Chapter 2
Magnetic Resonance Imaging

2.1 Bloch Equation

The concept of MRI physics is described by the bloch equations. Consider the weak magnetic field $\overrightarrow{M(t)}$ kept at an angle α (in the anticlockwise direction) with the strong magnetic field $\overrightarrow{B(t)}$ (which is kept in the z-direction as shown in the Fig. 2.1). The interaction between these magnetic fields end up with the torque (The rate of change of angular momentum $\overrightarrow{J(t)}$ is the torque) on the weaker magnetic field $\overrightarrow{B(t)}$ as mentioned in the Eq. (2.1).

$$\frac{\overrightarrow{J(t)}}{dt} = \overrightarrow{M(t)} \times \overrightarrow{B(t)} \tag{2.1}$$

Note that the magnetic moment is proportional to the angular momentum (i.e) $\overrightarrow{M(t)} = \gamma\ \overrightarrow{J(t)} \Rightarrow \frac{d\overrightarrow{M(t)}}{dt} = \gamma\ \overrightarrow{M(t)} \times \overrightarrow{B(t)}$, where γ is gyromagnetic ratio of the magnetic moment $\overrightarrow{M(t)}$.

Let $\overrightarrow{M(t)} = M_x(t)\widehat{i} + M_y(t)\widehat{j} + M_z(t)\widehat{k}$ and $\overrightarrow{B(t)} = B_x(t)\widehat{i} + B_y(t)\widehat{j} + B_z(t)\widehat{k}$ with $B_x(0) = 0$, $B_y(0) = 0$ and $B_z(0) = B0$

$$\Rightarrow \overrightarrow{M(t)} \times \overrightarrow{B(t)} = \begin{bmatrix} i & j & k \\ M_x(t) & M_y(t) & M_z(t) \\ 0 & 0 & B_0 \end{bmatrix}$$

This further implies,

$$\frac{dM_x(t)}{dt} = \gamma M_y(t) B_0 \tag{2.2}$$

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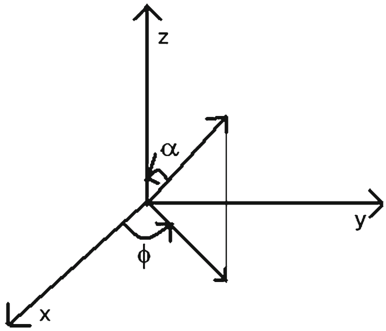

Fig. 2.1 Co-ordinate system illustrating Bloch equation

$$\frac{dM_y(t)}{dt} = -\gamma M_x(t) B_0 \tag{2.3}$$

$$\frac{dM_z(t)}{dt} = 0 \tag{2.4}$$

Let the projection of the intial magnetic moment $\overrightarrow{M}(0)$ with magnitude M_0 kept at an angle α with the magnetic moment $\overrightarrow{B(t)} = B_z(t)\widehat{k} = B_0\widehat{k}$ on the XY-plane is the vector with magnitude $M_{xy}(0) = M_0 \sin(\alpha)$ and it makes an angle ϕ (in the anti-clock wise direction) with the x-axis.

Note that the intial values of the magnetic moment $\overrightarrow{M}(t)$ (with initial magnitude M_0) projected on the three co-ordinates are mentioned as follows.

$$M_x(0) = M_{xy}(0)\cos(\phi) = M_0 \sin(\alpha)\cos(\phi) \tag{2.5}$$

$$M_y(0) = M_{xy}(0)\sin(\phi) = M_0 \sin(\alpha)\sin(\phi) \tag{2.6}$$

$$M_z(0) = M_0 \cos(\alpha) \tag{2.7}$$

To solve the Eq. (2.1), we assign $M_{xy}(t) = M_x + M_y j$, where $j = \sqrt{-1}$. Rewriting jointly the Eqs. (2.2) and (2.3), we get

$$\frac{dM_x(t)}{dt} + j\frac{dM_y(t)}{dt} = \gamma M_y(t)B_z(t) - j\gamma M_x(t)B_z(t)$$

$$\Rightarrow \frac{dM_{xy}(t)}{dt} = -j\gamma B_z(t) M_{xy}(t)$$

Note that $B_z(t)$ is constant and is represented as B_0.

$$\Rightarrow M_{xy}(t) = Ke^{-j\gamma B_0 t}$$

Applying the initial conditions (refer (2.5)–(2.7)) $M_{xy}(0) = M_0 \sin(\alpha)\cos(\phi) + jM_0 \sin(\alpha)\sin(\phi)$, we get

$$\Rightarrow M_{xy}(t) = (M_0 \sin(\alpha)\cos(\phi) + jM_0 \sin(\alpha)\sin(\phi))e^{-j\gamma B_0 t}$$

$$\Rightarrow M_{xy}(t) = M_0 \sin(\alpha)e^{j\phi}e^{-j\gamma B_0 t}$$

$$\Rightarrow M_x(t) = M_0 \sin(\alpha)\cos(\phi - \gamma B_0 t) = M_0 \sin(\alpha)\cos(-\gamma B_0 t + \phi) \quad (2.8)$$

$$M_y(t) = M_0 \sin(\alpha)\sin(\phi - \gamma B_0 t) = M_0 \sin(\alpha)\sin(-\gamma B_0 t + \phi) \quad (2.9)$$

$$M_z(t) = M_0 \cos(\alpha) \quad (2.10)$$

2.2 Comment on the Equations 2.8–2.10

When the weak initial magnetic moment $\overrightarrow{M(0)}$ with magnitude M_0 is kept at an angle α with the strong constant magnetic moment $\overrightarrow{B(t)} = B_z(t)\widehat{k} = B_0\widehat{k}$, due to bloch equation, magnetic moment in the z-direction remains unchanged. But the magnetic moment in the x-direction and the y-direction oscillates with the angular frequency of γB_0 radians or $\frac{\gamma B_0}{2\pi}$ Hz with maximum amplitude $M_0 \sin(\alpha)$. Thus at any particular time instant, the magnitude of the resultant magnetic moment on the X–Yplane is constant and is equal to $M_0 \sin(\alpha)$. Also note that at any particular time instant t, the resultant magnetic moment on the XY-plane is making an angle with magnitude $(-\gamma B_0 t + \phi)$ with the x-axis measured in the anti-clock wise direction. As time goes, the magnitude of the angle is increasing. This implies the resultant vector on the XY-plane rotates in the anti-clock wise direction (when viewed in the z direction) with the frequency γB_0. This frequecy is called larmor frequeny in radians and it is computed as $\frac{\gamma B_0}{2\pi}$ in Hz. For the constant strong magnetic moment B_0, the larmor frequency purey depends on the gyromagnetic ratio of the magnetic moment $\overrightarrow{B(t)}$. Note that the magnitude of the resultant magnetic moment in the XY-plane (transverse plane) is directly proportional to the angle α. Note:Clock wise direction is identified with respect to the view point in the direction of − z axis (refer Fig. 2.1).

2.3 The Larmor Frequency and the Tip Angle α

In general, resultant magnetic moment (without externel strong magnetic moment) obtained in the macroscopic level in the human body is zero. When the human body is kept under the constant strong magnetic moment of B_0 in the z-direction. The resultant magnetic moment in the macroscopic level is aligned to the direction of the external strong magnetic moment (i.e) z-direction. When it is disturbed to bring the resultant magnetic moment to make an angle α (measured anti-clock-wise direction) with the z-axis, there exists the resultant anti-clock-wise rotating magnetic moment in the transverse plane (due to bloch equations), that rotates with the frequency 42.58 Mhz, when B_0 is 1 Tesla.

2.3.1 Disturbance to obtain Non-Zero α Value

The external field (apart from the strong constant magnetic moment B_0) is applied for the short duration (τ) in such a way that the resultant magnetic moment $\overrightarrow{E(t)}$ is rotating exactly with the larmor frequency of the magnetic moment $\overrightarrow{M(t)}$ to be disturbed. It is noted that the macro magnetic moment $\overrightarrow{M(t)}$ obtained using the hydrogen atoms that are aligned in the z-direction due to the availibility of strong magnetic moment B_0 in the z-direction. The interaction between the magnetic moment $\overrightarrow{M(t)}$ aligned in the z-direction with magnitude M_0 and the rotating magnetic moment $\overrightarrow{E(t)}$ on the transverse plane is described by the bloch equations as described below. The strength of the magnetic moment $\overrightarrow{E(t)}$ is strong compared with the natural magnetic moment $\overrightarrow{M(t)}$ available in the human body that are aligned initially in the z-direction. Rewriting the bloch equation using $\overrightarrow{M(t)}$ and $\overrightarrow{E(t)}$, we get the following.

$$\frac{d\overrightarrow{J(t)}}{dt} = \overrightarrow{M(t)} \times \overrightarrow{E(t)} \tag{2.11}$$

$$\Rightarrow \overrightarrow{M(t)} \times \overrightarrow{E(t)} = \begin{bmatrix} i & j & k \\ 0 & 0 & M_z(t) \\ E_x(t) & E_y(t) & 0 \end{bmatrix} \Rightarrow$$

$$\frac{dM_x(t)}{dt} = \gamma E_y(t) M_0 = \gamma E_0 \cos(-\gamma B_0 t + \theta) M_0 \tag{2.12}$$

$$\frac{dM_y(t)}{dt} = \gamma E_x(t) M_0 = \gamma E_0 \sin(-\gamma B_0 t + \theta) M_0 \tag{2.13}$$

$$\frac{dM_z(t)}{dt} = 0 \tag{2.14}$$

Solving the Eqs. (2.12)–(2.14) as described in the Sect. 2.1, we still get the resultant magnetic moment $\overrightarrow{M(t)}$ lies only in the z-direction. It is noted from the equations that the transverse magnetic moment is zero due to the initial conditions $M_x(0) = 0,\ M_y(0) = 0$.

But in practice, due to the external field, there is the disturbance in the resultant magnetic moment and there exist very low magnitude M_x and M_y component that rotates in the larmor frequency due to the existance of strong field B_0 as described in the Sect. 2.1. Now consider the interaction between the magnetic fields $\overrightarrow{E(t)}$ (which has $E_x(t)$ and $E_y(t)$ components) and $\overrightarrow{B(t)}$ (which has $B_z(t) = B_0$ component) on the magnetic moment $\overrightarrow{M(t)}$ which have all the three components.

$$\frac{d\overrightarrow{M(t)}}{dt} = \gamma\ \overrightarrow{M(t)} \times (\overrightarrow{E(t)} + \overrightarrow{B(t)}) \tag{2.15}$$

The resultant magnetic moment $\overrightarrow{M(t)}$ depends upon the first term and the second term of the RHS of the (2.16) independently. The resultant $\overrightarrow{M(t)}$ due to the second term ends up with $M_x(t)$, $M_y(t)$ and $M_z(t)$ components as described in the Eqs. (2.9)–(2.11). Note that the z-component of the resultant vector is constant due to the second term. Now the magnetic moment $\overrightarrow{M(t)}$ due to the first term is obtained as follows. Rewriting (2.15) with only first term of the RHS as

$$\frac{d\overrightarrow{M(t)}}{dt} = \gamma \ \overrightarrow{M(t)} \ \times (\overrightarrow{E(t)}) \tag{2.16}$$

$$\Rightarrow \overrightarrow{M(t)} \times \overrightarrow{E(t)} = \begin{bmatrix} i & j & k \\ M_x(t) & M_y(t) & M_z(t) \\ E_x(t) & E_y(t) & 0 \end{bmatrix} \Rightarrow$$

$$\frac{dM_x(t)}{dt} = -\gamma E_y(t) M_z(t) = -\gamma E_0 \sin(-\gamma B_0 t + \theta) M_0 \cos(\alpha) \tag{2.17}$$

$$\frac{dM_y(t)}{dt} = \gamma E_x(t) M_z(t) = \gamma E_0 \cos(-\gamma B_0 t + \theta) M_0 \cos(\alpha) \tag{2.18}$$

$$\frac{dM_z(t)}{dt} = \gamma (M_x(t) E_y(t) - M_y(t) E_x(t)) \tag{2.19}$$

Solving the Eqs. (2.17)–(2.19) as described in Sect. 2.1, we get the following.

$$\frac{dM_x(t)}{dt} + j\frac{dM_y(t)}{dt} = \gamma E_0 M_0 \cos(\alpha)(-\sin(-\gamma B_0 t + \theta) + jcos(-\gamma B_0 t + \theta)) \tag{2.20}$$

$$\Rightarrow \frac{dM_{xy}(t)}{dt} = j\gamma E_0 M_0 \cos(\alpha) e^{j(-\gamma B_0 t + \theta)} \tag{2.21}$$

$$\Rightarrow M_{xy}(t) = \frac{-E_0 M_0 \cos(\alpha)}{B_0} e^{j(-\gamma B_0 t + \theta)} K,$$

where K is the constant.

As $M_{xy}(0) = M_0 \sin(\alpha)\cos(\phi) + jM_0 \sin(\alpha)\sin(\phi) = M_0 \sin(\alpha) e(j\phi)$ we get K as follows.

$$M_{xy}(0) = \frac{-E_0 M_0 \cos(\alpha)}{B_0} e^{(j\theta)} K$$

$$\Rightarrow \frac{-E_0 M_0 \cos(\alpha)}{B_0} \cos(\theta) K = M_0 \sin(\alpha)\cos(\phi)$$

Also,

$$\frac{-E_0M_0\cos(\alpha)}{B_0}\sin(\theta)K = M_0\sin(\alpha)\sin(\phi)$$
$$\Rightarrow K = \frac{-B_0}{E_0}tan(\alpha)e^{j(\phi-\theta)}$$
$$\Rightarrow M_{xy}(t) = \frac{-E_0M_0\cos(\alpha)}{B_0}e^{j(-\gamma B_0t+\theta)}\frac{-B_0}{E_0}tan(\alpha)e^{j(\phi-\theta)}$$
$$\Rightarrow M_{xy}(t) = M_0\sin(\alpha)e^{j(-\gamma B_0t+\phi)}$$

Note that, the transverse component is not changed due to the external field $\overrightarrow{E(t)}$. What we achieved is that the transverse magnetic moment $M_{xy}(t)$ due to the external field $\overrightarrow{E(t)}$ is in phase as that of the transvere component obtained using the static magnetic field B_0. The resultant transverse magnetic moment due to B_0 and $\overrightarrow{E(t)}$ is given as

$$M_{xy}(t) = 2M_0\sin(\alpha)e^{j(-\gamma B_0t+\phi)} \tag{2.22}$$

The effect of the external magnetic moment $\overrightarrow{E(t)}$ on the z-component of the magnetic moment $\overrightarrow{M(t)}$ is obtained by solving (2.19) as shown below.

$$\begin{aligned}\frac{dM_z(t)}{dt} &= \gamma(M_x(t)E_y(t) - M_y(t)E_x(t))\\ &= \gamma M_0E_0\sin(\alpha)(\sin(-\gamma B_0t+\theta)\cos(-\gamma B_0t+\phi)\\ &\quad - \cos(-\gamma B_0t+\theta)\sin(-\gamma B_0t+\phi))\\ &= \gamma M_0\sin(\alpha)E_0\sin(-\gamma B_0t+\theta-(-\gamma B_0t+\phi)\end{aligned}$$

Thus

$$\frac{dM_z(t)}{dt} = \gamma M_0\sin(\alpha)E_0\sin(\theta-\phi) \tag{2.23}$$
$$\Rightarrow M_z(t) = \gamma M_0\sin(\alpha)E_0\sin(\theta-\phi)t + M_z(0) \tag{2.24}$$

Applying the initial condition $M_z(0) = M_0\cos(\alpha)$ in (2.24), we get $M_z(t) = \gamma M_0\sin(\alpha)E_0\sin(\theta-\phi)t + M_0\cos(\theta)$. Recall that the z-component due to B_0 is $M_0\cos(\alpha)$ and hence resultant z-component is obtained as follows

$$M_z(t) = \gamma M_0\sin(\alpha)E_0\sin(\theta-\phi)t + 2M_0\cos(\alpha) \tag{2.25}$$

It is also noted that the resultant z-component with external field B_1 (instead of B_0) is given as

$$M_z(t) = \gamma M_0\sin(\alpha)E_0\sin(-\gamma(B_1-B_0)+\theta-\phi)t + 2M_0\cos(\alpha) \tag{2.26}$$

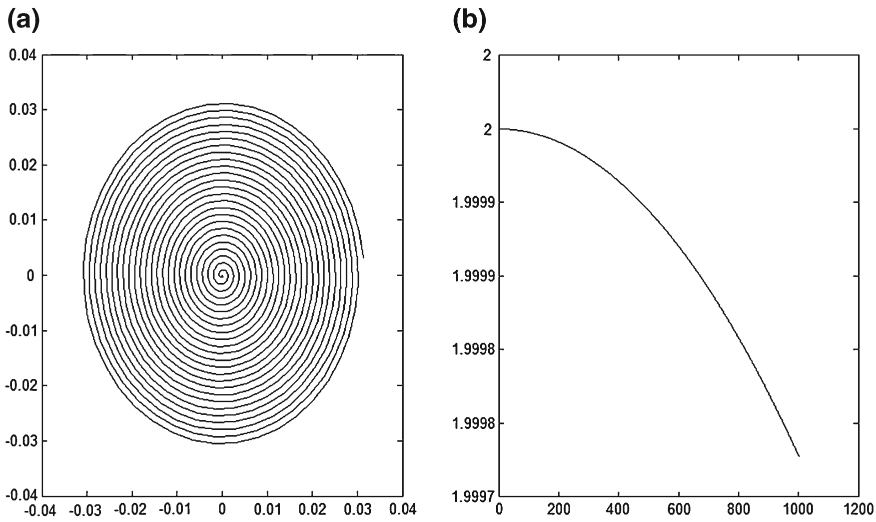

Fig. 2.2 **a** Trace of the transverse component of the resultantant magnetic moment for the duration of $t_{\frac{\pi}{200}}$ (refer Sect. 2.3.2) **b** Illustrating how the z-component of the magnetic moment is decreasing as time increases

2.3.2 Observation on (2.22) and (2.25)

1. Magnitude of the transverse component increases with α value (refer 2.22).
2. z-component decreases with incremental change in the time t (refer 2.25). This is equivalently viewed as the effective increase in α value.
3. This helps in further increasing the magnitude of the transverse component.
4. Note that the resultant magnetic moment is rotating with the larmor frequency.
5. Thus we can imagine that the rotating magnetic moment is moving towards the XY-plane. This is equivalently viewed as the spiral trajectory traced by the transverse component of the $\overrightarrow{M(t)}$ on the XY-plane, while z-component of the $\overrightarrow{M(t)}$ is decreasing along the z-direction (refer Fig. 2.2).
6. This in further looks like the helical movement of the resultant magnetic moment as shown in the Fig. 2.3.

Thus the external magnetic moment $\overrightarrow{E(t)}$ which is applied for the duration $T_{\pi/2}$ helps in bringing the magnetic moment $\overrightarrow{M(t)}$ to the XY-plane. Note that the $T_{\pi/2}$ is also the time duration to make the resultant α value $\frac{\pi}{2}$. It is computed as described below.

The magnitude of the magnetic moment making an angle α with the z-axis be M_0. The z-component of the magnetic moment is given as $M_0 \cos(\alpha)$. The rate at which z-component of the magnetic moment is decreasing is proportional to the rate at which the angle α is changing (refer 2.25). This implies

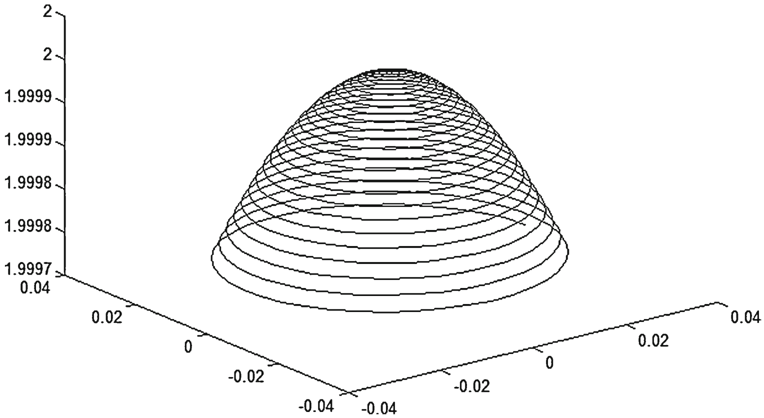

Fig. 2.3 Trace of the resultant magnetic moment in 3D for the duration of $t_{\frac{\pi}{200}}$ (refer Sect. 2.3.2)

$$-M_0 \sin(\alpha)\frac{d\alpha}{dt} = \gamma M_0 \sin(\alpha) E_0 \sin(\theta - \phi) \tag{2.27}$$

$$\Rightarrow \frac{d\alpha}{dt} = -\gamma E_0 \sin(\theta - \phi) \tag{2.28}$$

$$\Rightarrow \alpha(t) = -\int_0^t \gamma E_0 \sin(\theta - \phi) dt \tag{2.29}$$

If the external field is B_1, then $\alpha(t)$ is computed as follows (refer 2.26).

$$\alpha(t) = -\int_0^t \gamma E_0 \sin(-\gamma(B_1 - B_0) + \theta - \phi) dt \tag{2.30}$$

2.3.2.1 resmagneticmoment.m

```
gamma=2*pi*42.58*10^6;
E0=0.0001;
M0=1;
B0=1;
phi=0.1;
theta=0.2;
x=[];
y=[];
z=[];
duration=5.8713*10^(-5);
for t=0:(duration/100000):(duration/100)
alpha1= E0*gamma*t;
x=[x 2*M0*sin(alpha1)*cos(-gamma*B0*t+phi)];
```

```
y=[y 2*M0*sin(alpha1)*sin(-gamma*B0*t+phi)];
z=[z gamma*M0*sin(alpha1)*E0*sin(theta-phi)*t+2*M0*cos(alpha1)];
end
figure
plot3(x,y,z)
figure
subplot(1,2,1)
plot(x,y)
subplot(1,2,2)
plot(z)
```

2.4 Trick on MRI

The external magnetic moment $\overrightarrow{E(t)}$ is applied for the duration $T_{\pi/2}$ to bring the resultant magnetic moment to the transverse plane as described in the Sect. 2.3. After that, when $\overrightarrow{E(t)}$ is removed, the resultant magnetic moment has to rotate with constant magnitude in the XY-plane with the larmor frequency. But in nature, transverse component decreases and reaches zero after some time. This is called spin relaxation. This is due to the spin–spin interactions between the micro-level magnetic moments available in the human body. The time required to obtain $(1/e)$ times the initial value of the transverse component after the removal of the external magnetic moment (represented as T_2) depends on the characteristics of the tissues of human body. The resultant transverse magnetic moment during relaxation (free induction decay (FID)) is recorded using the receiver antenna. This is used to obtain T_2 MRI and proton-density MRI images.

In the same fashion, longituidanal component gradually increases and attains the maximum value. This is due to spin-lattice interactions in the micro level magnetic moments of the human body. The rate at which lognituidanal component reaches the maximum value is described by the time constant T_1 (depends on the characteristics of the tissues of the human body). Usually $T_1 \gg T_2$. After sufficient time (to nullify the influence of existing transverse component), longitudanal component is flipped to the transverse component and the corresponding FID is measured. This is used to obtain the T_1 MRI image. Note that in all the cases (T_1, T_2 and proton-density) MRI images, the receiver records only the transverse components of the magnetic field during relaxation. The complete description is given in Sect. 2.8.

2.5 Selecting the Human Slice and the Corresponding External RF Pulse

When the external RF frequency is same as that of the larmour frequency, we are able to get the transverse component of the magnetic field. When the complete human body is kept under the identical strong magnetic moment B_0, the recorded

FID signal corresponds to the complete human body. But we need to get the image of the particular slice. This is obtained using the concept of Gradient. Let us assume that we need to image the particular slice of the human body along the z-axis. We apply the gradient magnetic moment such that the z-component of the static magnetic field $B_z(z)$ is the function of z. The resultant z-component of the magnetic moment is given as $B_z(z) = G_z z + K$, where G_z is the z-gradient and K is the constant. The constant is chosen such that at the required point z, the magnitude of the magnetic moment is B_0. (i.e)

$$B_z(z) = G_z(z - \bar{z}) + B_0 \tag{2.31}$$

Recollect that the external field used to disturb the orignal magnetic moment $M(t)$ (to obtain non-zero alpha) which are kept under the strong magnetic field B_0 in the z-axis is given as $E_x(t) = -\gamma E_0 \sin(-\gamma B_0 t + \theta)$ and $E_y(t) = -\gamma E_0 \cos(-\gamma B_0 t + \theta)$ (refer 2.12 and 2.13). We can still use the same external field for disturbance. But it controls the slice corresponding to the frequency γB_0. In practice it is difficult to generate such signals. so the alternate technique is to choose the slice with pre-determined thickness. Let us assume, we need to image the slice corresponds to the magnetic field ranging between B_{z_1} and B_{z_2}, where $\bar{z} = \frac{(z_1+z_2)}{2}$. In this case, the external magnetic field is chosen such tha T_α for all the magnetic moments of the chosen slice is identical. It is given as follows.

$$E_x(t) = -A\Delta v \gamma \operatorname{sinc}(\Delta v t) \sin(-\gamma B_0 t + \theta) \tag{2.32}$$

$$E_y(t) = -A\Delta v \gamma \operatorname{sinc}(\Delta v t) \cos(-\gamma B_0 t + \theta) \tag{2.33}$$

It is noted that the external field is having only transverse comonents. Also note that the Δv is the bandwidth in Hz, which is computed as $\frac{\gamma(B_{z_1} - B_{z_2})}{2\pi}$. Using (2.31) we obtain, $\Delta v = \frac{\gamma G_z(z1 - z2)}{2\pi}$. It is noted $\Delta v \ll \frac{B_0 \gamma}{2\pi}$. Using this condition, bloch equations with external fields (refer Sect. 2.3.2 and appendix 1) is solved to obtain the following equation for $\alpha(t)$.

$$\alpha(t, z) = -\int_0^t A\Delta v \gamma \operatorname{sinc}(\Delta v t) \sin(-\gamma(B_z(z) - B_0) + \theta - \phi) dt \tag{2.34}$$

Note that the time instant at which $t = 0$ is the starting time at which the external field is applied. In (2.28), the amplitude (envelope) of the external field is constant. But in this case, the starting time is properly chosen as the envelope is the $A\Delta v \operatorname{sinc}(\Delta v t)$ function. The sinc function is given as $\operatorname{sinc}(\Delta v t) = \frac{\sin \pi \Delta v t}{\pi \Delta v t}$ and is maximum at $t = 0$.

Suppose let us assume the case we are applying the $A\Delta v \operatorname{sinc}(\Delta v t)$ pulse for the duration from $-\infty$ to ∞ (which is not practical). The obtained α values as the function of z (refer Sect. 2.3.2) is obtained as follows.

$$\alpha(z) = -\int_{-\infty}^{\infty} A\Delta v\gamma \operatorname{sinc}(\Delta vt)\sin(-\gamma G_z(z-\bar{z})t+\theta-\phi)dt \tag{2.35}$$

$$\Rightarrow \alpha(z) = Im\left(-\int_{-\infty}^{\infty} A\Delta v\gamma \operatorname{sinc}(\Delta vt)e^{j(-\gamma G_z(z-\bar{z})t+\theta-\phi)}dt\right) \tag{2.36}$$

Consider the following equation as the fourier transformation of the $A\Delta v\gamma$ sin c $(\Delta vt)e^{j(\theta-\phi)}$ with frequency $\gamma G_z(z-\bar{z})$ (in radians)

$$-\int_{-\infty}^{\infty} A\Delta v\gamma \operatorname{sinc}(\Delta vt)e^{j(-\gamma G_z(z-\bar{z})t+\theta-\phi)dt} \tag{2.37}$$

Solving we get the following.

$$\alpha(z) = -A\gamma\sin(\theta-\phi)\operatorname{rect}\left(\frac{\gamma G_z(z-\bar{z})}{2\pi\Delta v}\right) \tag{2.38}$$

$$\Rightarrow \alpha(z) = -A\gamma\sin(\theta-\phi)\operatorname{rect}\left(\frac{(z-\bar{z})}{\Delta z}\right) \tag{2.39}$$

where $\Delta z = \frac{2\pi\Delta v}{\gamma G_z}$

Thus the obtained α for the entire slice is constant. But in practice the sinc envelope is not applied for the infinite duration. It is applied during the duration $-\tau_p/2$ to $\tau_p/2$. This makes the $\alpha(z)$ profile not perfectly flat. From signal processing, we understand that $FT(\operatorname{rect}(\frac{t}{\tau_p})A\Delta v\gamma\operatorname{sinc}(\Delta vt))$ computed with frequency $\gamma G_z(z-\bar{z})$ is same as that of the convolution of $FT(\operatorname{rect}(\frac{t}{\tau_p})$ (computed with frequency $\gamma G_z(z-\bar{z})$) and $FT(A\Delta v\gamma\operatorname{sinc}(\Delta vt))$ (computed with frequency $\gamma G_z(z-\bar{z})$). This implies the following expression for $\alpha(z)$.

$$\alpha(z) = -A\gamma\sin(\theta-\phi)\operatorname{rect}(\frac{(z-\bar{z})}{\Delta z}) * \tau_p\operatorname{sinc}(\tau_p\gamma G_z(z-\bar{z})) \tag{2.40}$$

Thus using the external field (2.32) and (2.33), we obtain almost the identical α over the region of interest (slice region). Thus the particular slice of the human body along the z-axis is selected. Note that external field is applied over the duration $-\tau_p/2$ to $\tau_p/2$.

At the end of time instant $\tau_p/2$ (after acquiring required α value), $M_{xy}(t,z)$ is obtained as follows (refer (2.22)).

$$M_{xy}(\tau_p/2) = 2M_0\sin(\alpha_{\tau_p/2})e^{j(-\gamma B_z(z)\tau_p/2+\phi)} \tag{2.41}$$

$$\Rightarrow M_{xy}(\tau_p/2) = 2M_0\sin(\alpha_{\tau_p/2})e^{j(-\gamma(G_z(z-\bar{z})+B_0)\tau_p/2+\phi)} \tag{2.42}$$

Note that the phase component of (2.42) varies with z as $e^{j(-\gamma(G_z(z-\bar{z})\tau_p/2))}$. To nullify this, negative z-gradient $-G_z$ (along with the existent strong magnetic field B_0) is applied. This is known as Refocussing gradient. Note that the external field (2.32) and (2.33) are removed. When the resultant magnetic moment is having non-zero α and are kept with the strong magnetic field $-(G_z(z-\bar{z})+B_0)$ for the time duration $\tau_p/2$, rotating transverse component (after $\tau_p/2$) is obtained and is given as follows.

$$2M_0\sin(\alpha_{\tau_p/2})e^{j(-\gamma(G_z(z-\bar{z})+B_0)\tau_p/2+\phi)}e^{j(-\gamma(-G_z(z-\bar{z})+B_0)\tau_p/2)} \tag{2.43}$$

$$\Rightarrow 2M_0\sin(\alpha_{\tau_p/2})e^{j(-\gamma(B_0)\tau_p+\phi)} \tag{2.44}$$

Thus the resultant phase component is constant throughout the slice (not the function of z). But note that there is still strong magnetic field B_0 available in the z-axis. Hence transverse magnetic moments along the slice are having the same phase, having non-zero α value and are under the constant magnetic field B_0. Hence transverse components of the magnetic field along the slice follows the equation as mentioned below.

$$2M_0\sin(\alpha_{\tau_p/2})e^{j(-\gamma(B_0)\tau_p+\phi)}e^{j(-\gamma(B_0)t)} \tag{2.45}$$

In (2.45), $t=0$ corresponds to τ_p in the time scale t', where $t'=0$ corresponds to the middle of the sinc pulse applied. As described in the Sect. 2.4, the resultant transverse component (refer 2.46) gradually decreases due to spin–spin interations. This interactions start at the moment when there is non-zero α value. so at time $t=0$ (middle of the RF pulse) itself, the transverse components decreses with time constant T_2. If there is no refocussing gradient and other externally disturbing fields, the disturbance in the transverse component is only due to spin–spin interaction, which is completely described by the time constant T_2. For instance, after applying refocussing gradient, the transverse component of the magnetic moment (including the effect of spin–spin relaxation) is given as follows.

$$2M_0\sin(\alpha_{\tau_p/2})e^{j(-\gamma(B_0)\tau_p+\phi)}e^{j(-\gamma(B_0)t)}e^{\frac{-t}{T_2(x,y)}}e^{\frac{-\tau_p}{T_2(x,y)}} \tag{2.46}$$

Note that T_2 is the function of (x,y) due to different physical characteristics of the tissues. The transverse component of the signal is sampled at some time instant T_R (read-out time instant) depends on the factor $T_2(x,y)$ and helps for T_2-MRI imaging technique.

2.5.1 Summary of the Section 2.5

1. Apply positive z-gradient G_z to select the slice of the human body along the z-axis.
2. RF pulse $A\Delta v\gamma\,\mathrm{sinc}(\Delta vt)\mathrm{rect}(\frac{t}{\tau_p})$ is applied (Note that the duration is between $-\tau_p/2$ and $\tau_p/2$) to obtain the identical α throughout the slice.

3. Negative z-gradient $-G_z$ (refocussing gradient) (for the duration $\tau_p/2$ to τ_p) is applied to achieve the identical strong magnetic field throughout the slice, by nullifying the phase introduced during RF exitation.
4. Step 3 helps in obtaining the transverse magnetic components within the selected slice (in all (x, y) co-ordinates) to rotate with the larmour frequency with zero phase difference for a moment.
5. But the transverse component gradually decreases with time due to spin–spin interaction. This is known as relaxation. The rate at which the transverse components decreases depend on the location (with differnet tissue property at (x, y)) described by the time constant T_2.
6. Transverse components are assumed to start decaying from the time instant $t = 0$, which is the middle of the applied RF pulse.
7. We need to measure the transverse component during relaxation which acts as the first step to obtain MRI image as described in the Sect. 2.6

2.6 Measurement of the Transverse Component Using the Receiver Antenna

In general, the transverse magnetic moment and the longituidanal magnetic moment during the readout phase are given as follows.

$$M_{xy}(x, y, t) = M_{xy}(x, y, 0^+)e^{j(-\gamma(B_0)t+\phi)}e^{\frac{-t}{T_2(x,y)}}$$

$$M_{xy}(x, y, 0^+) = M_z(x, y, 0^-)\sin(\alpha(x, y))$$

$$M_z(x, y, t) = M_z(x, y, 0^+)e^{-t/T_1} + (1 - e^{-t/T_1})B_0$$

$$M_z(x, y, 0^+) = M_z(x, y, 0^-)\cos(\alpha(x, y))$$

To obtain the image that describes the $T_{x,y}$ property of the sliced XY plane (selected slice plane), readout time should be chosen such that the $\alpha(x, y)$ value must be constant throughout the plane. Rewriting the equations with constant $\alpha(x, y)$ is as follows.

$$M_{xy}(x, y, t) = M_{xy}(x, y, 0^+)e^{j(-\gamma(B_0)t+\phi)}e^{\frac{-t}{T_2(x,y)}} \tag{2.47}$$

$$M_{xy}(x, y, 0^+) = M_z(x, y, 0^-)\sin(\alpha) \tag{2.48}$$

$$M_z(x, y, t) = M_z(x, y, 0^+)e^{-t/T_1} + (1 - e^{-t/T_1})B_0 \tag{2.49}$$

$$M_z(x, y, 0^+) = M_z(x, y, 0^-)\cos(\alpha) \tag{2.50}$$

2.6.1 Observation on (2.47)–(2.50)

1. The time instant $t = 0$ in (2.47) is the starting time at which the receiver starts receiving the signal. This is otherwise called as the starting time instance of the readout phase.
2. Note that the transverse component is rotating with identical larmour frequency at all (x,y) positions. This is achieved with the identical strong magnetic field (in the z-axis) throughout the slice.
3. The amplitude $M_{xy}(x, y, 0^+)$ is the function of (x, y) as it involves the hidden term $e^{\frac{-t}{T_2(x,y)}}$ from the time instance of the middle of the RF pulse.
4. Hence if the receiver is designed to receive the transverse component as the function of (x, y), the image completely describes the $T_2(x, y)$ characteristics of the sliced tissue.

2.6.2 Receiver to Receive the Transverse Component

The transverse component $M(x, y, t)$ is represented as the vector $[M_{x,t}\, M_{y,t}]$. Usually $M_{x,t}$ component is sensed as the voltage induced in the the receiver coil as described below. When the receiver coil is excited with the external source to generate the transverse magnetic moment represented as the vector [1 0] and are kept in the transverse magnetic moment (to be sensed) represented as the vector $[M_{x,t}\, M_{y,t}]$, the voltage is induced in the coil as follows.

$$v(t) = -\frac{d}{dt}\int_x\int_y [M_{x,t}\, M_{y,t}] \cdot [1\ 0]dxdy \tag{2.51}$$

The generalized expression for the transverse component of the magnetic moment is given as

$$M(x, y, t) = (M_r + jM_i)e^{-j(\gamma B_0 t-\phi)} \tag{2.52}$$

$$\Rightarrow v(t) = -\frac{d}{dt}\int_x\int_y [(M_r\cos(\gamma B_0 t - \phi) + M_i\sin(\gamma B_0 t - \phi))$$

$$(-M_r\sin(\gamma B_0 t - \phi) + M_i\cos(\gamma B_0 t - \phi))] \cdot [1\ 0]dxdy \tag{2.53}$$

$$\Rightarrow v(t) = \gamma B_0\int_x\int_y (M_r\sin(\gamma B_0 t - \phi) - M_i\cos(\gamma B_0 t - \phi))dxdy \tag{2.54}$$

It is possible to obtain M_r and M_i as follows.

1. Multiply $v(t)$ with $\sin(\gamma B_0 t - \phi)$ and pass it through the low pass filter to obtain $K\gamma B_0 \int_x \int_y M_r dxdy$ component, where K is the constant. Note that phase lock is also required.
2. Similarly, multiply $v(t)$ with $-\cos(\gamma B_0 t - \phi)$ and pass it through the low pass filter to obtain $K\gamma B_0 \int_x \int_y M_i dxdy$ component.
3. Thus the complex numbr $C \int_x \int_y (M_r + jM_i)dxdy$ can be stored in the computer as the complex number, where C is real constant.
4. In MRI imaging, M_r, vM_i are usually the function of time which are represented as $M_r(t), M_i(t)$. The frequency content of the signals $M_r(t)$ and $M_i(t)$ are comparitively very less when compared with the frequency γB_0. Hence the same procedure (as described in 1 and 2) can be used to obtain the complex number $C_1 \int_x \int_y (M_r(t) + jM_i(t))dxdy$ as the function of time, where C_1 is some arbitrary real constant.
5. Sampling the signal at any time instant gives the constant complex number, which can be stored in the computer.

2.7 Sampling the MRI Image in the Frequency Domain

We understand that the the receiver is capable of receiving the real and imaginary component of the signal $s(t)$ (funtion of t) mentioned in (2.55). Recall that the $M_{xy}(x, y, 0^+)$ is the complex quantity.

$$s(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} M_{xy}(x, y, 0^+) e^{j(\phi)} e^{\frac{-t}{T_2(x,y)}} dxdy \tag{2.55}$$

Suppose that the external strong magnetic field (in the z-axis) is made as the function of x and y (i.e) $B_z(x, y) = B_0 + xg_x + yg_y$ (Note that g_x and g_y are constants), we get

$$s(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} M_{xy}(x, y, 0^+) e^{j(-\gamma(B_z(x,y))t+\phi)} e^{\frac{-t}{T_2(x,y)}} dxdy \tag{2.56}$$

$$\Rightarrow s(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} M_{xy}(x, y, 0^+) e^{j(-\gamma(B_0+xg_x+yg_y)t+\phi)} e^{\frac{-t}{T_2(x,y)}} dxdy \tag{2.57}$$

By varying the constants $g_x\, g_y$ described by the variables G_x, G_y respectively, the same receiver is now capable of obtaining the real and imaginary component of the signal $s(G_x, G_y, t)$.

$$s(G_x, G_y, t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} M_{xy}(x, y, 0^+) e^{j(-\gamma(xGx+yG_y)t+\phi)} e^{\frac{-t}{T_2(x,y)}} dxdy \tag{2.58}$$

Sample the obtained complex signal at the middle of the readout time duration ($T_{\text{readout}}/2$), which is the function of G_x and G_y). By vaying different values of G_x and G_y we obtain the real matrix $R(G_x, G_y)$ and the imaginary matrix $I(G_x, G_y)$. Note that G_x and G_y ranges from the $-G_{\max}/2$ to $G_{\max}/2$ with the step increment of $G_{\max}/L$, where L is the level number. Let us assume that the complete slice has to pictured with 101×101 pixel resolution, then the G_x and G_y must have $L = 101$. This is equivalent to sampling the image in frequency domain.

$$C(G_x, G_y) = R(G_x, G_y) + jI(G_x, G_y) \tag{2.59}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} M_{xy}(x, y, 0^+) e^{j(-\gamma(xG_x+yG_y)(T_{\text{readout}}/2)+\phi)} e^{\frac{-(T_{\text{readout}}/2)}{T_2(x,y)}} dxdy \tag{2.60}$$

Apply the linear map of the variable G_x to U and G_x to V as $G_x = (G_{\max}/(L - 1))U(G_{\max}/2)$ and $G_y = (G_{\max}/(L - 1))V - (G_{\max}/2)$ and rewrting the matrix $C(G_x, G_y)$ as $C1(U, V)$, where U ranges from 0 to $L-1$ and $V = 0$ to $L-1$, we get Discrete image in frequency domain. Thus applying the 2D-DFT, we get the discrete version of the following matrix.

$$\text{MRIIMAGE}(x, y) = M_{xy}(x, y, 0^+) e^{\frac{-(T_{\text{readout}}/2)}{T_2(x,y)}} e^{j\phi} \tag{2.61}$$

$$\text{MRIDISCRETEIMAGE}(r, c) = \sum_{0}^{L-1}\sum_{0}^{L-1} C1(U, V) e^{\frac{j2\pi rU}{L}} e^{\frac{j2\pi cV}{L}} \tag{2.62}$$

Note that the $r = 0$ and $c = 0$ indicate that top-left corner position of the discrete image. Also note that r and c ranges from 0 to $L-1$. Hence MRIDISCRETEIMAGE is obtained.

2.8 Practical Difficulties and Remedies in MRI

The Eqs. (2.47)–(2.50) completely describe the transverse components of the magnetic moment during the read-out duration. The equation is valid provided the slice under consideration must satisfy the following conditions.

- The α value and the strong magnetic field B_0 in the z-direction is constant along the z-axis. This is achived using RF exitation followed by refocussing gradient (as described in Sect. 2.5)
- There is no other sources that affect the magnetic field B_0

But in practice, due to perturbation of the magnetic field B_0 and other external magnetic field, the transverse components decreases at the faster rate with time constance T_2^* instead of T_2, with $T_2^* \ll T_2$. Hence the obtained discrete image described in the Sect. 2.7 is not completely due to T_2 characteristics (i.e spin–spin relaxation). If the signal $s(G_x, G_y, t)$ is sampled at time instant $t = 0$ (instead of $T_{\text{readout}}/2$), the obtained image is proton-density MRI image.

2.8.1 Proton-Density MRI Image using Gradient Echo

In practice, sampling the signal $s(G_x, G_y, t)$ at almost near to $t = 0$ gives proton-density image and is achieved as follows.

1. The z-gradient and the RF pulse are applied simultaneously for the duration $\tau_p/2$ to obtain the required α value throughout the selected slice in the z-direction. (refer Sect. 2.5). The value of the α is usually chosen as $\frac{\pi}{2}$.
2. Refocussing z-gradient is applied to obtain the identical magnetic field B_0 in the z-axis, throughout the slice.
3. The transverse component at this moment is given as $M_{xy}(x, y, t) = 2M_0 \sin(\alpha_{\tau_p/2}) e^{j(-\gamma(B_0)\tau_p+\phi)} e^{j(-\gamma(B_0)t)} e^{\frac{-t}{T_2(x,y)}}$
4. Apply the G_y gradient for the duration of τ_y, so that the transverse component becomes $M_{xy}(x, y, t) = 2M_0 \sin(\alpha_{\tau_p/2}) e^{j(-\gamma(B_0)\tau_p+\phi)} e^{j(-\gamma(B_0+G_y y)\tau_y)} e^{j(-\gamma(B_0 t))} \times e^{\frac{-(t+\tau_y)}{T_2(x,y)}}$.
5. Apply the $-G_x$ gradient for the duration of τ_x, so that the transverse component becomes $M_{xy}(x, y, t) = 2M_0 \sin(\alpha_{\tau_p/2}) e^{j(-\gamma(B_0)\tau_p+\phi)} e^{j(-\gamma(B_0+G_y y)\tau_y)} \times e^{j(-\gamma(B_0-G_x x)\tau_x)} e^{j(-\gamma(B_0 t))} e^{\frac{-(t+\tau_y+\tau_x)}{T_2(x,y)}}$ (It is assumed that there is no significant change in α value.)
6. The read-out phase starts at this moment. Postive gradient G_x is applied during the read-out phase for the duration of $2\tau_x$. The resultant magnetic moment during the read-out phase is given as mentioned in (2.63).

$$M_{xy}(x, y, t) = 2M_0 \sin(\alpha_{\tau_p/2}) e^{j(-\gamma(B_0)\tau_p+\phi)} e^{j(-\gamma(B_0+G_y y)\tau_y)} e^{j(-\gamma(B_0-G_x x)\tau_x)} \times e^{j(-\gamma(B_0+G_x x)t)} e^{\frac{-(t+\tau_y+\tau_x)}{T_2(x,y)}}. \tag{2.63}$$

7. The phase component introduced due to G_x cancels in the middle of the read-out phase. This is known as Gradient echo. This helps to synchronize the hardware to sample the real and imaginary part of the signal at the end of the read-out phase ($2\tau_x$) to obtain the sample value of the magnitude of the signal $s(t)$ corresponding to the particular location in the K-space ((i.e) G_x, G_y).
8. Wait for the complete relaxation ((i.e) equillibrium) until all the longituidal components reaches B_0. This can also be done using spoiler gradient in modern techniques.

9. The steps 1–7 are repeated for the complete range of G_x and G_y (refer Sect. 2.7) and hence discrete MRI image in the frequency domain is obtained.
10. Apply the inverse 2D-DFT to obtain MRI image. The image thus obtained corresponds to proton-density image. This gives the proton-density (refer Sect. 3.1 for illustration) of every pixel of the image slice.

2.8.2 T_2 MRI Image Using Spin–Echo and Carteisian Scanning

T_2 MRI principles are explained with the micro-level behaviour of the randomly oriented individual magnetic moments (with various rate at which the phase is changing) at every point (x,y) across the slice. Due to the slice selection (along the z-axis) by applying the z-gradient, followed by RF exitation and refocussing gradient we are able to obtain the in-phase resultant magnetic moment making an angle α with the z-axis (measured anti-clockwise when viewed along the z-axis). The corresponding transverse component is making an angle ϕ with the x-axis (measured anti-clockwise when viewed along the z-axis).

The individual magnetic moments at the particular position (x, y) after the release of RF exitation, starts to experience different phase (even though it was made inphase due to external RF exitation). This is the natural phenomenon due to spin–spin interaction. The phase achived by the individul magnetic moment over the time helps in decreasing the resultant transverse component. The rate at whcich the the phase of the individual magnetic moment is changing purely depends on the tissue. The rate at which the resultant transverse magnetic moment is decreasing is described by the factor $T_2(x, y)$ and hence $T_2(x, y)$ plays the important role in knowing the characteristics of the tissue and hence image is obtained. But in practice the rate at which the resultant transverse component is characterized by the factor T_2^*. Even after the transverse component becomes zero due to the factor T_2^*, the dephasing operation still continuos. This leads to the technique called spin echo (described below) to obtain the non-zero transverse component, (even after reaching zero due to T_2^*). This in further helps to obtain the frequency sample of the MRI image highlighting the T_2 values as described below.

1. Steps 1–4 are performed similar to the technique mentioned in the Sect. 2.8.1.
2. Thus the currently obtained transverse component is given by $M_{xy}(x, y, t) = 2M_0 \sin(\alpha_{\tau_p/2})e^{j(-\gamma(B_0)\tau_p+\phi)}e^{j(-\gamma(B_0+G_y y)\tau_y)}e^{j(-\gamma(B_0 t))}e^{\frac{-(t+\tau_y)}{T_2(x,y)}}$.
3. Apply the G_x gradient for the duration of τ_x, so that the transverse component becomes (2.65). $M_{xy}(x, y, t) = 2M_0 \sin(\alpha_{\tau_p/2})e^{j(-\gamma(B_0)\tau_p+\phi)}e^{j(-\gamma(B_0+G_y y)\tau_y)} \ldots$

$$e^{j(-\gamma(B_0+G_x x)\tau_x)}e^{j(-\gamma(B_0 t))}e^{\frac{-(t+\tau_y+\tau_x)}{T_2(x,y)}} \tag{2.64}$$

It is assumed that there is no significant change in α value.

4. The exponentially decreasing term $e^{\frac{-t}{T_2(x,y)}}$ in (2.64) describes the micro-level behaviour of the individual magnetic moments (spin–spin interactions) at (x, y). Thus the equation can also be written with micro-level behaviour of the individual magnetic moments as follows.

$$\sum_n M_{xy}(x, y, t, n) = \sum_n 2M_0 \sin(\alpha_{\tau_p/2}) e^{j(-\gamma(B_0)\tau_p+\phi_n(t,x,y))} e^{j(-\gamma(B_0+G_y y)\tau_y)} \times e^{j(-\gamma(B_0+G_x x)\tau_x)} e^{j(-\gamma(B_0 t))}.$$

where, $M_{xy}(x, y, t, n)$ is the nth micro-level magnetic moment which is the function of x, y and t.

5. Now apply the 180° RF pulse. This is not same as that of the RF pulse. This helps in changing the phase component of the transverse component from arbitrary ρ to $-\rho$. Note that the selection gradient(G_z) is applied while applying 180° pulse.
6. After applying 180° pulse, the resultant transverse magnetic moment is given as

$$\sum_n M_{xy}(x, y, t, n) = \sum_n 2M_0 \sin(\alpha_{\tau_p/2}) e^{j(+\gamma(B_0)\tau_p-\phi_n(t,x,y))} e^{j(+\gamma(B_0+G_y y)\tau_y)} \times e^{j(+\gamma(B_0+G_x x)\tau_x)} e^{j(-\gamma(B_0 t))}.$$

7. The magnitude of the signal $M_{xy}(x, y, t)$ at every pixel corresponding to the transverse component in step 5 is decreasing gradully with time (due to dephasing). Hence the magnitude of $M_{xy}(x, y, t)$ corresponding to the transverse component in step 6 increases with time (refer Sect. 3.2.2 for illustration) and reaches maximum after some time duration. This is known as spin–echo. Spin–echo guarantees the existence of required amplitude of MRI signal for sampling.
8. Read-out phase starts immediate after some time (required time for rephasing) after applying 180° pulse along with the positive x-gradient G_x for the duration of τ_x. The resultant transverse component during read-out phase is given as

$$\sum_n M_{xy}(x, y, t, n) = \sum_n 2M_0 \sin(\alpha_{\tau_p/2}) e^{j(+\gamma(B_0)\tau_p-\phi_n(x,y,t))} e^{j(+\gamma(B_0+G_y y)\tau_y)} \times e^{j(+\gamma(B_0+G_x x)\tau_x)} e^{j(-\gamma((B_0+G_x x)t))}.$$

9. After time duration of τ_x, there is the cancellation of the phase introduced due to G_x gradient (upto step 8). This is known as Gradient echo. This helps to synchronize the hardware and sample the magnitude of $s(t)$ at the end of the reading phase $2\tau_x$ corresponding to the particular location in the K-space. This step is same as that of the one used in proton-density imaging using phase

gradient. But what we achived is the sampled value gives the information about $T_2(x, y)$. (refer Sect. 3.2) for illustration.

10. Note that the sampled value corresponds to $(G_x, -G_y)$, not (G_x, G_y).

2.8.3 T_2 MRI Image Using Spin–Echo and Polar Scanning

1. Steps 1–3 are performed similar to the technique mentioned in the Sect. 2.8.1.
2. Thus the currently obtained transverse component is given by

$$M_{xy}(x, y, t) = 2M_0 \sin(\alpha_{\tau_p/2})e^{j(-\gamma(B_0)\tau_p+\phi)}e^{j(-\gamma(B_0)t)}e^{\frac{-t}{T_2(x,y)}}.$$

3. Apply both G_x and G_y gradient simulataneously for the time duration τ_{xy}, so that the transverse components become the following.

$$M_{xy}(x, y, t) = 2M_0 \sin(\alpha_{\tau_p/2})e^{j(-\gamma(B_0)\tau_p+\phi)}e^{j(-\gamma(B_0+G_y y+G_x x)\tau_{xy})} \times e^{j(-\gamma(B_0 t))}e^{\frac{-(t+\tau_{xy})}{T_2(x,y)}}$$

4. The technique used in the steps 4–7 of the Sect. 2.8.2 are adopted.
5. Read-out phase starts immediate after some time (required time for rephasing) after applying 180° pulse along with the positive x-gradient G_x and positive y-gradient G_y for the duration of τ_{xy}. The resultant transverse component during read-out phase is given as

$$\sum_n M_{xy}(x, y, t, n) = \sum_n 2M_0 \sin(\alpha_{\tau_p/2})e^{j(+\gamma(B_0)\tau_p-\phi_n(x,y,t))} \times e^{j(+\gamma(B_0+G_x x+G_y y)\tau_{xy})}e^{j(-\gamma((B_0+G_x x+G_y y)t))}.$$

6. After time duration of τ_{xy}, there is the cancellation of the phase introduced due to G_x and G_y gradient (upto step 4). This is gradient echo. This helps to synchronize the hardware to sample the magnitude of $s(t)$ at the end of the reading phase $2\tau_x$ corresponding to the particular location in the K-space. This step is same as that of the one used in phase gradient. But what we achived is the sampled value gives the information about $T_2(x, y)$. (refer Sect. 3.3) for illustrations.
7. Note that the sample value corresponds to the point $(-G_x, -G_y)$.
8. This is the polar version of the technique used in Sect. 2.8.2. The main difference is that the gradients are applied simultaneously. Also the value of G_x and G_y are changed in such a way that the samples of the k-space (frequency domian) are uniformly scanned over the variables r and θ, where $r = \sqrt{(G_x^2 + G_y^2)}$ and $\theta = tan^{-1}\left(\frac{G_y}{G_x}\right)$.

2.8.4 T_1 MRI Image

We understand that there is the natural relaxation in the transverse component of the resultant magnetic moment due to spin–spin interaction. This is known as T_2 relaxation. We exploit the different relaxation time and different proton-density (refer Sects. 2.8.1–2.8.3) for the different tissues to obtain MRI image (T_2 and proton-density respectively). In the same way, due to spin–lattice interaction, the longintu-idanal components gradually increases. This is the another natural relaxation known as T_1 relaxation (which is independent of T_2 relaxation). Different tissues have different T_1 relaxation time. This helps to obtain the another type of MRI image known as T_1 MRI image. Usually T_1 is very larger when compared with the corresponding T_2. Exploiting this property, the following trick is used to obtain the T_1 MRI image.

1. Steps 1–3 in Sect. 2.8.1 is performed.
2. Wait for the time to get almost zero transverse-component. But by this time, longituidal component gradually increases (not reached maximum).
3. At this moment, flip the longituidanal component by an angle α (usually 90°) to obtain the transverse component. Sample the transverse component immediate after obtaining the transverse component (i.e at $t = 0$ during read-out phase) to obtain the K-space of the T_1 image for the particular G_x and G_y. The operation is repeated for the required range of G_x and G_y values. Apply 2D-IDFT to obtain the T_1 image. Instead of sampling the transverse component at time instant $t = 0$, gradient echo can also be used to measure the transverse component as described in the Sect. 2.8.1.
4. Note that T_1 image is obtained by measuring the transverse component. But the measured transverse component is mainly due to longituidanal relaxation obtained from spin–lattice interaction. The trick is to flip the longituidal component, which are in relaxation state to obtain the transverse component. The magnitude of the transverse component describes the T_1 characteristics of the tissue. Thus the image obtained is known as T_1 image (refer Sect. 3.4 for illustration).

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