Chapter 2
Introducing Lesson Play

In Chap. 1, we offered an example of a lesson plan that satisfied many of the goals of reform-based teaching. Of course, as we know, there can be an enormous distance between planned lessons and implemented lessons. Indeed, when working with prospective teachers, we noticed that they were able to produce impressive lesson plans but, when we observed them teaching mathematics, the careful attention to the use of manipulatives, to problem-based learning and to group work was almost swept away by their actual interactions with students. In these interactions, we saw the same kind of moves that have been reported in the literature such as:

- An emphasis on procedural thinking (Crespo et al. 2010)
- A tendency to ask fact-based questions rather than questions that invite mathematical reasoning (Vacc 1993)
- The use of misleading or erroneous mathematical explanations
- A tendency to position the textbook or the teacher as the mathematical authority in the classroom (Herbel-Eisenmann and Wagner 2007)

These observations led us to believe that prospective teachers needed help in developing more strategies needed to achieve their global goal of reform-based teaching. They needed to think about and pay attention to the way in which they asked questions, responded to students, and provided direction. In the next section, we provide a brief overview of how we came to develop the idea of lesson play that is used in this book. We then provide an example of a lesson play and point to the particular opportunities it offers for helping teachers develop the kinds of moves they need to respond to the complex environment of the reform-based mathematics classroom.

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Developing the "Lesson Play"

As mentioned above, the idea of lesson play grew out of our frustration with 'good' lesson plans that did not attend, or had no place to attend, to what we consider important features in planning for instruction. Over the past 7 years it evolved from a general instruction to "write a play as an imagined interaction" to an explicit request to attend to a presented problematic, the way it could have emerged and the way it could be resolved. This alternative attends to John's (2006) suggestion that "the lesson plan should not be viewed as a blueprint for action, but should also be a record of interaction" (p. 495). In Chap. 3 we outline the evolution of the lesson play task from infancy to the stage of its current implementation. However, in the next section we invite the reader to consider several potential in-class interactions and an example of a lesson play.

Potential Interactions

Imagine the following interaction, in which a teacher is asking students to identify whether different numbers are prime.

Teacher: Everyone finished? Good. Let's check the rest of the numbers. How about 91?
Rita: 91 is prime.

Although the student is an imaginary one, her statement is not uncommon, as evidenced in the literature (Zazkis and Campbell 1996a). How might you respond to this student? You are unlikely to follow-up in this manner:

Teacher: Everyone finished? Good. Let's check the rest of the numbers. How about 91?
Rita: 91 is prime
Teacher: You are wrong. 91 is 7 times 13.

Instead, you will probably want to let Rita engage in some mathematical reasoning. We challenge you to take 5 minutes and actually write down the next five or six exchanges. Perhaps you want to incorporate the voices of other students in the class. We think you will find that actually selecting the words that you use to respond to the student takes some thought, and you will probably find yourself editing your first attempt. You will certainly notice that there are many options available, perhaps more than you had first considered. For example, consider the two options offered below.

Prime Follow-up A		Prime Follow-up B	
Teacher:	Everyone finished? Good. Let's check the rest of the numbers. How about 91?	Teacher:	Everyone finished? Good. Let's check the rest of the numbers. How about 91?
Rita:	91 is prime.	Rita:	91 is prime.
Teacher:	What is a prime number?	Teacher:	I'm going to ask you to add one more column of your 12 by 12 multiplication table.

In Prime Follow-up A, the teacher's question carries with it the assumption that Rita does not understand what it means to be a prime number. The teacher's imagined trajectory looks like this: first, establish a correct definition of prime number; then, when Rita uses this definition for 91, she will find that it is not prime. Presumably, Rita has already encountered the definition for prime number, but the teacher might assume she does not remember it. In Prime Follow-up B, the teacher assumes that Rita thinks that numbers not in the multiplication table are prime. The teacher's imagined trajectory is thus to extend the multiplication table, which will enable Rita to see the number 91 appear, which will lead her to recognize that 91 is not prime. We note that both options communicate the fact that Rita is wrong, without saying so explicitly. But each option will play out very differently in the classroom and affect the way Rita will think of prime numbers and, even, the way she thinks of mathematics—in Prime Follow-up A, mathematics is framed as an activity based on definitions while in Prime Follow-up B, it is an activity involving computation.

While the lesson plan makes quite clear the content in focus (identifying prime numbers), the lesson play and the dialogue between the teacher and the students draws much more attention to the process through which that content will be communicated in the classroom. At a mathematical level, the imagined verbal exchanges necessarily bring into focus both the actual use of mathematical language in communicating and the forms in which ideas are explained or justified. At the pedagogical level, the imagined exchange articulates assumptions about how students are thinking and how their thinking might be changed; it also articulates possible teaching trajectories. And, as shown in the two options above, the lesson play suggests something about the very nature of learning without falling into any pre-fixed pedagogical "ism".

In our work with prospective teachers, we ask them to continue the exchange far beyond the follow-ups exemplified above. Not only do they have to imagine what they would say, as teachers, but also how students might respond. We also invite them to imagine what might have happened *before* a given prompt. So, just as we provided, in Chap. 1, a model lesson plan, we offer here a model lesson play based on the prompt offered at the beginning of the section (which appears at the beginning of Scene 2, in this play). As you read, we invite you to think about the different assumptions the teacher made about the students and to try to identify the general teaching trajectory that the playwright had in mind. What do you notice about the way the teacher asks questions or responds to the students? What choices has the teacher made about her use of mathematical language?

A Sample Lesson Play

Scene 1

1		*(Students were given a list of numbers and asked to determine which ones are prime and which ones are composite, and to explain their decisions. After about* 5 *min of silent individual work, some students are half way through the task, while others are hesitating. The teacher decides to check some of the work to assure students are on the right track.)*
2	Teacher	So, class, let's check what we have come up with so far. Please pay attention, I know you have not finished, you can continue later. Let's start with the first number on our list—23. Is it prime or composite? Yes, Susan.
3	Susan	Prime.
4	Teacher	Okay, and why do you say this?
5	Susan	Because nothing goes into it.
6	Teacher	Goes into?
7	Susan	I mean nothing divides it.
8	Teacher	Nothing? Nothing at all?
9	Maria	She means no numbers other than 23 and 1. You can write it as 23 times 1, but no other options.
10	Teacher	Good. So rather than "nothing", we say 23 has exactly 2 divisors, 23 and 1.
11	Susan	And also when we worked with chips we could only put them in one long line, and you could not make another rectangle without leftovers.
12	Teacher	Indeed, excellent. Let's move on. How about 34, is it prime or composite? Yes, Jamie.
13	Jamie	Composite.
14	Teacher	And you say this because …
15	Jamie	Because it is even.
16	Teacher	So? Please explain.
17	Jamie	We know it is even, right, and if it is even it has 2 in it.
18	Teacher	Has 2 in it? Hmm, I see 34, I see a 3 and a 4. Where is the 2?
19	Maria	What he means is 2 is a factor. Even numbers have 2 as a factor, so it cannot be prime.
20	Teacher	So you are saying that an even number cannot be prime?
21	Maria	Sure. All even numbers are 2 times something, so they are not prime. Primes are odd.
22	Teacher	And what about the number 2?
23	Jamie	2 is prime, and 2 is even.
24	Teacher	So I am confused here. Can you help?
25	Maria	Sure. No need for confusion. What I mean to say is 2 is an exception. It is the only even prime because it is in the very beginning. The other primes are odd. 2 is the only exception.

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26	Teacher	Okay, good. We figured this out. Let us proceed—68?
27	Marty	Composite of course. We just said that even numbers, not 2, but bigger even numbers cannot be prime. So no need to go over even numbers on the list, they are all composite.
28	Teacher	Does everyone agree? Great, so this makes our work easier, of course. Let's go over odd numbers only. The next on our list is 19, Kevin?
29	Kevin	It is composite because … it almost looks like prime but then I remembered in my times tables it is 7 times 7. And the same is with the next one, 63, it is 7 times 9.
30	Teacher	Very good. Your multiplication tables helped you decide. Okay. Now let us take a few more minutes and complete the work. If you have already decided whether each number is prime or composite, please turn to problem 7 on page 106.

Scene 2

31		*(Students continue to work on their own. Some are just finishing up with the list of numbers provided while others have moved onto working on the problem in the textbook.)*
32	Teacher	Everyone finished? Good. Let's check the rest of the numbers. How about 91?
33	Rita	91 is prime.
34	Teacher	And you say so because?
35	Rita	It is not anywhere on the times tables.
36	Teacher	Interesting. So are you saying that only composite numbers are on our multiplication tables?
37	Rita	*(hesitating)* That's what Kevin said and you said "Okay."
38	Teacher	What exactly did Kevin say?
39	Rita	That 49 is 7 times 7 and 63 is 7 times 9 on the times tables. And he is right, and you said "Okay", and 91 is not there.
40	Teacher	I see. When do we say that a number is prime?
41	Students	2 factors only, no factors other than itself and 1.
42	Teacher	So if 63 is 7 times 9, what do we know about its factors?
43	Tina	We know it has 7 and 9 as its factors.
44	Teacher	Exactly, that is why it cannot be prime. But is it possible that 91 has factors that are not on our multiplication table?
45	Rita	*(hesitating)* No, I think, because it is smaller than 100.
46	Teacher	Let's look at 34. Can you find it on the table *(pointing to a* 12 by 12 *multiplication table mounted on the wall).*
47	Tina	It is not there, but it is even. So for even numbers no need to look at the table. We KNOW they are not prime. Like 38 is also not on the tables but it is not prime.
48	Teacher	So we cannot find 34 and 38 on the tables, but they are not prime. Isn't this strange?
49	Rita	Yeah, because they are even, but 91 is not even.
50	Teacher	I see. Let's look at… look at *(thinking)* an odd number … 39.
51	Tina	It is not on the tables.
52	Teacher	So what are you saying?
53	Rita	I say it is 3 times 13, so I say it is composite.
54	Teacher	Isn't it interesting! Can we find another ODD number that is NOT on the tables, but is composite?

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55	Kevin	51?
56	Mary	65 and 75 and 85 and 95!
57	Teacher	Anything else?
58	Mark	57.
59	Teacher	Good. Let's gather all these numbers you found, that are not on the tables and are odd and composite, and write them as products, show them in multiplication. So we have 39, 51, 57, 65, 75, 85, and 95.
60	Mark	Mary's are easy, because they all are 5 times something.
61	Teacher	Nice observation, but let's work out all of them.
62	Students	*(pause)* $39 = 3 \times 13$, $51 = 3 \times 17$, $57 = 3 \times 19$, $65 = 5 \times 13$, $75 = 5 \times 15$, $85 = 5 \times 17$, $95 = 5 \times 19$.
63	Teacher	Very nice. Now, I look carefully at all these COMPOSITE numbers, and I wonder, why are they not on our multiplication table?
64	Rita	Because there are big numbers you are timesing by, and the table does not go that far.
65	Teacher	So where does this bring us with respect to 91?
66	Rita	That what we said, it is not on the times tables, was wrong. I mean it is right that it is not there, but it does not mean it is prime. So this was wrong. It is 7×13. It is not prime, it is composite. Actually, all the people at my table said it was prime, but now we figured it out. It is not prime because it is 7×13, so it has these factors.
67	Teacher	Excellent, Rita. Is it clear to everyone what she said?
68	Mark	She said that we cannot use the times tables to decide what is prime.
69	Teacher	*(smiles)* Yes, that's basically it. Right. So NOW I have a challenge for the class. Let us find ALL the composite numbers that are ODD and that DO NOT appear anywhere on the multiplication table.

Of course, the lesson plan that led to this particular interaction could also have led to millions of others. Thus, what we are interested in here are the particular goals, choices, and assumptions that can be seen within the imagined interactions. We focus first on the mathematical features of the interaction, and then turn our attention to the pedagogical ones. Our intention is not to separate the mathematical from the pedagogical, but to use these two lenses as ways of analyzing the lesson play.

In terms of the mathematical features then, we elaborate on two main points. First, the lesson play deals explicitly with the use of mathematical language. The teacher is constantly attending to the students' language. For example, the teacher repeats Susan's use of the vague phrase "goes into" [5, 6] in an effort to prompt more precise mathematical language. Later, the same thing happens with Jamie's use of "has 2 in it" [17, 18]. Both Jamie and Susan may see the teacher's words as simple synonyms for their own, but in the lesson play, the teacher offers the more precise vocabulary that will be needed for effective communication about prime numbers, not just for Jamie and Susan, but for their classmates as well. The teacher's responses not only offer alternative ways of talking about composite numbers, but also show how nonmathematical language such as "has 2 in it" can be communicatively misleading (since 34 clearly has no 2 in it). This close attention to language, and to the need for precision in communication cannot be separated from the content in question, but it is specific to the way in which the content is worked on in the classroom. Broadly, we might say that the teacher works to bridge the students' everyday language to formal mathematical language (see Herbel-Eisenmann 2002). While such a goal might be included in a lesson plan, the lesson play offers the specific details of how and when this happens.

In addition to the language focus, the lesson play also makes explicit the various forms of mathematical reasoning that might emerge in the classroom. For instance, when Maria makes the argument that "all even numbers are 2 times something, so they are not prime" [21], the teacher evaluates the argument and proposes a counter-example [22]. This occurs again with respect to Rita's claim about composite numbers appearing on the times table [35, 36, 50]. In both cases, the students have made quite a reasonable inference, perhaps even a necessary one given their current experiences, and the teacher must recognize them and then devise ways in which the students can come to more appropriate inferences. The actual counter-examples used by the teacher (2 for Maria and 39 for Rita) are highly specific in their responsiveness, and emerge directly from the dialogue.

In the lesson play, we can also identify specific "pedagogical moves" that the teacher makes in order to sustain the interaction. We have already noted the attention to language, but the teacher's way of working with language involves some "re-voicing" of students' statements. This move enables the teacher to acknowledge the student's statement while also offering a mathematically preferable rendition. So, for example, the teacher re-voices Maria's statement about prime numbers by saying "Good. So rather than 'nothing,' we say 23 has exactly 2 divisors, 23 and 1" [10]. Another example of re-voicing comes later on, when the teacher re-voices Rita's response as a conjecture (that numbers not on the times table are prime [36]) that Rita can then investigate.

In addition to instances of re-voicing, we can also attend to the kinds of questions that the teacher asks. We know from research that teachers tend to ask fact-based questions that require little reasoning (Vacc 1993). For example, after Rita says that 91 is a prime, the teacher might ask fact-based questions such as "Is 91 on the times table?" or "What is 91 divided by 13?" The first requires the student to scan her times table and the second requires her to undertake a calculation. Neither necessarily involves reasoning. In this lesson play, the teacher chooses to ask the question "And you say so because?" [34]. By asking this question, the teacher is able to elicit the student's reasoning and use it to help Rita see how this reasoning leads to a contradiction. Unlike in Prime Follow-up B, the teacher does not *assume* that Rita's error involves the multiplication table. Further, unlike Prime Follow-up B, the teacher does not immediately engage Rita in calculation but, instead, re-voices Rita's response as a conjecture.

Re-voicing and probing student thinking (through reasoning-based questions) are two of the "talk moves" that Chapin et al. (2009) identify as promoting classroom discussion. In many mathematics classrooms, the interaction follows what is known as the IRE format (initiation-response-evaluation), which leads students through a predetermined set of information and does little to encourage students to express their thinking (Cazden 2001; Nystrand 1997). In promoting "talk moves", Chapin et al. seek alternative interactions that engage students and foster reasoning. So, while we can focus on re-voicing or reasoning-based questioning as talk moves, it is important also to zoom out somewhat and consider the kind of interaction that follows from these moves.

In her work on mathematics discussions in the classroom, Wood (1998) identifies two forms of classroom interaction: focusing and funneling. Similar to IRE, funneling occurs when the teacher asks a series of questions that guide the students through a procedure or to a desired end. In this situation, the teacher is engaged in cognitive activity and the student is merely answering the question to arrive at the solution, often without seeing the connection among the questions. Consider how the following lesson play differs from the one offered above.

Teacher	Everyone finished? Good. Let's check the rest of the numbers. How about 91?
Rita	91 is prime.
Teacher	I am going to ask you to add one more column of your 12 by 12 multiplication table.
Rita	Okay. I will add the column for 13.
Teacher	And what do you notice?
Rita	I see that 91 is there.
Teacher	What are its factors?
Rita	13 and 7.
Teacher	So is it prime?
Rita	No.

In this example, although the teacher asks some open questions (such as "What do you notice"), the teacher is focused on getting Rita to find the factors of 91. In this scene of a lesson play, the teacher does not find out why Rita thinks 91 is prime. Nor does the teacher enable Rita to make sense of her generalization from the previous class (drawing on the interaction with Kevin). Indeed, in examining the lesson plays written by prospective teachers, based on a prompt in which a student mistakenly identified 91 as prime (see Chap. 6) we have found that the vast majority of them lead students through a process of extending the multiplication table. This is not, of course, an incorrect method, but it leads to a funneled discussion in which the interaction is necessarily pre-determined—which does not make for a very interesting discussion!

In contrast to funneling, focusing requires the teacher to listen to the students' responses and to guide them based on what the students are thinking rather than how the teacher would solve the problem. Achieving this kind of focusing interaction can be very challenging, and requires the use of moves that go beyond simple initiation and feedback. Indeed, in the model lesson play we offered, the teacher needs to deal with Kevin's generalization, with multiples of 2 and 5 that are not on the times table, as well as with counter-examples involving composite numbers that are odd. Instead of having a fixed endpoint to the discussion, the teacher must remain responsive to the student and open to the possibility that the student pursues a method of solving the problem that is initially unknown. This does not mean that the teacher does not have a goal. Indeed, we can see in the model lesson play that the teacher wants to help Rita see that there are many numbers that are composite—and that Rita knows are composite—that are not on the times table. The teaching trajectory is thus to help Rita refute the implicit conjecture about the times table by considering the numbers that are not on it and thus revisiting the idea of what it means to be prime.

In terms of the pedagogical features of the lesson play, we wish also to draw attention to some aspects of its format. The structure of the lesson play—as a dialogue occurring over time with possibilities for different points of view—allows for the portrayal of the messy, sometimes repetitive interactions of an inquiry-based classroom. This structure stands in stark contrast to a necessarily ordered and simplified list of actions such as: take up homework, state definition, provide examples, give problems, and evaluate solutions. In this lesson play, we see the teacher revisiting definitions of “prime” and “composite” that were used in Scene 1 with the help of new ideas that emerge in Scene 2, such as the multiplication table. The lesson play communicates the fact that the meanings of definitions change for students as they encounter new examples or problems. It also probes the way in which student interpretations can lead to unexpected consequences.

For example, at the beginning of Scene 2, we see Rita defending her claim that 91 is prime because it is not on the multiplication table: “That’s what Kevin said and you said ‘Okay’. ” [37]. Here the teacher has the option of proposing a counter-example, returning to the definition of prime, or arguing about the context of her response to Kevin. The lesson play tests out these different options by ‘running’ them like a script and seeing how Rita (and other students) might respond. Being interpretations, these different options can now be critiqued, so that decisions can be evaluated. In contrast to a lesson plan, which may be “good” or “bad”, the lesson play, as an interpretation, invites questioning about the different ways in which teachers might respond to students, and the different conditions under which students might build understandings.

This leads to a final point about the lesson play that relates to its ‘playfulness’. By its very nature, the lesson play requires a focus on specific and particular imagined interactions. In a lesson plan, one can include directives such as “call on different students to answer questions”. In a lesson play, those students must be named, individually, and the playwright has to decide quite explicitly whether, for example, Tina or Rita will answer a teacher’s question. The playwright is forced to consider whether it is more important to make Tina follow through or to give Rita a chance to participate. This may, at one level, sound trivial, but we see it as part of the imaginative work that teachers must do to prepare and practice for the classroom—much the same way children practice routines of communication in their self talk.

By being forced to make a choice, one must follow through with the consequences of each option, and one might even find it necessary to evaluate the outcomes of different choices. Further, the playwright must do this imaginative work not only for the teacher (the role she will eventually play), but also for the students—the playwright must try to think or talk like a student. We conjecture that this type of role-playing might help teachers develop better models of students' conceptual schemes (see Steffe and Thompson 2000). While crafting lesson plays cannot replace real experiences of teaching or of listening to student ideas, it can help teachers develop a larger repertoire of possible actions and reactions.

Virtual Planning: What the Lesson Might Be

Lesson planning is limited in its ability to allow teachers to prepare for teaching. Its very structure is built around generalities and well laid plans in the absence of students' questions and alternate conceptions of the topic being taught. Having realized these limitations, teacher educators have attempted to introduce prospective teachers to students' thinking by other means. Analysis of video-clips—which has gained popularity with the advances of video technology—is one way to draw attention to the detail of communication and is considered to be an effective tool in teacher education (Maher 2008). This may include the study of effective teaching and the revisiting of one's own teaching. Analysis of video-clips helps prospective teachers examine the relationship between a teacher's actions and students' learning, study subtle details of classroom interactions and, as a result, become more aware of their practice and inform their future planning.

While not diminishing the importance of discussion and reflection provided by the examination of video-clips, we feel that lesson play requires prospective teachers to practice and play in the particulars of their own. Centrally, the lesson play provides an opportunity to imagine the future, being informed by the past, rather than reexamine the past. Its structure is built around the specific conceptions of a particular student, or group of students, learning the details of a mathematical concept, with the preciseness of mathematical language, through the relationship of teaching. It is not a description of how things will occur in the classroom, but an imagined account of how things might occur in a virtual space. We hypothesize that through several instances of detailed planning for such detailed encounters a prospective teacher can build up general strategies that allow for improvisation in other contexts.

In this chapter, we have drawn primarily on a model lesson play that we designed ourselves. One problem with such a model play is that it masks some of the challenges that a teacher experiences when attempting to create lesson plays (and to teach!). Our goal, however, was to provide an example of what a lesson play looks like and how it can evoke aspects of teaching that are not made explicit in traditional lesson planning. In the next section of the book, however, we will be looking at the plays that prospective teachers have created and use them as a lens into their images of teaching in general and images of teaching mathematics in particular.

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