multiplied by a factor representing a purchasing power index. Finally, we conclude by summarizing the key points and mentioning a few areas in which more research is needed.

## 2.2 Higher-Order Attitudes as Risk Apportionment

We start by reintroducing the well-known concept of risk aversion, which is a second-order risk attitude. An individual has an initial wealth  $W > 0$ . The individual is assumed to prefer more wealth to less wealth. Let  $k_1 > 0$  and  $k_2 > 0$  be positive constants. Consider the following two lotteries expressed via probability trees, as shown in Fig. 2.1. We assume that all branches have a probability of occurrence of one-half and that all variables are defined so as to maintain a strictly positive total wealth. This latter assumption avoids complications to the model associated with bankruptcy.

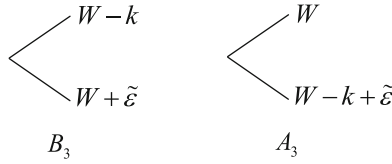
In lottery  $B_2$ , the individual always receives one of the two “harms,” either a sure loss of  $k_1$  or a sure loss of  $k_2$ . The only uncertainty in lottery  $B_2$  is which of the two losses will occur. In lottery  $A_2$ , the individual has a 50–50 chance of either receiving both harms together (losing both  $k_1$  and  $k_2$ ) or receiving neither one. An individual is defined as being risk averse if he/she prefers lottery  $B_2$  to lottery  $A_2$  for every arbitrary  $k_1, k_2$ , and  $W$  satisfying the above constraints. Put differently suppose that the consumer already has the lottery paying  $W$  in state 1 and paying  $W - k_1$  in state 2, where each state has a probability of 0.5. If forced to add a second loss  $k_2$  in one of the two states, a risk averter always prefers to add the second loss in state 1, the state where  $k_1$  does not occur.

The risk averter prefers to “apportion” the sure losses  $k_1$  and  $k_2$  by placing one of them in each state. [Eeckhoudt and Schlesinger \(2006\)](#), who define the concept of risk apportionment, describe this type of behavior as a preference for “disaggregating the harms.” It is trivial for the reader to verify that the above definition of risk aversion can only be satisfied with a concave utility function, if preferences are given by expected utility. It is also easy to verify that lottery  $A_2$  is riskier than lottery  $B_2$  in the sense of [Rothschild and Stiglitz \(1970\)](#).<sup>1</sup>

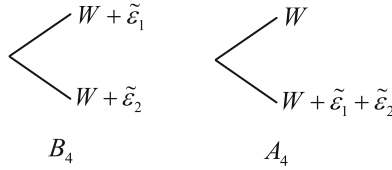


**Fig. 2.1** Lottery preference as risk aversion

<sup>1</sup>The lottery  $A_2$  is easily seen to be a simple mean-preserving spread of the lottery  $B_2$ .



**Fig. 2.2** Lottery preference as prudence



**Fig. 2.3** Lottery preference as temperance

To view the third-order risk attitude of prudence, let  $k > 0$  denote a positive constant and let  $\tilde{\varepsilon}$  denote a zero-mean random variable. Someone who is risk averse will dislike the random wealth variable  $\tilde{\varepsilon}$ . We assume that  $W - k + \varepsilon > 0$  for all realizations of the random variable  $\tilde{\varepsilon}$ . Although we do not need risk aversion to define prudence, it makes the interpretation a bit simpler, since in this case we now have a new pair of “harms,” namely, losing  $k$  and adding  $\tilde{\varepsilon}$ . A prudent individual is one who always prefers to disaggregate these two harms. This is illustrated in Fig. 2.2.

In lottery  $B_3$ , the individual always receives one of the two “harms,” either a sure loss of  $k$  or the addition of a zero-mean random wealth change  $\tilde{\varepsilon}$ . In lottery  $A_3$ , the individual has a 50–50 chance of either receiving both harms together or of receiving neither one. [Eeckhoudt and Schlesinger \(2006\)](#) define an individual as being *prudent* if he/she always prefers lottery  $B_3$  to lottery  $A_3$ . Alternatively, one could describe the behavior as preferring to attach the zero-mean lottery  $\tilde{\varepsilon}$  to the state with the higher wealth vis-à-vis the state with the lower wealth.<sup>2</sup> Equivalently, we could describe it as preferring to attach the sure loss  $k$  to the state with no risk, as opposed to the state with the risk  $\tilde{\varepsilon}$ . Although this definition is not specific to expected utility, if we assume a model with differentiable utility, prudence is equivalent to a positive third derivative of the utility function, as we show in the next section.

Once again, our definition is expressed in terms of risk apportionment: a prudent individual prefers to apportion the two harms by placing one in each state. To define temperance, which is a fourth-order effect, [Eeckhoudt and Schlesinger \(2006\)](#) simply replace the “harm” of losing the fixed amount of wealth  $k$  with the “harm” of a second zero-mean risk. To this end, let  $\tilde{\varepsilon}_1$  and  $\tilde{\varepsilon}_2$  be two distinct zero-mean risks, where we assume that  $\tilde{\varepsilon}_1$  and  $\tilde{\varepsilon}_2$  are statistically independent of one another. An individual is defined as being temperate, if he/she always prefers to apportion the two harms ( $\tilde{\varepsilon}_1$  and  $\tilde{\varepsilon}_2$ ) by placing one in each state.

In Fig. 2.3, again with equally likely states of nature, the temperate decision maker always prefers lottery  $B_4$  to lottery  $A_4$ . Again, this is a preference for “disaggregating the harms.” Given a risk in one of these two states, the individual prefers to locate a second independent risk in the other state.<sup>3</sup>

<sup>2</sup>A similar observation was made by [Eeckhoudt et al. \(1995\)](#) and by [Hanson and Menezes \(1971\)](#), who all confined their analysis to EU.

<sup>3</sup>The rationale for statistical independence here should be apparent. For example, if  $\tilde{\varepsilon}_1$  and  $\tilde{\varepsilon}_2$  were identically distributed and perfectly negatively correlated, every risk averter would prefer to have the two risks in the same state, since they would then “cancel” each other.

Note that all of the definitions as given above are not dependent on expected utility or any other specific model of preferences. In a certain sense, these definitions are “model free” and can be examined within both expected-utility and non-expected-utility types of models.<sup>4</sup>

By nesting the above two types of lotteries in an inductive way, [Eeckhoudt and Schlesinger \(2006\)](#) generalize the concepts of prudence and temperance to even higher orders.<sup>5</sup> In our view, this nesting makes everything a bit less transparent, but the idea of risk apportionment remains the same. Although our focus in this chapter will be on risk attitudes no greater than order four (temperance), we introduce a way to view even higher-order risk attitudes later in the chapter, when we discuss a generalization that involves combining “good” with “bad” outcomes.

### 2.3 Risk Attitudes and Expected Utility

Suppose that preferences can be expressed using expected utility and let the individual’s utility of wealth be given by the strictly increasing function  $u$ . We assume that  $u$  is continuous and is continuously differentiable.<sup>6</sup> Of course, risk aversion is equivalent to having  $u$  be a concave function, as is well known. Under our differentiability assumption, this implies that  $u'' < 0$ .<sup>7</sup>

Within an expected-utility framework, prudence is equivalent to  $u''' > 0$ , exactly as in [Kimball \(1990\)](#), and temperance is equivalent to  $u'''' < 0$ , as in [Kimball \(1992\)](#). The “tool” in deriving these results is the utility premium, measuring the degree of “pain” involved in adding risk. To the best of our knowledge, the first direct look at the utility premium was the work of [Friedman and Savage \(1948\)](#). Although this measure actually predates more formal analyses of behavior under risk, as pioneered by [Arrow \(1965\)](#) and [Pratt \(1964\)](#), it has been largely ignored in the literature.<sup>8</sup> One reason for ignoring the utility premium is that it cannot be used to compare individuals. However, our interest here is examining choices made by a single individual. As such, the utility premium turns out to be an extremely useful tool.

We define the utility premium for the risk  $\tilde{\epsilon}_1$ , given initial wealth  $W$  as follows:

$$v(W) \equiv Eu(W + \tilde{\epsilon}_1) - u(W). \quad (2.1)$$

The utility premium is the amount of utility added by including the risk  $\tilde{\epsilon}_1$  with initial wealth. Of course, for a risk averter, the individual loses utility by adding the zero-mean risk  $\tilde{\epsilon}_1$ ; hence,

<sup>4</sup>It is easy to see in [Fig. 2.2](#) that the means and variances for  $A_3$  and  $B_3$  are identical, but  $B_3$  has a higher skewness (is more right skewed). For two distributions with the same first two moments, it can be shown that it is impossible for every prudent individual to prefer the distribution with a lower skewness. If the two zero-mean risks in [Fig. 2.3](#) are symmetric, then the first three moments of  $A_4$  and  $B_4$  are identical, but with  $A_4$  having a higher kurtosis (fatter tails). For two distributions with the same first three moments, it can be shown that it is impossible for every temperate individual to prefer the distribution with a higher kurtosis.

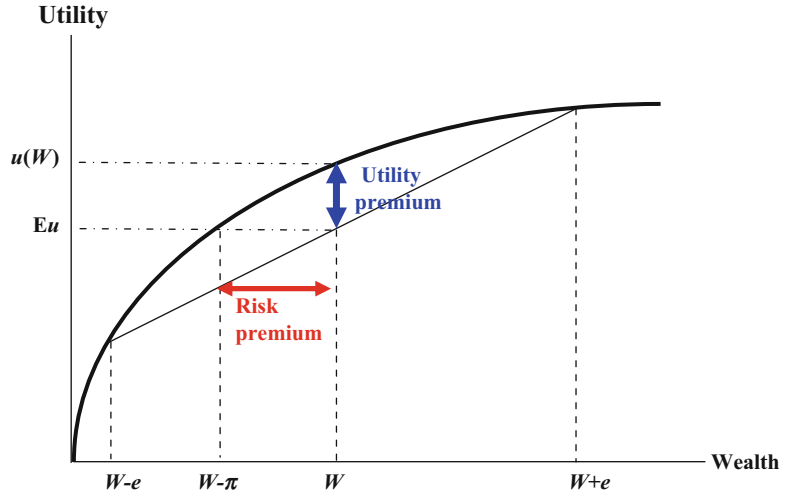
<sup>5</sup>These higher orders are already known to be important in various contexts. For example, standard risk aversion as defined by [Kimball \(1993\)](#), as well as risk vulnerability as defined by [Gollier and Pratt \(1996\)](#), each require temperance. [Lajeri-Chaherli \(2004\)](#) looks at a rationale to use 5th-order risk attitudes. Both of these higher-order risk preferences are also given intuitive economic interpretations by [Courbage and Rey \(2010\)](#).

<sup>6</sup>Although utility-based models can also be derived without differentiability, most of the literature assume that these derivatives exist.

<sup>7</sup>For the mathematically astute, we admit that this is a slight exaggeration. Strict risk aversion also allows for  $u'' = 0$  at some wealth levels, as long as these wealth levels are isolated from each other. See [Pratt \(1964\)](#) for more details.

<sup>8</sup>An article by [Hanson and Menezes \(1971\)](#) made this same observation more than 40 years ago!

**Fig. 2.4** Utility premium and risk premium



$v(W) < 0$ . This follows easily from Jensen's inequality since  $u$  is concave.<sup>9</sup> To the extent that utility is used to measure an individual's welfare, the utility premium measures the level of "pain" associated with adding risk  $\tilde{\varepsilon}_1$  to wealth, where "pain" is measured as the loss of utility.

An example of the utility premium is illustrated in Fig. 2.4 for the case where  $\tilde{\varepsilon}_1$  is a 50–50 chance of either gaining or losing wealth  $e$ . In Fig. 2.4,  $Eu$  denotes the expected utility of wealth prospect  $W + \tilde{\varepsilon}_1$ . Pratt's (1964) risk premium, denoted here by  $\pi$ , is the amount of wealth that the individual is willing to give up to completely eliminate the risk  $\tilde{\varepsilon}_1$ . The utility premium (which is the negative of the amount drawn in Fig. 2.4) shows exactly how much utility is lost by the addition of  $\tilde{\varepsilon}_1$ . Since the utility function representing an individual's preferences is not unique, the utility premium will change if the utility scale changes.<sup>10</sup> For example, if we double all of the utility numbers, the utility premium will also double. Pratt's risk premium, on the other hand, is invariant to such changes. For this reason, we can use Pratt (1964) to compare preferences between individuals, but we cannot use the utility premium.

We can use the utility premium to easily show how our earlier definitions of prudence and temperance relate to expected utility. To this end, differentiate the utility premium with respect to initial wealth to obtain

$$v'(W) \equiv Eu'(W + \tilde{\varepsilon}_1) - u'(W). \quad (2.2)$$

Using only Jensen's inequality, it follows from (2.2) that  $v'(W) > 0$  whenever  $u'$  is a convex function, i.e., when  $u'''(y) > 0 \forall y$ . Since the utility premium is negative, we interpret  $v'(W) > 0$  as meaning that the size of the utility premium gets smaller as initial wealth  $W$  increases.

Now consider our earlier definition of prudence. A prudent individual would prefer to attach the zero-mean risk  $\tilde{\varepsilon}_1$  to the state with the higher wealth  $W$ , as opposed to attaching it to the state with the lower wealth,  $W - k$ . This is due to the fact that  $\tilde{\varepsilon}_1$  causes less "pain" at the higher wealth level, where pain in our expected-utility model is measured via utility. In other words, prudence is equivalent to saying that the size of our utility premium decreases with wealth, i.e.,  $u''' > 0$ .

More formally, a decreasing utility premium,  $v'(W) > 0$ , is equivalent to saying that, for all  $k > 0$ ,

<sup>9</sup>In the original article by Friedman and Savage (1948), the risks that were considered had positive expected payoffs and could thus have a positive utility premium, even for a risk averter. In this chapter, we only consider zero-mean risks.

<sup>10</sup>The utility function is only unique up to a so-called affine transformation. See Pratt (1964).

$$\text{Eu}(W + \tilde{\varepsilon}_1) - u(W) > \text{Eu}(W - k + \tilde{\varepsilon}_1) - u(W - k). \quad (2.3)$$

Rearranging (2.3) and multiplying by 1/2 yields

$$\frac{1}{2}[\text{Eu}(W + \tilde{\varepsilon}_1) + u(W - k)] > \frac{1}{2}[u(W) + \text{Eu}(W - k + \tilde{\varepsilon}_1)], \quad (2.4)$$

which is the expected-utility representation of the lottery preference depicted in Fig. 2.2.

To show that temperance is equivalent to assuming that  $u'''' < 0$ , we need to first differentiate the utility premium a second time with respect to wealth to obtain

$$v''(W) \equiv \text{Eu}''(W + \tilde{\varepsilon}_1) - u''(W). \quad (2.5)$$

It follows from (2.5), using Jensen's inequality, that  $v''(W) < 0$  whenever  $u''$  is a concave function, i.e., whenever the fourth derivative of utility is negative,  $u'''' < 0$ . If we also have a decreasing utility premium (prudence), this can be interpreted as saying that the rate of decrease in the utility premium lessens as wealth increases.

We will still let  $v(W)$  denote the utility premium for adding the risk  $\tilde{\varepsilon}_1$  to wealth  $W$ . To understand how this relates to temperance, we need to consider adding a second independent zero-mean risk  $\tilde{\varepsilon}_2$ . Consider the change in the utility premium from this addition of  $\tilde{\varepsilon}_2$ . We are particularly interested in the case where the presence of risk  $\tilde{\varepsilon}_2$  exacerbates the loss of utility from risk  $\tilde{\varepsilon}_1$ .<sup>11</sup> Since the utility premium is negative, this condition is equivalent to

$$E v(W + \tilde{\varepsilon}_2) - v(W) < 0, \quad (2.6)$$

which itself holds for all  $W$  and for all zero-mean  $\tilde{\varepsilon}_2$  if and only if  $v$  is a concave function. From (2.5), we see that the inequality in (2.6) holds whenever  $u'''' < 0$ . We can now use the definition of the utility premium in (2.2) to expand the left-hand side of the inequality (2.6). Rearranging the result and multiplying by 1/2 shows that the inequality in (2.6) is equivalent to

$$\frac{1}{2}[\text{Eu}(W + \tilde{\varepsilon}_1) + u(W + \tilde{\varepsilon}_2)] > \frac{1}{2}[u(W) + \text{Eu}(W + \tilde{\varepsilon}_1 + \tilde{\varepsilon}_2)]. \quad (2.7)$$

Of course, the inequality in (2.7) is simply the expected-utility representation of the lottery preference depicted in Fig. 2.3.

## 2.4 Pairing Good Outcomes with Bad Ones

Another approach to viewing higher-order risk attitudes extends the concept of “mitigating the harms,” as was discussed in Sect. 2.2. To implement this approach, we first need to provide a definition of an  $N$ th-degree increase in risk, as introduced by Ekern (1980). Assume that all random variables only take on values strictly between  $a$  and  $b$ . Consider a random wealth variable with cumulative distribution function  $F(x)$ . Define  $F^{(1)}(x) \equiv F(x)$  and then define  $F^{(i)}(x) \equiv \int_a^x F^{(i-1)}(t)dt$  for all  $i \geq 2$ .

<sup>11</sup>Replacing the second condition in the definition with  $F^{(i)}(b) \leq G^{(i)}(b)$  yields a definition of  $N$ th-order stochastic dominance. The results in this section easily extend to stochastic dominance, as shown by Eeckhoudt et al. (2009).



**Definition:** The distribution  $G$  is an  $N$ th-degree increase in risk over  $F$  if  $F^{(N)}(x) \leq G^{(N)}(x)$  for all  $a \leq x \leq b$  and  $F^{(i)}(b) = G^{(i)}(b)$  for  $i = 2, \dots, N - 1$ .<sup>12</sup>

As an example that might be more familiar to some readers, when  $N = 2$ , a second-degree increase in risk is identical to a “mean-preserving increase in risk” as defined by [Rothschild and Stiglitz \(1970\)](#). As another example, for  $N = 3$ , a third-degree increase in risk is identical to an “increase in downside risk” as defined by [Menezes et al. \(1980\)](#).

From the definition above, it follows that the first  $N - 1$  moments of  $F$  and  $G$  are identical. For  $N = 2$ , if  $G$  is a second-degree increase in risk over  $F$ ,  $G$  must have a higher variance than  $F$ . However, the reverse implication does not hold: for two distributions with the same mean, a higher variance for  $G$  does not necessarily imply that  $G$  is a second-degree increase in risk over  $F$ .

Before proceeding further, we require the following result, which is due to [Ekern \(1980\)](#).

**Theorem (Ekern):** The following two statements are equivalent:

- (i)  $G$  is an increase in  $N$ th-degree risk over  $F$ .
- (ii)  $\int_a^b u(t) dF \geq \int_a^b u(t) dG$  for all functions  $u$  such that  $\text{sgn}[u^{(N)}(t)] = (-1)^{N+1}$ .

As a matter of notation, if the random variables  $\tilde{X}$  and  $\tilde{Y}$  have distribution functions  $F$  and  $G$ , respectively, where  $G$  is an increase in  $N$ th-degree risk over  $F$ , we will write  $\tilde{X} \succ_N \tilde{Y}$ . Now consider four random variables, each of which might possibly be a degenerate random variable (i.e., a constant):  $\tilde{X}_1, \tilde{Y}_1, \tilde{X}_2, \tilde{Y}_2$ . We assume that  $\tilde{X}_1 \succ_N \tilde{Y}_1$  and  $\tilde{X}_2 \succ_M \tilde{Y}_2$  for some  $N$  and  $M$ . From Ekern’s theorem, we see that  $\tilde{X}_1$  is preferred to  $\tilde{Y}_1$  for any individual with  $\text{sgn}[u^{(N)}(t)] = (-1)^{N+1}$ . In a certain sense, we can thus think of  $\tilde{X}_1$  as being “good” relative to  $\tilde{Y}_1$ , which is relatively “bad.” In a similar manner,  $\tilde{X}_2$  is preferred to  $\tilde{Y}_2$  for any individual with  $\text{sgn}[u^{(M)}(t)] = (-1)^{M+1}$ , so that  $\tilde{X}_2$  is “good” relative to  $\tilde{Y}_2$  for this person.

Now consider a choice between two lotteries. The first lottery, lottery  $B$ , is a 50–50 chance of receiving either  $\tilde{X}_1 + \tilde{Y}_2$  or  $\tilde{Y}_1 + \tilde{X}_2$ . The second lottery, lottery  $A$ , is a 50–50 chance of receiving either  $\tilde{X}_1 + \tilde{X}_2$  or receiving  $\tilde{Y}_1 + \tilde{Y}_2$ . In other words, lottery  $B$  always yields one “good” outcome added to one “bad” outcome. Lottery  $A$ , on the other hand, yields either the sum of both “good” outcomes or the sum of both “bad” outcomes. The following result, which is due to [Eeckhoudt et al. \(2009\)](#) formalizes a certain type of preference for combining “good” with “bad.”<sup>13</sup>

**Proposition 1.** Given  $\tilde{X}_1, \tilde{Y}_1, \tilde{X}_2, \tilde{Y}_2$  with the lotteries  $A$  and  $B$  as described above, lottery  $A$  has more  $(N + M)$ th degree risk than lottery  $B$ . In other words,  $B \succ_{N+M} A$ .

From Ekern’s theorem, Proposition 1 implies that anyone with utility satisfying  $\text{sgn}[u^{(N+M)}(t)] = (-1)^{N+M+1}$  will prefer lottery  $B$  to lottery  $A$ . To see how this proposition generalizes the results of Sect. 2.3, consider the following examples. In each of the examples below, we assume that  $\tilde{\varepsilon}_1$  and  $\tilde{\varepsilon}_2$  are statistically independent zero-mean risks.

*Example 1. (Risk aversion)* Let  $\tilde{X}_1 = W$ ,  $\tilde{Y}_1 = W - k_1$ ,  $\tilde{X}_2 = 0$ ,  $\tilde{Y}_2 = -k_2$ . Lotteries  $A$  and  $B$  are thus identical to the lotteries  $A_2$  and  $B_2$  in Fig. 2.1. It is easy to see from the definition that  $\tilde{X}_1 \succ_1 \tilde{Y}_1$  and  $\tilde{X}_2 \succ_1 \tilde{Y}_2$ . Thus,  $N = M = 1$  in applying Proposition 1. Hence, everyone who is risk averse, with  $u^{(2)}(t) < 0 \forall t$ , will prefer lottery  $B$  to lottery  $A$ .

*Example 2. (Prudence)* Let  $\tilde{X}_1 = W$ ,  $\tilde{Y}_1 = W - k$ ,  $\tilde{X}_2 = 0$ ,  $\tilde{Y}_2 = \tilde{\varepsilon}$ . Lotteries  $A$  and  $B$  are then identical to the lotteries  $A_3$  and  $B_3$  in Fig. 2.2. It follows from the definition that  $\tilde{X}_1 \succ_1 \tilde{Y}_1$  and

<sup>12</sup>[Kimball \(1993\)](#) refers to the two risks in this case as “mutually aggravating.” [Pratt and Zeckhauser \(1987\)](#) came very close to making this same observation. Their basic difference was considering independent risks  $\varepsilon_i$  that were disliked by a particular individual, rather than zero-mean risks, which are disliked by every risk averter. [Menezes and Wang \(2005\)](#) offer an example that is also quite similar and refer to this case as “aversion to outer risk.”

<sup>13</sup>These authors also provide a proof of this result, which we do not reproduce here.



$\tilde{X}_2 \succ_2 \tilde{Y}_2$ . Thus,  $N = 1$  and  $M = 2$  in applying Proposition 1. Hence, everyone who is prudent, with  $u^{(3)}(t) > 0 \forall t$ , will prefer lottery  $B$  to lottery  $A$ .

*Example 3.* (Temperance) Let  $\tilde{X}_1 = W$ ,  $\tilde{Y}_1 = W + \tilde{\varepsilon}_1$ ,  $\tilde{X}_2 = 0$ ,  $\tilde{Y}_2 = \tilde{\varepsilon}_2$ . Lotteries  $A$  and  $B$  are thus identical to the lotteries  $A_4$  and  $B_4$  in Fig. 2.3. In this example, we have  $\tilde{X}_1 \succ_2 \tilde{Y}_1$  and  $\tilde{X}_2 \succ_2 \tilde{Y}_2$ . Thus,  $N = M = 2$  in applying Proposition 1. Hence, everyone who is temperate, with  $u^{(4)}(t) < 0 \forall t$ , will prefer lottery  $B$  to lottery  $A$ .

In each of these examples, the “bad” outcome is either losing a fixed amount of money or adding a zero-mean risk. We can view the absence of the harm as a relatively “good” outcome and the inclusion of the harm as a relatively “bad” outcome. So our former description of “disaggregating the harms” is now reinterpreted as a preference for “mixing good with bad outcomes.” But notice how the current story allows for additional applications of “good” and “bad.” Moreover, this approach often allows for alternative interpretations. Take, for example, the case of temperance. Instead of using  $N = M = 2$  in applying Proposition 1, we can also let  $N = 1$  and  $M = 3$  as in the following example:

*Example 4.* (Temperance) Let  $\tilde{X}_1 = W$ ,  $\tilde{Y}_1 = W - k$ ,  $\tilde{X}_2 = \tilde{\theta}_1$ ,  $\tilde{Y}_2 = \tilde{\theta}_2$ . Here we assume that  $E\tilde{\theta}_1 = E\tilde{\theta}_2 = 0$  and that  $\text{Var}(\tilde{\theta}_1) = \text{Var}(\tilde{\theta}_2)$ , but that  $\tilde{\theta}_2 \succ_3 \tilde{\theta}_1$ , i.e., that  $\tilde{\theta}_2$  has more 3rd-degree risk (more “downside risk”) than  $\tilde{\theta}_1$ . By the definition above, this implies that  $\tilde{\theta}_2$  must be more skewed to the left than  $\tilde{\theta}_1$ . Proposition 1 implies that a temperate individual would prefer to add  $\tilde{\theta}_2$  in the state with higher wealth, with  $\tilde{\theta}_1$  added to the state with lower wealth, as opposed to reversing the locations of the two  $\tilde{\theta}$  risks. Again we see how this interpretation can be made with regard to apportioning the risks.

Proposition 1 extends easily to the more common ordering of stochastic dominance, as shown by [Eeckhoudt et al. \(2009\)](#). We note here that the random variable  $\tilde{Y}$  has more  $N$ th-degree risk than the random variable  $\tilde{X}$ , which implies that  $\tilde{X}$  dominates  $\tilde{Y}$  via  $N$ th-order stochastic dominance and that the first  $N-1$  moments of  $\tilde{X}$  and  $\tilde{Y}$  are identical. The extension of Proposition 1 to stochastic dominance can be written as follows:

**Corollary 1.** *Given  $\tilde{X}_1, \tilde{Y}_1, \tilde{X}_2, \tilde{Y}_2$  with the lotteries  $A$  and  $B$  as defined in Proposition 1, lottery  $B$  dominates lottery  $A$  via  $(N+M)$ th-order stochastic dominance.*

## 2.5 Precautionary Motives

Since this topic is dealt with elsewhere in this handbook, we only wish to give some insight into the logic behind precautionary motives. To the best of our knowledge, the first articles dealing with this topic in an expected-utility framework were by [Leland \(1968\)](#) and [Sandmo \(1970\)](#). Both considered the effect of risky future income on current saving. To the extent that future risk increased the level of current saving, this additional saving was referred to as “precautionary saving.” The notion of this precautionary motive for saving was introduced by [Keynes \(1930\)](#), and it was embedded into the macroeconomics literature on the permanent income hypothesis by [Bewley \(1977\)](#).

Both Leland and Sandmo discovered that a precautionary-saving motive would be ensured if and only if the consumer’s differentiable utility function exhibited prudence,  $u''' > 0$ . However, since the term “prudence” did not exist prior to [Kimball \(1990\)](#) and since the requirement  $u''' > 0$  might need some motivation at the time, both Leland and Sandmo were quick to point out that the well-accepted principle of DARA was sufficient to obtain their results. Although DARA is actually a stronger property, it had an intuitive economic rationale and thus was probably easier to justify.

However, as we now see from (2.2), the size of the utility premium for adding a zero-mean risk to some initial wealth level will always be decreasing in the wealth level if and only if  $u'$  is a convex

function, i.e., if and only if  $u''' > 0$  when utility is differentiable. Before examining the rationale for a precautionary motive, let us first be careful to note the distinction between prudence and DARA. For example, if utility exhibits the well-known property of constant absolute risk aversion (CARA), we will still have prudence,  $u''' > 0$ . Indeed, under CARA, the size of the utility premium, as defined in (2.1), is decreasing wealth. Thus, the level of pain from a zero-mean risk will decrease as the individual becomes wealthier. At first thought, this might seem counter to the basic property under CARA that the individual's willingness to pay to completely eliminate the risk is independent of his/her wealth level. However, one needs to also consider the fact that our individual is risk averse, which implies that the marginal utility of money is decreasing in wealth. Under CARA, a zero-mean risk will cause less pain as the individual becomes wealthier. However, as the individual becomes wealthier, his/her willingness to pay to remove each unit of pain will increase (since money is worth less at the margin). Under CARA, these two effects exactly offset and the individual pays the same total amount to remove the risk at every wealth level. See, for example, [Eeckhoudt and Schlesinger \(2009\)](#).

As another example, consider the often used quadratic form of the utility function  $u(w) = w - bw^2$ , where  $b > 0$  and we restrict  $w < (2b)^{-1}$ . Since  $u'''(w) = 0$  for all  $w$ , it follows from (2.2) that the level of pain from adding a zero-mean risk will be constant at all levels of initial wealth. However, since we still have decreasing marginal utility, the willingness to pay to eliminate each unit of pain will increase as the individual becomes wealthier. This leads to the undesirable property of increasing absolute risk aversion for this utility function, as is well known.

Now let us consider a different interpretation for the risk apportionment story. Rather than consider the 50–50 lotteries, such as those in Fig. 2.2, let us consider sequentially receiving each of the two lottery outcomes, one in each period. Denote these two outcomes as  $\tilde{x}_1$  and  $\tilde{x}_2$ , with the understanding that the outcomes might or might not both be random. We previously considered the expected utility of a lottery, which was defined as  $\frac{1}{2}\text{Eu}(\tilde{x}_1) + \frac{1}{2}\text{Eu}(\tilde{x}_2)$ . But if we simply add the utility from the two outcomes,  $\text{Eu}(\tilde{x}_1) + \text{Eu}(\tilde{x}_2)$ , we can reinterpret the model as a two-period (undiscounted) lifetime utility.

Thus, from Fig. 2.2, we see that a preference for  $B_3$  over  $A_3$  implies that the individual prefers to have more wealth in the time period with the zero-mean risk, whenever the individual is prudent. Worded differently, the individual can decrease the pain from this zero-mean risk by shifting wealth to the period with the risky income. In the precautionary-saving model, this implies shifting more wealth to the second period via an increase in saving.

[Eeckhoudt and Schlesinger \(2008\)](#) extend this reasoning to cases where the risk in the second period changes. Since increasing wealth in the second period via additional saving is itself a first-order change, we can apply the results of Sect. 2.4 to consider  $N$ th-degree changes in the riskiness of second-period income. If the second-period income is risky, but riskiness increases via a second-degree increase in risk, the individual can mitigate some of this extra pain by increasing saving if the individual is prudent. The application of Proposition 1 is identical to that used in Example 2 of the previous section, with  $N = 1$  and  $M = 2$ .

However, the link between prudence and precautionary change is broken if we consider other types of changes in the riskiness of future income. Suppose, for example, that future risky income undergoes a first-degree deterioration. This would be the case, for instance, when there is an increased risk of being unemployed in period 2. We can then apply Proposition 1 as in Example 1, with  $N = M = 1$ . Any risk-averse individual would increase his/her saving in response to such a change in the riskiness of future income. Thus, prudence is no longer necessary to induce precautionary saving. On the other hand, suppose that the first two moments of risky future income remained unchanged, but that there was a third-degree deterioration in the risk connoting more downside risk. In that case, we apply our Proposition 1 with  $N = 1$  and  $M = 3$ . Thus, prudence is no longer sufficient to induce precautionary saving, and we need to assume temperance to guarantee an increase in saving.

Obviously, models employing joint decisions about saving and insurance will find all of the above analysis useful. However, precautionary motives can also be found in decision models that do not

include saving. For example, consider a simple model of insurance with two loss states: loss and no-loss, where a loss of size  $L$  occurs with probability  $p$ .<sup>14</sup> The individual's initial wealth is  $W > 0$ . Coinsurance is available that pays a fraction  $\alpha$  of any loss for a premium of  $\alpha(1 + \lambda)pL$ , where  $\lambda \geq 0$  denotes the so-called premium loading factor. It is straightforward to show that the first-order condition for the choice of an optimal level of coinsurance in an expected-utility framework is

$$\frac{dEu}{d\alpha} = pL[1 - (1 + \lambda)p]u'(y_1) - pL[1 - (1 + \lambda)p + \lambda]u'(y_2) = 0, \quad (2.8)$$

where  $y_1 \equiv W - \alpha(1 + \lambda)pL - L + \alpha L$  and  $y_2 \equiv W - \alpha(1 + \lambda)pL$ .<sup>15</sup> Let  $\alpha^*$  denote the optimal level of insurance chosen.

Now suppose that we introduce an additive noise term  $\tilde{\varepsilon}$ , with  $E\tilde{\varepsilon} = 0$ , but that this noise only occurs in the loss state. Examining the derivative in the first-order condition (2.8), but with the noise term added, yields

$$\left. \frac{dEu}{d\alpha} \right|_{\alpha^*} = pL[1 - (1 + \lambda)p]Eu'(y_1 + \tilde{\varepsilon}) - pL[1 - (1 + \lambda)p + \lambda]u'(y_2). \quad (2.9)$$

If the individual is prudent, we know that  $Eu'(y_1 + \tilde{\varepsilon}) > u'(y_1)$ . Comparing (2.9) with (2.8), it follows that  $\left. \frac{dEu}{d\alpha} \right|_{\alpha^*} > 0$ , so that more insurance will be purchased when the noise is present.

Note that the extra insurance does nothing to protect against the loss  $L$ . Rather, the extra insurance lowers the “pain” from the zero-mean noise that exists only in the loss state. Although there is no saving in this model, the individual can increase his/her wealth in the loss state by increasing the level of insurance purchased. This additional insurance is thus due solely to a precautionary motive, and it is dependent on having such a precautionary motive, which in this case requires prudence.

The reader can easily examine the case where the zero-mean loss occurs only in the no-loss state. In that case, if we assume prudence, a precautionary effect induces the individual to increase his/her wealth in the no-loss state. In our insurance model, this is achieved by reducing the level of insurance and thus spending less money on the insurance premium.

## 2.6 Multivariate Preferences

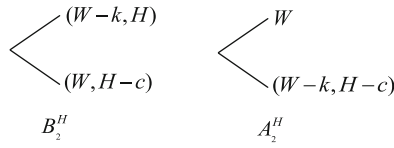
In this section, we examine an extension of the model that has much applicability in insurance models, namely, the case where preferences depend on more than just wealth. Quite often, preferences over wealth in the loss state are not the same as in the no-loss state. As a concrete example, consider one's health. To this end, let  $y$  denote the individual's wealth and  $h$  denote the individual's health status. To make the model viable, we need to assume that  $h$  is some objective measure, such as the remaining number of years of life.<sup>16</sup> We also assume that an increase in  $h$  is always beneficial and that the individual is risk-averse in  $h$ , so that the individual would always prefer to live another 10 years for certain as opposed to having a 50–50 chance of living either 5 years or 15 years.

Suppose that an individual with initial wealth  $W$  and initial health  $H$  faces a loss of size  $k > 0$  in wealth and a loss of size  $c > 0$  in health. Consider the following two 50–50 lotteries as shown in Fig. 2.5.

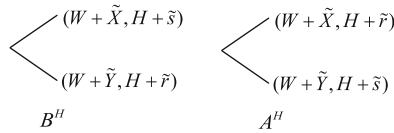
<sup>14</sup>This example is adapted from Fei and Schlesinger (2008).

<sup>15</sup>The second-order sufficient condition for a maximum follows trivially if we assume risk aversion.

<sup>16</sup>For another interesting application, see Gollier (2010), who lets  $h$  denote the quality of the planet's environment.



**Fig. 2.5** Lottery preference as correlation aversion



**Fig. 2.6** Lottery preference as multivariate risk apportionment

In lottery  $B_2^H$ , the individual incurs either a reduction in wealth or a reduction in health, each with a 50% chance. In lottery  $A_2^H$ , the individual either has neither reduction or has a simultaneous reduction in both wealth and health. If we extend the earlier concept of mitigating the two “harms,” then the individual would prefer lottery  $B_2^H$  to lottery  $A_2^H$ . The individual prefers to apportion the two “harms” by placing them in separate states of nature. Likewise, we can interpret this lottery preference as preference for combining “good with bad.”

Such preference is defined as “correlation aversion” by [Epstein and Tanny \(1980\)](#). To the best of our knowledge, this concept was first introduced to the literature by [Richard \(1975\)](#), who used a different terminology. For preferences represented by a bivariate utility function  $u(y, h)$ , [Richard \(1975\)](#) and [Eeckhoudt et al. \(2007\)](#) show that this preference follows if and only if the cross-partial derivative  $u_{12} \equiv (\partial^2 u)/(\partial y \partial h)$  is everywhere negative.

We should note that such preference is not one that is universally assumed in the literature. In fact, the empirical evidence is mixed on the direction of the lottery preference. Indeed, this topic has been debated in the literature, as summarized well by [Rey and Rochet \(2004\)](#). The main thrust of the counterargument is that one cannot enjoy wealth in poor states of health, so that it might be better to pair lower wealth with lower health. In other words, a case can be made that it might be preferable to pair bad with bad and pair good with good, counter to the arguments made above. If this preference always occurs, then the cross-partial derivative  $u_{12} \equiv (\partial^2 u)/(\partial y \partial h)$  is everywhere positive in an expected-utility setting.

The implication of such assumptions can have a big impact in models of insurance choice. For instance, consider our two state insurance example from the previous section, without any noise. Assume further that the financial loss of size  $L$  occurs only when the individual also receives a reduction in his/her health status from  $H$  to  $H - c$ . This additional assumption requires only that we adapt the first-order condition (2.8) by changing  $u'(y_1)$  to  $u_1(y_1, H - c)$  and by changing  $u'(y_2)$  to  $u_1(y_2, H)$ . Without losing generality, we can scale utility so that, if the individual is correlation averse, we have  $u_1(y_1, H - c) > u'(y_1)$  and  $u_1(y_2, H) < u'(y_2)$ . It then follows in a straightforward manner from (2.8) that the optimal level of insurance would need to be increased, when the financial loss of size  $L$  is accompanied by a loss in health status of amount  $c$ . Note that this additional insurance provides a bit more wealth in the loss state and that wealth in the loss state now provides the additional benefit of reducing the “pain” due to the lower health status.

Although the risk attitude of correlation aversion has existed in the literature since [Richard \(1975\)](#), it has only recently begun to receive much attention. Moreover, the concept has been extended by [Eeckhoudt et al. \(2007\)](#), [Tsetlin and Winkler \(2009\)](#), and others to higher orders of multivariate risk attitudes.

As we did in Sect. 2.4, consider the (possibly degenerate) random wealth variables  $\tilde{X}$  and  $\tilde{Y}$ . We assume that  $\tilde{Y}$  has more  $N$ th-degree risk than  $\tilde{X}$ . Let  $\tilde{r}$  and  $\tilde{s}$  denote two (possibly degenerate) health-status variables, where  $\tilde{s}$  has more  $M$ th-degree risk than  $\tilde{r}$ . Hence, we can view  $\tilde{X}$  and  $\tilde{r}$  as each being relatively “good,” whereas  $\tilde{Y}$  and  $\tilde{s}$  are relatively “bad.” Consider the 50–50 lotteries in Fig. 2.6.

In lottery  $B^H$ , the individual mixes good with bad. In lottery  $A^H$ , the individual mixes good with good and mixes bad with bad. A preference for  $B^H$  over  $A^H$  thus represents a type of multivariate risk apportionment.<sup>17</sup>

Consider the case where  $N = M = 1$ . This case corresponds to correlation aversion. Indeed, as shown by Tsetlin and Winkler (2009), in an expected-utility model, this preference can be guaranteed to hold if and only if  $u_{12}(y, h)$  is everywhere negative. In Fig. 2.6, our earlier definition of correlation aversion is illustrated by setting  $\tilde{X} = \tilde{r} = 0$ ,  $\tilde{Y} = -k$ , and  $\tilde{s} = -c$ . Once again, both definitions turn out to be equivalent.

The case where  $N = 1$  and  $M = 2$  is labeled “cross prudence in wealth” by Eeckhoudt et al. (2007). For an individual displaying such preference, more wealth mitigates the “harm” of a riskier health, where “riskier” means more second-degree risk. The case in which  $N = 2$  and  $M = 1$  is labeled “cross prudence in health.” This preference implies that a riskier (in the second degree) wealth is better tolerated when the individual is healthier. If  $N = M = 2$ , we obtain what Eeckhoudt et al. (2007) label “cross temperance.” Their interpretation considers the special case where  $\tilde{X} = \tilde{r} = 0$  and where  $\tilde{Y}$  and  $\tilde{s}$  are zero-mean risks. Such an individual would prefer a 50–50 lottery with either risky wealth or risky health, as compared to 50–50 lottery with simultaneous risky wealth and risky health versus no risk.

In each of the above settings, the lottery  $B^H$  is necessarily preferred to lottery  $A^H$  in an expected-utility framework if and only if  $(-1)^{N+M-1} \frac{\partial^{N+M} u(y, h)}{\partial N_y \partial M_h} > 0$ . Several examples of how these results can be applied to decision problems can be found in Eeckhoudt et al. (2007). As an insurance example, consider the two-state insurance model of Sect. 2.5, where a financial loss of size  $L$  occurs with probability  $p$ . Here we first assume that only wealth is random and that health status is constant at level  $H$ . The first-order condition for an optimal choice of coinsurance is thus

$$\frac{dEu}{d\alpha} = pL[1 - (1 + \lambda)p]u_1(y_1, H) - pL[1 - (1 + \lambda)p + \lambda]u_1(y_2, H) = 0. \quad (2.10)$$

Denote the solution to (2.10) by the insurance level  $\alpha^*$ .

Now suppose that the mean health status is not affected, but that health status becomes noisy in the state where there is a financial loss. In particular, health status in this state becomes  $H + \tilde{s}$ , where  $\tilde{s}$  is a zero-mean random health variable. Thus,

$$\frac{dEu}{d\alpha} \big|_{\alpha^*} = pL[1 - (1 + \lambda)p]Eu_1(y_1, H + \tilde{s}) - pL[1 - (1 + \lambda)p + \lambda]u_1(y_2, H). \quad (2.11)$$

Comparing (2.11) with (2.10), it follows that the level of insurance will increase whenever  $Eu_1(y_1, H + \tilde{s}) > u_1(y_1, H)$ . From Jensen’s inequality, this will hold whenever the function  $u_1(y, h)$  is convex in  $h$ , i.e., whenever  $u_{122} > 0$ , which by definition is whenever the individual is cross prudent in wealth. Intuitively, the extra insurance in this case helps to mitigate the “pain” caused by introducing noise into the health status. Note that if we had  $u_{122} < 0$ , denoting cross-imprudence, it would follow that less insurance is purchased under a noisy health status.

<sup>17</sup>This analysis is based on a generalization and extension of the results in Tsetlin and Winkler (2009), who confine themselves to expected-utility models.

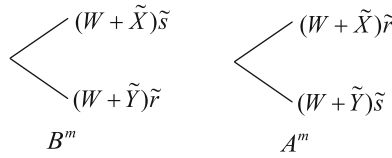


Fig. 2.7 Lottery preference as multiplicative risk apportionment

## 2.7 Multiplicative Risks

In the first five sections of this chapter, preferences were univariate over wealth alone. Moreover, the various components of wealth were all additive. In Sect. 2.6, we considered general multivariate preferences. Let us change the multivariate notation slightly so that the utility function in the multivariate case is written  $\mathcal{U}(y, h)$ . The additive univariate model can be obtained as special case by simply interpreting  $h$  as a second additive wealth term and then defining utility equal to  $\mathcal{U}(y, h) = u(y + h)$ . In this setup, for example, it is easy to see that  $\mathcal{U}_{112}(y, h) = \mathcal{U}_{122}(y, h) = u'''(y + h)$ . Thus, both multivariate cases of “cross prudence” correspond to the simple univariate additive case of “prudence,” with the simple requirement that  $u''' > 0$ . Other higher-order risk attitudes over wealth can be similarly derived in the same manner.

In several applications of decision making, there are two (or more) sources of risk that are multiplicative. For example, stochastic wealth might be multiplied by a stochastic price deflator or stochastic portfolio returns in a foreign currency might be adjusted via multiplying by a stochastic exchange rate factor. When preferences are univariate over wealth, but the components are multiplicative, we can model this as another special case of multivariate preference.

As we did in Sect. 2.4, consider the (possibly degenerate) random wealth variables  $\tilde{X}$  and  $\tilde{Y}$ , where  $\tilde{Y}$  has more  $N$ th-degree risk than  $\tilde{X}$ . We also consider  $\tilde{r}$  and  $\tilde{s}$  as two (possibly degenerate) additional variables that are used to rescale overall wealth, where  $\tilde{s}$  has more  $M$ th-degree risk than  $\tilde{r}$ . Hence, we can view  $\tilde{X}$  and  $\tilde{r}$  as each being relatively “good,” whereas  $\tilde{Y}$  and  $\tilde{s}$  are relatively “bad.” Consider the 50–50 lotteries in Fig. 2.7.

In lottery  $B^m$ , the individual mixes good with bad. In lottery  $A^m$ , the individual mixes good with good and mixes bad with bad. A preference for  $B^m$  over  $A^m$  thus represents a type of multiplicative risk apportionment. Let us consider first the case of correlation aversion. Here we have  $N = M = 1$ . For example, Eeckhoudt et al. (2009) consider the special case where  $\tilde{X} = 0$ ,  $\tilde{Y} = -k$ ,  $\tilde{r} = 1$ , and  $\tilde{s} = c < 1$ . As illustrated in Fig. 2.7, an individual who exhibits multiplicative correlation aversion prefers lottery  $B^m$  to lottery  $A^m$ . From Sect. 2.6, this behavior follows if and only if  $\mathcal{U}_{12}(y, h) = u'(yh) + yhu''(yh) < 0$ . Straightforward manipulation shows that this last inequality is equivalent to having relative risk aversion be everywhere larger than one, i.e.,  $-yhu''(yh)/u'(yh) > 1$ .

Eeckhoudt and Schlesinger (2008) and Eeckhoudt et al. (2009) show that for  $N = 1$  and  $M = 2$ , “cross prudence,”  $\mathcal{U}_{112}(y, h) = \mathcal{U}_{122}(y, h) = u'''(y + h)$ , holds for all  $y$  and  $h$  if and only if relative prudence is greater than 2, i.e.,  $-yhu'''(yh)/u''(yh) > 2$ .<sup>18</sup> In this setting, we can let  $\tilde{X} = 0$ ,  $\tilde{Y} = -k$  and let  $\tilde{s}$  and  $\tilde{r}$  be random variables with  $\tilde{s}$  exhibiting more second-degree risk than  $\tilde{r}$ . For example, let both  $\tilde{s}$  and  $\tilde{r}$  have a mean of one, so that  $W - \tilde{X} = W$  and  $W - \tilde{Y} = W - k$  represent expected wealth in the two states of nature. For instance,  $\tilde{r}$  might take values of 0.95 or 1.05—either adding or losing five percent of total wealth—each with a 50–50 chance, and  $\tilde{s}$  might take on equally likely values of 0.90 or 1.10—either adding or losing 10 % of total wealth. Since  $\tilde{r}$  has less second-degree risk, multiplying any wealth level by  $\tilde{r}$ , as opposed to  $\tilde{s}$ , is preferred by every risk averter.

<sup>18</sup>For a generalization of the multiplicative case to any arbitrary order  $n$ , see Wang and Li (2010).



From what we learned about precautionary motives in Sect. 2.5, we know that the “pain” from a (second-degree) riskier wealth in the state with  $\tilde{s}$  can be mitigated by having *more* wealth in that state. On the other hand, having more wealth, this state means that the dollar risk will be higher, since  $\tilde{s}$  is multiplied by a higher dollar amount. In other words, the dollar risk could be reduced by having less *wealth* in the state with the higher risk  $\tilde{s}$ . In order to have more wealth in the state with  $\tilde{s}$  be the better of the two alternatives, the precautionary effect must be strong enough to dominate. The result above makes this notion precise, by telling us that this precautionary effect will always dominate if and only if the measure of absolute prudence is everywhere larger than two.<sup>19</sup>

## 2.8 Concluding Remarks

In this handbook chapter, we introduce some fundamentals about higher-order risk attitudes. Although much is known about risk aversion (a second-order risk attitude) and a bit is known about prudence (a third-order risk attitude), much less is known about higher orders. The analysis by [Eeckhoudt and Schlesinger \(2006\)](#) marked a break in the direction of research in this area. Whereas most research had focused on specific choice problems and their comparative statics, this new direction focused on preferences between pairs of simple lotteries. This direction is a bit similar to the way in which [Rothschild and Stiglitz \(1970\)](#) characterized risk aversion as an aversion to mean-preserving spreads.

Within an expected-utility framework, our lottery preference typically relates to the sign of various derivatives of the utility function. These lottery preferences also can be described as preferences for “risk apportionment,” which tell us a general rule for how an individual likes to combine various components of risk. For example, risk aversion was seen as a preference for “disaggregating the harms,” where the harms were two potential sure losses of wealth. By redefining the “harms” in a particular way, we can obtain all of the higher-order risk attitudes. Equivalently, these attitudes were shown to be a preference for combining “good” with “bad,” with good and bad being defined via  $N$ th-degree differences in risk à la [Ekern \(1980\)](#). Not surprisingly, at least to us, extensions to multivariate preferences also depended upon the signs of the derivatives, often the cross-partial derivatives, of the multivariate utility function.

The analysis becomes a bit more complicated if we consider the analysis about multiplicative risks, in Sect. 2.7. In that section, note that we were not able to equate higher-order risk attitudes based on lottery preference with only signs of the derivatives of the original utility function. In particular, the signs of the cross derivatives of the bivariate utility function depended on more than just the signs of derivatives of the univariate utility function. For example, we showed that  $\mathcal{U}_{12}(y, h) < 0$  requires that relative risk aversion of the utility function  $u$  exceeds unity. In a similar vein,  $\mathcal{U}_{112}(y, h) = \mathcal{U}_{122}(y, h) > 0$  requires relative prudence exceeding two. Thus, our lottery preference depends not only on the individual’s being risk averse or being prudent but also on the degree of risk aversion or magnitude of prudence.

The value of measuring intensities of risk aversion was introduced by [Pratt \(1964\)](#) and [Arrow \(1965\)](#). The analysis was extended to intensity measures of prudence by [Kimball \(1990\)](#).<sup>20</sup> Essentially, these measures were used to aid in determining the qualitative changes of decisions made within specific choice problems. For example, when will some small change in the initial conditions lead to the purchase of more insurance?

<sup>19</sup>Note that for commonly used CRRA utility functions, relative prudence always equals the measure of relative risk aversion plus one, so that relative risk aversion exceeding one is equivalent to relative prudence exceeding two.

<sup>20</sup>[Caballé and Pomansky \(1996\)](#) further extended these measures to arbitrarily high orders.



However, the literature on higher-order risk has shown that other intensity measures can be important for comparative statics in decision problems.<sup>21</sup> How these alternative measures relate to lottery preference is an interesting area of current research, for which we do not yet know very many answers.

Given the analysis presented in this chapter, empirical-relevance issues remain. Are individuals prudent? Are they temperate? Obviously, behavioral issues complicate the situation. For example, most all of the experimental evidence shows that risk aversion does not occur universally, although risk aversion is generally accepted as a relevant trait for models of decision making. The extant empirical evidence seems to show that individuals behave in a mostly prudent manner. Likewise, most of the evidence lean towards temperate behavior.<sup>22</sup>

Over the years, we have progressively learned much about risk aversion, and that knowledge has permeated models of decision making under risk, such as models of insurance choice. As we continue to learn more and more about higher-order risk attitudes, such knowledge will become more important as it integrates into insurance economics and other areas of risky decision making. We are quite curious ourselves to see where this all takes us over the next decade or two.

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<sup>21</sup>A short summary of these existing measures is provided by Eeckhoudt (2012).

<sup>22</sup>See Tarazona-Gomez (2004), Deck and Schlesinger (2010), Ebert and Wiesen (2011, 2012), Maier and Rüger (2011) and Noussair et al. (2013).

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