

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Finite Markov Chains</b>	<b>5</b>
2.1	A Few Examples	5
2.2	Transition Probability Matrix	10
	Two-Step (Three-Step, etc.) Transition Probabilities	10
2.3	Long-Run Properties	13
2.4	Classification of States	18
2.5	Periodicity of a Class	22
2.6	Regular Markov Chains	29
2.A	Inverting Matrices	31
	Inverting (Small) Matrices	31
	Inverting Matrices (of Any Size)	32
	Exercises	33
<b>3</b>	<b>Finite Markov Chains II</b>	<b>39</b>
3.1	Absorption of Transient States	39
	Lumping of States	39
	Reducing Recurrent Classes to Absorbing States	40
	Large Powers of a Stochastic Matrix	50
3.2	Reversibility	56
3.3	Gambler's Ruin Problem	57
	Game's Expected Duration	59
	Corresponding Variance	61
	Distribution of the Game's Duration	63
3.A	Solving Difference Equations	64
	Nonhomogeneous Version	66
	Complex-Number Arithmetic	67
	Exercises	69

<b>4</b>	<b>Branching Processes</b>	73
4.1	Introduction and Prerequisites	73
	Compound Distribution	74
4.2	Generations of Offspring	75
	Generation Mean and Variance	76
4.3	Ultimate Extinction	79
	Total Progeny	81
4.A	Probability-Generating Function	86
	Exercises	88
<b>5</b>	<b>Renewal Theory</b>	91
5.1	Pattern Generation	91
	Runs of $r$ Consecutive Successes	92
	Mean and Variance	95
	Second, Third, etc. Run of $r$ Successes	96
	Mean Number of Trials (Any Pattern)	97
	Breaking Even	99
	Mean Number of Occurrences	101
5.2	Two Competing Patterns	103
	Probability of Winning	104
	Expected Duration	106
5.A	Sequence-Generating Function	109
	Exercises	109
<b>6</b>	<b>Poisson Process</b>	111
6.1	Basics	111
	Correlation Coefficient	115
6.2	Various Modifications	115
	Sum of Two Poisson Processes	116
	Two Competing Poisson Processes	116
	Nonhomogeneous Poisson Process	118
	Poisson Process in More Dimensions	120
	$M/G/\infty$ Queue	122
	Compound (Cluster) Poisson Process	125
	Poisson Process of Random Duration	127
	Exercises	129
<b>7</b>	<b>Birth and Death Processes I</b>	133
7.1	Basics	133
7.2	Pure-Birth Process	135
	Yule Process	136
7.3	Pure-Death Process	138
7.4	Linear-Growth Model	140
	Mean and Standard Deviation	141
	Extinction	141

7.5	Linear Growth with Immigration	144
7.6	$M/M/\infty$ Queue	147
7.7	Power-Supply Problem	148
7.A	Solving Simple PDEs	150
	Extension	153
	Exercises	158
<b>8</b>	<b>Birth-and-Death Processes II</b>	161
8.1	Constructing a Stationary Distribution	161
	More Examples	164
8.2	Little's Formulas	166
8.3	Absorption Issues	167
8.4	Probability of Ultimate Absorption	169
8.5	Mean Time Till Absorption	171
	Exercises	174
<b>9</b>	<b>Continuous-Time Markov Chains</b>	177
9.1	Basics	177
9.2	Long-Run Properties	180
	Stationary Probabilities	180
	Absorption Issues	182
9.A	Functions of Square Matrices	187
	Multiple Eigenvalues	189
	Applications	190
	Exercises	193
<b>10</b>	<b>Brownian Motion</b>	197
10.1	Basics	197
10.2	Case of $d = 0$	198
	Reaching $a$ Before Time $T$	199
	Reaching $y$ While Avoiding 0	200
	Returning to 0	203
10.3	Diffusion with Drift	205
10.4	First-Passage Time	208
	Inverse Gaussian Distribution	210
	Exercises	212
<b>11</b>	<b>Autoregressive Models</b>	215
11.1	Basics	215
	White Noise	216
	Markov Model	216
11.2	Yule Model	220
	Stability Analysis	225
	Partial Serial Correlation	227
11.3	General Autoregressive Model	228

11.4	Summary of $AR(m)$ Models . . . . .	230
11.5	Parameter Estimation . . . . .	233
	Markov Model . . . . .	234
	Maximum-Likelihood Estimators . . . . .	235
	Yule Model . . . . .	235
11.A	Normal Distribution and Partial Correlation . . . . .	237
	Univariate Normal Distribution . . . . .	237
	Bivariate Normal Distribution . . . . .	238
	Conditional Distribution . . . . .	238
	Multivariate Normal Distribution . . . . .	239
	Finding MLEs of $\mu$ and $\Sigma$ . . . . .	242
	Partial Correlation Coefficient . . . . .	245
	General Conditional Distribution . . . . .	246
	Exercises . . . . .	249
<b>12</b>	<b>Basic Probability Review</b> . . . . .	251
12.1	Probability . . . . .	251
	Boolean Algebra . . . . .	251
	Probability . . . . .	252
	Random Variable . . . . .	254
	Multivariate Distribution . . . . .	254
	Moments . . . . .	257
	Probability-Generating Function . . . . .	260
	Moment-Generating Function . . . . .	260
	Convolution and Composition of Two Distributions . . . . .	261
12.2	Common Distributions . . . . .	264
	Discrete Type . . . . .	264
	Continuous Type . . . . .	265
<b>13</b>	<b>MAPLE Programming</b> . . . . .	269
13.1	Working with Maple . . . . .	269
	Maple Worksheet . . . . .	270
	Library Commands . . . . .	271
	Lists and Sequences . . . . .	271
	Integral Calculus . . . . .	272
	Plotting . . . . .	273
	Loops . . . . .	274
	Linear Algebra . . . . .	275
	Statistics . . . . .	277
	Typical Mistakes . . . . .	278
	<b>References</b> . . . . .	281
	<b>List of Abbreviations</b> . . . . .	283
	<b>Index</b> . . . . .	285

<http://www.springer.com/978-1-4614-4056-7>

Informal Introduction to Stochastic Processes with  
Maple

Vrbik, J.; Vrbik, P.

2013, X, 287 p. 54 illus. With online files/update.,

Softcover

ISBN: 978-1-4614-4056-7