

Chapter 2

Methods of Macroeconomic Analysis and Parametric Control of Equilibrium States in a National Economy

Conducting a stabilization policy on the basis of the results of macroeconomic analysis of a functioning market economy is an important economic function of the state.

The $AD-AS$, IS , LM , $IS-LM$, $IS-LM-BP$ models, as well as the Keynesian model of common economic equilibrium for a closed economy and the model of a small country for an open economy [39], include one of the efficient instruments of macroeconomic analysis of the functioning of a national economy.

In previously published literature, we can see how these models are used for carrying out a macroeconomic analysis of the conditions of equilibrium in national economic markets. But there are no published results in the context of the estimation of optimal values of the economic instruments on the basis of the Keynesian model of common economic equilibrium and the model of an open economy of a small country in the sense of some criteria, as well as an analysis of the dependence of the optimal criterion value on exogenous parameters.

Based on the dependence of a solution to a system of algebraic equations on its coefficients, we propose an approach to parametric control of the static equilibrium of a national economy that reduces to the estimation of optimal values of economic instruments as a solution of some respective mathematical program.

In this chapter, we construct IS , LM , and $IS-LM$ models of the Keynesian all-economic equilibrium and a small open national economy. We also present results of macroeconomic analysis and parametric control of the static equilibrium of a national economy.

2.1 Macroeconomic Analysis of a National Economic State Based on IS, LM, and IS-LM Models, Keynesian All-Economy Equilibrium. Analysis of the Influence of Instruments on Equilibrium Solution

This section elaborates the construction of the IS, LM, and IS-LM models as well as the Keynesian model of common economic equilibrium using an example of the economy of the Republic of Kazakhstan, analysis of the influence of economic instruments to the equilibrium conditions in the respective markets, as well as the estimation of optimal values of economic instruments on the basis of the Keynesian mathematical model of common economic equilibrium [39].

2.1.1 Construction of the IS Model and Analysis of the Influence of Economic Instruments

Let's introduce the notation for economic indices used for model construction: T is the tax proceeds (to the state budget, in billions of tenge); S is the net savings, billions of tenge; I is the investment in the capital asset, billions of tenge; G is the public expenses, billions of tenge; Y is the gross national income, billions of tenge; and C is the household consumption, billions of tenge.

Macroeconomic estimation of the equilibrium conditions in the wealth market can be done on the basis of the IS model [39, p. 76] represented as

$$T + S = I + G. \quad (2.1)$$

The tax proceeds T to the state budget represented by the expression $T = T_Y Y$ has the following econometric estimation based on statistical information for the years 2000–2008:

$$T = 0.2207 Y. \quad (2.2)$$

(0.000)

The statistical characteristics of model (2.2) are as follows: the determination coefficient $R^2 = 0.986$; the standard error $Se = 209.5$; the approximation coefficient $A = 10.47\%$; the Fisher statistics $F = 581.66$. The statistical significance of the coefficient of regression (2.2), as well as the regressions estimated below, is given within parentheses under the respective coefficients of the regressions in the form of p -values.

The net savings S represented by the expression $S = a + S_y Y$ has the following econometric estimation:

$$S = -366.055 + 0.222 Y. \quad (2.3)$$

(0.000) (0.000)

The statistical characteristics of model (2.3) are as follows: The determination coefficient $R^2 = 0.994$; the standard error $Se = 69.2$; the approximation coefficient $A = 11.47\%$; the Fisher statistics $F = 1287.2$; the Durbin–Watson statistics $DW = 1.96$.

The investment in capital assets represented by the expression $I = a + I_i i$ after estimating the parameters of this model using statistical information becomes the following:

$$I = 1367.9 - 81.3 i + 0.2751 Y_{\text{mean}}. \quad (2.4)$$

(0.02) (0.03) (0.00)

The statistical characteristics of model (2.4) are as follows: The determination coefficient $R^2 = 0.99$; the standard error $Se = 126.8$; the approximation coefficient $A = 4.2\%$; the Fisher statistics $F = 326.48$; the Durbin–Watson statistics $DW = 1.72$. Substituting into (2.4) the value of the mean nominal gross national income for the years 2000–2008 in billions of tenge, $Y_{\text{mean}} = 6662.7$, finally yields the following model for the investment:

$$I = 3202 - 81.3 i. \quad (2.5)$$

Substituting expressions (2.2), (2.3), and (2.5) into (2.1), we obtain the IS model representation in the following form:

$$-366.055 + 0.222Y + 0.2207Y = 3202 - 81.3 i + G_{200X}, \quad (2.6)$$

which allows us to determine the equilibrium value of i for the given values of Y and G_{200X} . From macroeconomic theory, a method [39, p. 77] of plotting the IS curve exists, which is the set of combinations of the equilibrium values of Y and i (Fig. 2.1).

From the model IS₂₀₀₇ (Fig. 2.1), it follows that the equilibrium GNI₂₀₀₇ with interest rate 13.6% equals 11,602.75 billion tenge, and the real GNI₂₀₀₇ with interest rate 13.6% equals 11,371 billion tenge, which shows a lack of wealth in the considered market. From the model IS₂₀₀₈ (Fig. 2.1), it follows that the equilibrium GNI₂₀₀₈ with interest rate 15.3% equals 13,957.91 billion tenge, while the real GNI₂₀₀₈ with the interest rate 15.3% equals 13,734 billion tenge, which also shows a lack of wealth within that market.

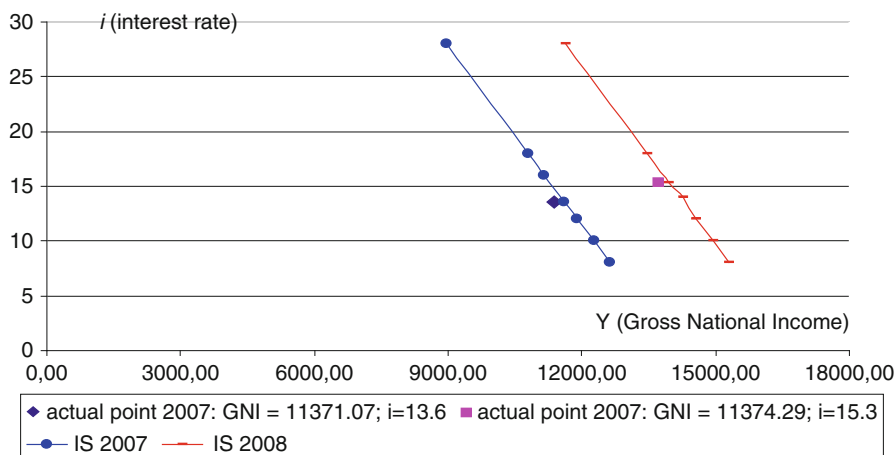


Fig. 2.1 Plots of IS_{2007} and IS_{2008} models

Table 2.1 Consequences of changing public expenses and taxation

Consequence	Action	
	Public expenses increase by ΔG	Taxes decrease by ΔT
National income increases by	$\frac{1}{T_y + S_y} \Delta G = 2.26 \Delta G$	$\frac{C_{y^v}}{T_y + S_y} \Delta T = 1.3 \Delta T$
Budgeted deficit increases by	$\left(1 - \frac{T_y}{T_y + S_y}\right) \Delta G = 0.5 \Delta G$	$\left(1 - \frac{T_y C_{y^v}}{T_y + S_y}\right) \Delta T = 0.7 \Delta T$

To estimate the multiplicative effects [41, p. 78] of the economic instruments T_y and G , we'll construct an econometric model of the consumption of households C , which, on the basis of statistical information for the years 2000–2008, is given by

$$C = 428.68 + 0.552Y^v, \\ (0.000) \quad (0.000)$$

where $Y^v = Y - T_y Y$, $C_{y^v} = 0.552$. The statistical characteristics of this model are as follows: The determination coefficient $R^2 = 0.999$; the standard error $SE = 68.92$; the approximation coefficient $A = 1.78\%$; the Fisher statistics $F = 5394$; the Durbin–Watson statistics $DW = 1.53$.

Table 2.1 presents the expressions and values of the multipliers [39, p. 83] of the instruments T_y and G derived on the basis of the IS model (2.6).

Let's estimate the multiplicative effects of the instruments T_y and G based on the data for the year 2008. According to that data, we have $G = 3859.98$, $Y = 13,734.3$, and $T_y = 0.2207$. Now, let's change G to $\Delta G = 579$. This change, in accordance with the multiplier of ΔG , results in an increment of GNI by the value $\Delta Y = 1308.54$.

Table 2.2 Value of the money aggregate $M3$ and the velocity of money

Year	GNI	Value of money aggregate $M3$	V , velocity of money
2007	11,371	4,629.8	2.5
2008	13,734	6,266.4	2.2

Also, from the data of year 2008, we have $G = 3859.98$, $Y = 13,734.3$, and $T_y = 0.2207$. Let's replace T_y by $\Delta T_y = -0.01$. This change, in accordance with the multiplier of ΔT_y , results in an increment of GNI by the value $\Delta Y = 328.37$. The derived results agree with the macroeconomic theory that considers the influence of the economic instruments on the changes in the domestic national income, which is represented by Table 2.1, "Consequences of changing public expenses and taxation" [39, p. 83].

2.1.2 Macroeconomics of Equilibrium Conditions in the Money Market

The macroestimation of equilibrium conditions in the money market can be realized on the basis of the LM model represented as follows [41, p. 111]:

$$M = l_{pr} + l_{tr}, \quad (2.7)$$

where M is the money supply, in billions of tenge; l_{pr} is the volume of property (deposits in deposit organizations by sectors and currencies), in billions of tenge; l_{tr} is the volume of transaction [the volume of credits given by second-level banks (SLB) taking into account the money velocity], in billions of tenge.

To estimate the money velocity, let's use the Fisher equation [39, p. 112]:

$$MV = Y,$$

where V is the money velocity, Y is the nominal GNI, and the money aggregate $M3$ is accepted in the Fisher equation as the active money volume M .

Estimation of the money velocity by the expression $V = Y/M$ on the basis of the statistical information for the years 2007–2008 is presented in Table 2.2.

The value of the money supply represented in the Fisher equation by the aggregate $M3$ can be checked again through its estimation determined by yearly values of the money base and the money multiplier μ .

The money multiplier μ is defined by the following relation [39, p. 99]:

$$\mu = \frac{1 + \gamma(1 - \alpha - \beta)}{\alpha + \beta + \gamma(1 - \alpha - \beta)},$$

Table 2.3 Values of multipliers

Year	α	β	γ	Values of multipliers		
				Deposit	Credit	Money
2007	0.143	0.043	0.250	2.565	2.087	3.087
2008	0.045	0.069	0.252	2.969	2.632	3.632

Table 2.4 Calculated values of money supply and values of money aggregate

Years	Calculated values of money supply	Values of money aggregate $M3$
2007	4,519.9	4,629.8
2008	5,343.6	6,266.4

where $\alpha = RR/D$ is the normative of minimal reserve;

$\beta = ER/D$ is the coefficient of cash remainders of the commercial banks;

$\gamma = CM/K$ is the share of money in cash in the total sum of credits of the commercial banks;

RR the minimal reserves;

D is the check (current) deposits (we used the information about deposits in the deposit organizations by sectors and currencies);

ER is the excess reserves;

K is the credits of the commercial banks accepted in accordance with the expression K_1/V ;

K_1 is the statistical information about the given credits;

CM is the active money in cash.

Estimates of the money supply M by the money bases for the years 2007–2008 and values of μ for the same period are respectively equal to the following: For the year 2007, $M = \mu H = 4519.9$ billion tenge; for the year 2008, $M = \mu H = 5343.6$ billion tenge.

Table 2.4 presents the calculated values of the money supply and the values of the money aggregate $M3$ by years. Table 2.4 shows that the calculated values of M and values of the money aggregate $M3$ are of the same order and close to each other. Taking into consideration this fact, together with the result on the money velocity derived above, in this specific analysis we accept the calculated values as the money supply, and the actual values of credits of the second-level banks are corrected subject to the money velocity.

The property demand represented by the expression $l_{pr} = e^{a+l_i i}$ has the following econometric estimate:

$$l_{pr} = 438883.3 \times 0.66^i. \quad (2.8)$$

(0.000) (0.01)

The regression coefficients are statistically significant, although we have the coefficient of determination $R^2 = 0.33$, the standard error $Se = 0.6$, and the Fisher

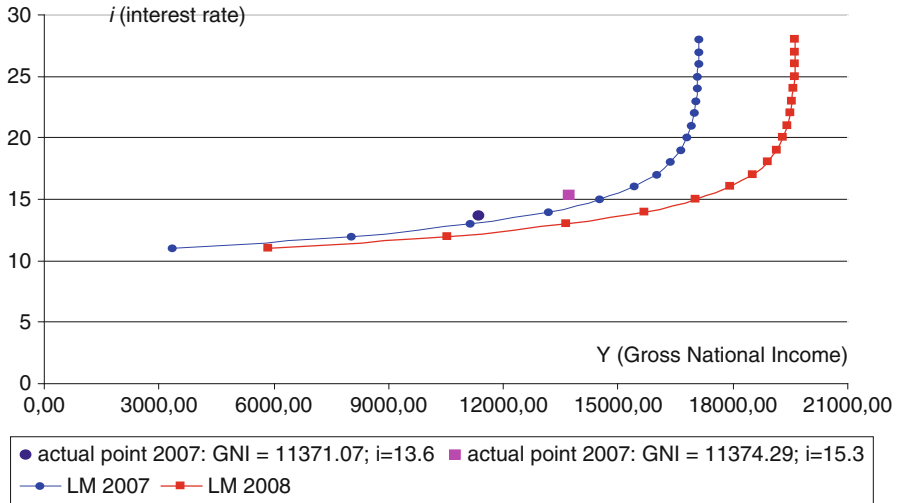


Fig. 2.2 Plots of LM_{2007} and LM_{2008} models

statistics $F = 67$. The demand for money for transactions represented by the expression $l_{tr} = a + bY$ describes the following econometric estimation:

$$l_{tr} = -1062.85 + 0.326 Y. \quad (2.9)$$

(0.0005) (0.0000)

The statistical characteristics of model (2.9) are as follows: The determination coefficient $R^2 = 0.965$, the standard error $SE = 267$, and the Fisher statistics $F = 193.7$.

Substituting expressions (2.8) and (2.9) into (2.7), we obtain the representation of the LM model in the following form:

$$M_{200X} = 438883.3 \times 0.66^i - 1062.85 + 0.326 Y, \quad (2.10)$$

which allows us to determine the equilibrium value of i for the given values of Y and M_{200X} . In macroeconomic theory, a method exists [39, p. 113] to plot the LM curve, which is the set of combinations of the equilibrium values of Y and i . Figure 2.2 presents the plots of the LM models for the years 2007 and 2008.

In accordance with the obtained results and plotted LM_{2007} , LM_{2008} , one can conclude that the actual values of Y and i for the years 2007–2008 are situated above the respective curves LM_{2007} , LM_{2008} , which shows the relatively low demand for the monetary assets.

The alarming aspect is that the actual state in which the money market found itself in the year 2008 corresponds to a higher mean market interest rate than in the year 2007, whereas the whole line LM for 2008 is situated below and to the right of

Table 2.5 Joint equilibrium and actual values of Y and i

	Actual values		Joint equilibrium conditions	
	i , interest rate of SLB, %	Y , gross domestic income, billion tenge	i^*	Y^* , Keynesian effective demand
2007	13.6	11,371.1	13.23	11,670.89
2008	15.3	13,734.3	13.29	14,327.31

the respective line for 2007; that is, the same volume of GNI corresponds to a lower equilibrium interest rate than that of a year before. This is an indirect indicator that the government has regulated the money market based on the necessity of making money cheaper, but the second-level banks reacted to those signals in the opposite way, raising the commercial rate. Exactly the same situation occurred in 2008 in most developed countries on the threshold of the economic crisis.

2.1.3 Macroestimation of the Mutual Equilibrium State in Wealth and Money Markets. Analysis of the Influence of Economic Instruments

On the basis of the derived IS and LM models, the model for macroestimation of the joint equilibrium state in the wealth and money markets can be represented by the following system:

$$\begin{cases} -366.055 + 0.222Y + 0.2207Y = 3202 - 81.3i + G_{200x}, \\ M_{200x} = 438833.3 \times 0.66^i - 1062.85 + 0.326Y. \end{cases} \quad (2.11)$$

The results of solving system (2.11) to estimate the joint equilibrium state in the wealth and money market for the years 2007 and 2008 are presented in Table 2.5. The plots of the IS and LM models in the same period are shown in Fig. 2.3.

From Fig. 2.3, it follows that the coordinates of the effective demand point for years 2007 and 2008 are respectively represented by $Y^*_{2007} = 11,670.89$; $i^*_{2007} = 13.23$, and $Y^*_{2008} = 14,327.31$; $i^*_{2008} = 13.29$. The points of the actual state of the economy of the Republic of Kazakhstan in 2007 and 2008 are respectively situated to the left of the corresponding IS_{2007} and IS_{2008} plots and above the respective LM_{2007} and LM_{2008} plots. Such location of the points of the actual economic state means a respective lack of wealth market and an excess of money in the money market in 2007 and 2008.

Let's estimate the influence of the instruments G and M on the joint equilibrium conditions using the data for the year 2008.

By the results of the solution of system (2.11), on the basis of the data from 2008, we have that $G = 3,859.98$ and $M = 5,343.6$. Let's now increase G by $\Delta G = 579$. With unchanged M , this fluctuation results in an increase of the Keynesian effective demand – GNI up to 15,522 billion tenge and an increase of the interest rate up to

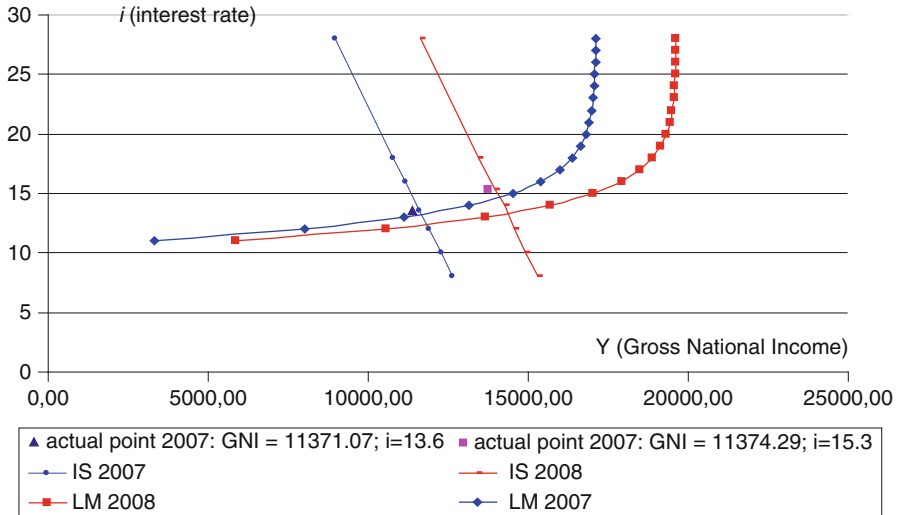


Fig. 2.3 Plots of IS_{2007} , LM_{2008} , LM_{2007} , and LM_{2008} models

13.9% due to the shift of IS to the right as a result of the multiplicative effect from increasing the public expenses.

Let's now increase M_{2008} by $\Delta M = 534$. With unchanged G_{2008} , this fluctuation results in an increase of GNI up to 14,438.6 billion tenge and a decrease of the interest rate to 12.7% due to the shift of IS to the right as a result of the multiplicative effect from increasing the money supply.

The obtained results also agree with the macroeconomic theory on the influence of the economic instruments in the wealth and money markets [39, pp. 78, 114].

2.1.4 Macroestimation of the Equilibrium State on the Basis of the Keynesian Model of Common Economic Equilibrium. Analysis of the Influence of Economic Instruments

The Keynesian mathematical model of common economic equilibrium on the basis of the IS and LM models, as well as the econometric function of the labor supply price and the econometric expression of the production function, is given by the following [39, p. 223]:

$$\begin{cases} T(Y) + S(Y) = I(i) + G, & (2.12) \end{cases}$$

$$\begin{cases} M = l(Y, i), & (2.13) \end{cases}$$

$$\begin{cases} W^S(N, P) = PY_N, & (2.14) \end{cases}$$

$$\begin{cases} Y = Y(N), & (2.15) \end{cases}$$

where $W^s(N, P)$ is the function representing the labor supply price, Y_N is the derivative of the production function, and $Y(N)$ is the production function.

Equations (2.12) and (2.13) of the common economic equilibrium model are given by the respective IS and LM equations (2.11).

The econometric representation of the labor supply price using the statistical data for the years 2000–2008 is given by

$$W^s(N, P) = 60.12P - 0.007N, \quad (2.16)$$

(0.000) (0.000)

where P is the level of prices for year 2000, and N is the busy population in thousands per capita. The respective p -values (of t -statistics) in the equation in W^s are presented in parentheses below the regression coefficients. The results of the analysis of the statistical significance of the model for W^s are as follows: The determination coefficient $R^2 = 0.99$, the standard error $Se = 3.37$, the Fisher statistics $F = 522.6$, and the approximation coefficient $A = 7.4\%$.

The econometric representation of the production function $Y(N)$ using the statistical data for the years 2000–2008 is given by

$$Y = -5.654N + 0.0009N^2. \quad (2.17)$$

(0.000) (0.000)

The results of analysis of statistical significance of the model for W^s are as follows: The determination coefficient $R^2 = 0.98$, the standard error $Se = 1227$, and the Fisher statistics $F = 172$.

The Keynesian model of common economic equilibrium on the basis of relations (2.11), (2.16), and (2.17) is given by

$$\begin{cases} -366.055 + S_y Y + T_y Y = 3202 - 81.30i + G_{200X}, \\ M_{200X} = 438883.3 \times 0.66^i - 1062.85 + 0.326Y, \\ 60.12P - 0.00698N = -5.65P + 0.0018NP, \\ Y = -5.65N + 0.0009N^2. \end{cases} \quad (2.18)$$

In this system describing the behavior of the macroeconomic subjects, the exogenously given parameters include the value of public expenses G and the nominal values of the money in cash M . The values of five endogenous parameters, Y^* , i^* , P^* , N^* , and W^* , that result in attaining equilibrium simultaneously in all three mentioned markets are determined from the solution of this system of equations.

Substituting the actual values of G_{200X} and M_{200X} of the respective year and solving system (2.18), we obtain the values of variables that are in equilibrium simultaneously in all three markets.

Table 2.6 Comparative analysis of actual and equilibrium values of GNI, interest rate, level of prices, busy population

		Y	i	P	N
2007	Actual	11,371.10	13.60	1.789	7,631.10
	Equilibrium	11,670.89	13.23	1.050	7,751.60
	Deviation	2.64%	-0.37	-0.740	1.58%
2008	Actual	13,734.30	15.30	1.959	7,857.20
	Equilibrium	14,327.30	13.30	1.103	8,048.80
	Deviation	4.32%	-2.00	-0.900	2.44%

Table 2.6 presents the equilibrium values of the endogenous parameters by using the solution of system (2.18) on the basis of the data for the years 2007 and 2008.

Let's estimate the influence of instruments G and M on the Keynesian common economic equilibrium from the data from 2008.

Increasing G by $\Delta G = 579$ while keeping the values of M results in an increase of the GNI to 15,522.6 billion tenge and a decrease in the interest rate to 13.9%, while at the same time unemployment drops by 1.6% and the level of prices increases to 1.12.

Increasing M_{2008} by $\Delta M = 534.4$ while keeping the values of G results in an increase of the GNI to 14,438.56 billion tenge and a decrease of the interest rate to 12.68%, while unemployment is reduced by 0.15%, and the level of prices increases insignificantly to 1.105.

Increasing G by $\Delta G = 579$ and increasing M_{2008} by $\Delta M = 534.4$ result in an increase of the GNI to 15,658.85 billion tenge and a decrease of the interest rate to 13.15%, while unemployment is reduced by 1.77% and the level of prices increases to 1.13.

2.1.5 Parametric Control of an Open Economic State Based on the Keynesian Model

Estimation of the optimal values of the instruments M and G for the given external exogenous parameters S_y , T_y on the basis of model (2.18) for the year 2008 in the sense of the GNI criterion gives

$$Y \rightarrow \max. \quad (2.19)$$

This estimate can be obtained by solving the following mathematical programming problem.

Problem 1

On the basis of mathematical model (2.18), find the values of (M, G) maximizing criterion (2.19) under the constraints

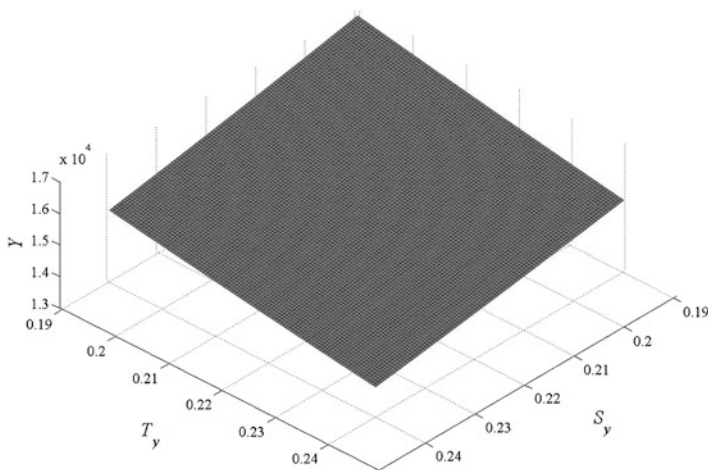


Fig. 2.4 Plot of dependence of optimal values of criterion Y on parameters T_y , S_y

$$\begin{cases} |M - M^*| \leq 0.1M^*, \\ |G - G^*| \leq 0.1G^*, \\ |N - N^*| \leq 0.1N^*, \\ |P - P^*| \leq 0.1P^*, \\ |i - i^*| \leq 0.1i^*, \\ |Y - Y^*| \leq 0.1Y^*. \end{cases} \quad (2.20)$$

Here M^* and G^* are the respective actual values of the money and public expenses supplies in 2008. The symbol (*) for the unknown variables of system (2.20) corresponds to the equilibrium values of these variables with fixed values of M^* and G^* .

For Problem 1, the optimal values of the parameters are $M = 5877.96$, $G = 4245.98$, which ensure attaining the maximum value of the criterion $Y = 15,255.9$. The value of this criterion without control is equal to 14,327.3. For the optimal values of the instruments M and G that were obtained, the equilibrium values of the other endogenous variables turn out to be $N = 8148.539$, $P = 1.1210$, and $i = 12.986$. Here we should also note that solving this optimization problem results in an increase of the working segment of the population by approximately 100,000 people.

On the basis of Problem 1, we carry out the analysis of the dependence of the optimal values of criterion Y on the pair of the exogenous parameters $\{T_y, S_y\}$ given in their respective regions. The obtained plot of the optimal values of criterion (2.19) is presented in Fig. 2.4.

2.2 Macroeconomic Analysis and Parametric Control of the National Economic State Based on the Model of a Small Open Country

Ensuring a double equilibrium, that is, a common economic equilibrium in conditions of full employment with a planned (assumed zero) balance of payments, is an urgent problem in the conditions of an open economy, when the country is engaged in the exchange of goods and capital with the outside world.

All of the remaining states in the national economy differing from the double equilibrium represent various kinds of nonequilibrium states. Hence, unemployment remains the same in spite of an excess in the balance of payments. Unemployment can be accompanied by an excess in the balance of payments. The excess of employment can be accompanied by both the excess and deficiency of the balance of payments. Therefore, public economic policy aims at attainment of a double equilibrium. The estimation of the equilibrium conditions for an open economy can be partially considered on the basis of the model of a small country [39, p. 433].

This section is devoted to the construction of a mathematical model of an open economy of a small country using the example of the Republic of Kazakhstan, to the analysis of the influence of economic instruments on the conditions of common economic equilibrium and state of the balance of payments, and to the estimation of the optimal values of the economic instruments on the basis of the model of an open economy of a small country, as well as an analysis of the dependencies of the optimal values of the criteria on the values of one, two, and three parameters from the set of the external economic parameters given in the respective regions.

2.2.1 Construction of the Model of an Open Economy of a Small Country and Estimation of Its Equilibrium Conditions

Let's introduce the following notation for the economic indices used for the model construction: Y is the gross national income (GNI); C is the household consumption; I is the investment in capital assets; G is public expenses; NE is the net export of wealth; P is the level of prices of RK; P_z is the level of prices abroad; l is the real cash remainder; i is the interest rate of second-level banks; N is the number of employed; dY/dN is the derivative of the gross national income as a function of the number of employed; W^s is the level of wages; NKE is the net capital export; e is the rate of exchange of the national currency; e^e is the expected rate of exchange of the national currency; $\hat{e}^e$ is the expected rate of increase of the exchange rate of the national currency [39, p. 121]; M is the money supply determined from [39, p. 412] by the formula $M = \mu H$, where H is the money base of each year; μ is the money multiplier calculated from the balance equations of the banking system and defined by the formula

$$\mu = (1 + \gamma(1 - \alpha - \beta)) / (\alpha + \beta + \gamma(1 - \alpha - \beta)), \quad (2.21)$$

where $\alpha = RR/D$ is the norm of the minimal reserve;

$\beta = ER/D$ is the coefficient of the cash remainder of the second-level banks;

$\gamma = CM/K$ is the share of cash in the whole sum of the credits of second-level banks;

RR is the minimal reserve;

ER is the excessive reserve;

D is the check deposits;

CM is the active money in cash;

K is the credits of second-level banks corrected subject to the velocity of money.

Let's begin to construct a mathematical model of an open economy of a small country by estimating the money multiplier, real cash remainders, and economic functions characterizing the national economic state.

The estimations of values of the money multiplier calculated by formula (2.21) using the statistical data for the period of years 2006–2008 are presented below:

Year	2006	2007	2008
μ	2.372	3.087	3.632

The real cash remainder l is determined by the formula

$$l = l_{pr} + l_{tr}, \quad (2.22)$$

where l_{pr} is the property volume [deposits in the deposit organizations (by sectors and kinds of currency)], billions of tenge, and l_{tr} is the volume of the transaction (the volume of the credits given by second-level banks subject to the money velocity), billions of tenge.

The estimation of the money velocity is calculated by the Fisher equation [42]:

$$MV = Y,$$

where V is the money velocity, and M is the quantity of the active money usually represented by the money aggregate $M3$ in the Fisher equation.

From the latter formula, the estimates of the money velocity calculated by the formula $V = Y/M$ on the basis of the statistical information for 2006–2008 [37] are presented in Table 2.7.

In the macroeconomic theory, the behavior of the national economy is characterized by the following functions constructed by econometric methods [1] on the basis of official statistical information.

The consumption C represented by the expression $C = a + C_Y Y$ has the following econometric estimation derived on the basis of the statistical information of the Republic of Kazakhstan for the period 2000–2008:

$$C = 555.8 + 0.4101Y. \quad (2.23)$$

(0.00) (0.00)

Table 2.7 Values of GNI (billions of tenge), money aggregate $M3$ (billions of tenge), and money velocity V

Year	GNI	$M3$	V
2007	11,371	4,629.8	2.5
2008	13,734	6,266.4	2.2

The statistical characteristics of the constructed model of the consumption C are as follows: The determination coefficient $R^2 = 0.994$, and the approximation coefficient $A = 1.8\%$. The statistical significance of the coefficients of regression (2.23), as well as of the regressions estimated below, is presented in parentheses under the respective regression coefficients as the p -values.

The consumption of the imported wealth Q_{im} is represented by the regression equation

$$Q_{im} = a_1 Y + b_1 e P^Z / P$$

or, in estimated form,

$$Q_{im} = 0.4076Y - 2.6059eP^Z / P \quad (2.24)$$

(0,00) (0,17)

with the determination coefficient $R^2 = 0.975$ and the approximation coefficient $A = 10\%$.

The model of the demand of the real cash remainder is given by $l = a_2 + b_2 Y + b_3 i + b_4 e$ or, after estimating the parameters of this model using the statistical information,

$$l = 1.3320Y - 138.1i - 22.6e. \quad (2.25)$$

(0,01) (0,08) (0,03)

In constructing model (2.25), the values of l calculated in accordance with formula (2.22) are accepted as the data for the left-hand side. The determination coefficient is given by $R^2 = 0.999$, and the approximation coefficient is $A = 0.2\%$. The statistical insignificance of the latter model concerns the fact that in the model there are correlated factors.

The model of the labor supply price is given by $W^S = b_5 N + b_6 P_{\text{mean}}$, where $P_{\text{mean}} = (1 - \alpha)P + \alpha e P^Z / e^0$ has the following econometric estimation derived on the basis of the statistical information:

$$W^S = -0.0219N + 156.9 P_{\text{cp}}, \quad (2.26)$$

(0,00) (0,00)

where $P_{\text{mean}} = 0.6P + 0.4eP^z/e^0$, e^0 is the currency exchange rate within the base period (year 2000), and α is the share of the imported goods in their entire volume accepted at the level of 0.4. We also have the determination coefficient $R^2 = 0.98$ and the approximation coefficient for 2007–2008 at the 5% level.

The model of the net capital export is given by $NKE = b_7e(i^Z + \hat{e}^e - i)$ or, after estimating the parameters of this model by using the statistical information,

$$NKE = -0.3349e(i^Z + \hat{e}^e - i), \quad (2.27)$$

(0,00)

with the determination coefficient $R^2 = 0.51$.

The production function is represented in the regression pair $Y = a_3 + b_8 N$ or, in the estimated form,

$$Y = -17,409.0 + 3.0866 N, \quad (2.28)$$

(0,00) (0,00)

with the determination coefficient $R^2 = 0.93$ and the coefficient approximated based on 2007–2008 data not exceeding $A = 3.5\%$.

The model of investment in capital assets is given by

$$I_t = a_4 + b_9 Y_{t-1} + b_{10} i_t,$$

where I_t and i_t are the values of the investments in the current period, and Y_{t-1} is the value of the gross national income in the preceding period.

After estimating the latter model parameters by the statistical data, the following expression is derived:

$$I_t = 862.8 + 0.3122 Y_{t-1} - 48.4 i_t. \quad (2.29)$$

(0.34) (0.00) (0.41)

At that, the determination coefficient $R^2 = 0.93$ and the approximation coefficient $A = 5\%$.

Substituting the value $Y_{t-1} = Y_{2007}$ to (2.29), finally we obtain the following model of investment in the year 2008:

$$I_{2008} = 2,846.7 - 48.4 i. \quad (2.30)$$

Similarly, substituting the value $Y_{t-1} = Y_{2006}$ into (2.29) for investment in 2007, we obtain the following model:

$$I_{2007} = 2,737.3 - 48.4 i. \quad (2.31)$$

The wealth export model is a regression of the form $Q_{\text{ex}} = b_{11}eP^Z/P$. After estimating the parameters, this model becomes

$$Q_{\text{ex}} = 17.87eP^Z/P. \quad (0, 01) \quad (2.32)$$

The determination coefficient is $R^2 = 0.65$.

On the basis of derived econometric estimate (2.23), (2.24), (2.25), (2.26), (2.27), (2.28), (2.29), (2.30), (2.31) and (2.32) characterizing the state of the national economy, let's proceed to the construction of a model of an open economy of a small country for year 2008.

Within the framework of the IS curve, we constructed the function $Y = C + I + G + Q_{\text{ex}} - Q_{\text{im}}$, which, subject to (2.23), (2.24), (2.29), (2.30), (2.31) and (2.32), becomes

$$\begin{aligned} Y &= 555.8 + 0.4101Y + 2,846.7 - 48.4i + G + 20.47eP^Z/P - 0.4076Y \text{ or} \\ Y &= 3,410.9 - 48.4i + 20.52eP^Z/P + 1.0024G. \end{aligned} \quad (2.33)$$

The equation of the LM line $M/P = l$ subject to the econometric model (2.25) becomes

$$M/P = 1.3320Y - 138.1i + 22.6e,$$

from which one can derive the following relation:

$$i = -0.1640e + 0.0096Y - 0.0072M/P \quad (2.34)$$

Substituting (2.34) into (2.33), we obtain the value of the aggregate demand Y^D :

$$Y^D = 2,324.0 + 5.42e + 13.98eP^Z/P + 0.6830G + 0.2392M/P. \quad (2.35)$$

Let's substitute (2.33) into (2.34) and determine the function of the domestic commercial interest rate:

$$i = 22.42 - 0.1117e - 0.0049M/P + 0.1349eP^Z/P + 0.0065G \quad (2.36)$$

The condition of equilibrium in the labor market is given by $P dY/dN = W^S$ [41, p. 435], which, subject to the econometric functions (2.26) and (2.28), can be represented by the expression

$$7,5P = -0.0219N + 156.87(0.6P + 0.4eP^Z/e^0). \quad (2.37)$$

From (2.37) we obtain the following relation for N :

$$N = 4,165.8P + 20.20eP^Z \quad (2.38)$$

Substituting expression (2.38) into the production function (2.28), we obtain the function of the aggregate supply:

$$Y^S = -17,408.6 + 12,858.2P + 62.36eP^Z \quad (2.39)$$

The balance of payments has a zero balance if the net wealth export equals the net capital export, i.e., $NE = NKE/P$, is valid. The econometric representation of the latter equality on the basis of (2.24), (2.27), and (2.32) is given by

$$17.87eP^Z/P - (0.4076Y - 2.6059eP^Z/P) = -0.3349e(i^Z + \bar{e}^e - i)/P.$$

Substituting the value of the domestic interest rate (2.36) into the latter equality, after some transformation we obtain the following equation of the curve of the zero balance of payments:

$$\begin{aligned} Y^{ZBO} = & 50.23 \frac{eP^Z}{P} - 0.8215 \frac{ei^Z}{P} - 0.8215 \frac{e^e}{P} + 19.24 \frac{e}{P} \\ & - 0.0918 \frac{e^2}{P} - 0.0041 \frac{eM}{P^2} + 0.111 \frac{e^2P^Z}{P^2} + 0.005 \frac{eG}{P}. \end{aligned} \quad (2.40)$$

Thus, the model of an open economy of a small country in the year 2008 is given by the following system of equations:

$$\left\{ \begin{array}{l} Y^D = 2324.0 + 5.42e + 13.98 \frac{eP^Z}{P} + 0.6830G + 0.2392 \frac{M}{P}, \\ Y^S = -17408.6 + 12858.2P + 62.36eP^Z, \\ Y^{ZBO} = 50.23 \frac{eP^Z}{P} - 0.8215 \frac{ei^Z}{P} - 0.8215 \frac{e^e}{P} + 19.24 \frac{e}{P} - 0.0918 \frac{e^2}{P} - \\ \quad - 0.0041 \frac{eM}{P^2} + 0.111 \frac{e^2P^Z}{P^2} + 0.005 \frac{eG}{P}, \\ Y^D = Y^S = Y^{ZBO}. \end{array} \right. \quad (2.41)$$

Similarly to (2.41), a model of an open economy of a small country in 2007 can be constructed.

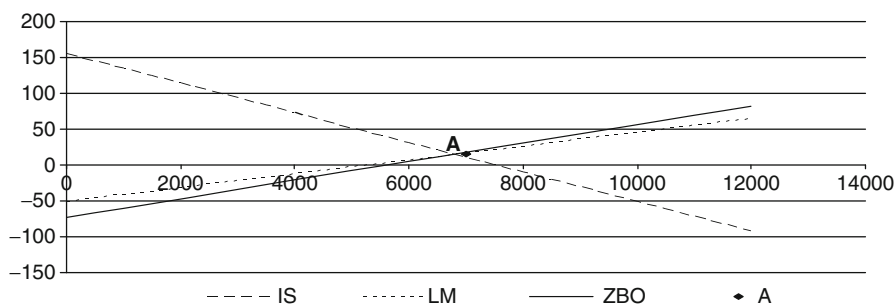


Fig. 2.5 Double equilibrium by 2008

Solving system (2.41) with prescribed values of the external economic indexes P^Z , i^Z , e^e and the economic instruments M and G , we'll determine the equilibrium conditions of the gross national income $Y^* = Y^D = Y^S = Y^{ZBO}$, level of prices P^* , and exchange rate of the national currency e^* . The equilibrium values of the credit interest rate of the second-level banks i^* and the number of employed are calculated by formulas (2.36) and (2.38), respectively.

The following equilibrium values of the endogenous variables are obtained by solving system (2.41) for the given external uncontrolled economic indices P^Z , i^Z , e^e and the controlled economic instruments M and G :

- by year 2007: $Y^* = 6,383.1$; $P^* = 1.2054$; $e^* = 110.2$; $i^* = 16.4$; $N^* = 7,708.0$
- by year 2008: $Y^* = 6,785.4$; $P^* = 1.2099$; $e^* = 114.7$; $i^* = 14.7$; $N^* = 7,838.4$.

Figure 2.5 presents the double equilibrium state, where the point of intersection of the IS–LM–ZBO curves corresponds to a simultaneous equilibrium in the wealth, money, and labor markets with full employment and zero balance of payments in 2007. All combinations of the values of the national income and interest percent, except $i = 14.7\%$, $Y = 6785.4$, offer different types of non-equilibrium states. In 2008 Kazakhstan, as per the presented figure, also has unemployment and deficit of balance payments. In the graph of Fig. 2.2.1 this situation is represented by point A ($Y_{2008} = 70098.0$; $i_{2008} = 15.3\%$). However one can note that according to official statistics in 2008 Kazakhstan had a surplus of balance payments.

Taking into account the obtained equilibrium values, the equilibrium values of the economic indices C , I , and others calculated by econometric models are constructed above. We present the results of comparison of the equilibrium indices with the actual values of these indices in 2007. Table 2.8 shows similar results for 2008.

Table 2.8 Equilibrium and actual values of indices in 2008

Indices	2008			
	Equilibrium value of Y^*	Equilibrium value of Y_{actual}	Deviation $Y_{\text{actual}} - Y^*$	
			Absolute	%
Level of prices P	1.2099	1.96	0.7501	38.8
Currency exchange rate e	114.6	120.3	5.6687	4.7
Interest rate of SLB i	14.7	15.3	0.6246	4.1
National income Y	6785.4	7009.8	224.4	3.2
Consumption C	3338.2	3395.1	56.9	1.7
Import Q_{im}	2467.5	2326.4	-141.2	-6.1
Investment I	2137.043	2149.2	12.2	0.6
Export Q_{ex}	2045.384	4370.6	2325.2	53.2

2.2.2 Influence of Economic Instruments on Equilibrium Solutions and Payment Balance States

Below, we'll estimate the influence of economic instruments, namely, the money supply and public expenses, on the conditions of common economic equilibrium and the state of the balance of payments using the following algorithm:

1. Changing the value M_{2008} by $\Delta M = 0.01 M_{2008}$ while keeping the values G_{2008} and i^Z_{2008} , P^Z_{2008} , e^e_{2008} unchanged, define the values $(M\Delta Y^*)/(Y^*\Delta M)$, $(M\Delta P^*)/(P^*\Delta M)$, $(M\Delta e^*)/(e^*\Delta M)$, and $(M\Delta i^*)/(i^*\Delta M)$ that show the percentage by which the equilibrium values of the indices Y^* , P^* , e^* , i^* change with variation of M_{2008} by 1%.
2. Changing the value G_{2008} by $\Delta G = 0.01 G_{2008}$ while keeping the values M_{2008} and i^Z_{2008} , P^Z_{2008} , e^e_{2008} unchanged, define the values $(G\Delta Y^*)/(Y^*\Delta G)$, $(G\Delta P^*)/(P^*\Delta G)$, $(G\Delta e^*)/(e^*\Delta G)$, and $(G\Delta i^*)/(i^*\Delta G)$ that show the percentage by which the equilibrium values of the indices Y^* , P^* , e^* , i^* change with variation of G_{2008} by 1%.
3. Changing the value M_{2008} by $\Delta M = 0.01 M_{2008}$ and the value G_{2008} by $\Delta G = 0.01 G_{2008}$ while keeping the values i^Z_{2008} , P^Z_{2008} , e^e_{2008} unchanged, define the values $100\Delta Y^*/Y^*$, $100\Delta P^*/P^*$, $100\Delta e^*/e^*$, and $100\Delta i^*/i^*$ that show the percentage by which the equilibrium values of the indices Y^* , P^* , e^* , i^* change with simultaneous variation of M_{2008} and G_{2008} by 1%.

The results of computations carried out by the above algorithm are given in Tables 2.9, 2.10, and 2.11.

According to the proposed algorithm, first we estimate the influence of the economic instruments, namely, the money supply and public expenses, on the conditions of the common economic equilibrium and the state of the balance of payment individually. From Tables 2.9 and 2.10, it follows that increasing G_{2008} by ΔG while keeping the value M_{2008} results in growth of the national income and an

Table 2.9 Influence of the money supply instrument on the equilibrium state of national economy in 2007 for $\Delta M = 0.01M_{2008}(\%)$

$(M\Delta Y^*)/(Y^*\Delta M)$	$(M\Delta P^*)/(P^*\Delta M)$	$(M\Delta e^*)/(e^*\Delta M)$	$(M\Delta i^*)/(i^*\Delta M)$
0.359	-0.1128	0.4827	-1.4692

Table 2.10 Influence of the public expenses instrument on the equilibrium state of national economy in 2007 for $\Delta G = 0.01G_{2008}(\%)$

$(G\Delta Y^*)/(G^*\Delta M)$	$(G\Delta P^*)/(P^*\Delta G)$	$(G\Delta e^*)/(e^*\Delta G)$	$(G\Delta i^*)/(i^*\Delta G)$
0.1892	0.0345	0.0865	0.8017

Table 2.11 Influence of money supply and public expenses instruments on the equilibrium state of the national economy in 2007 for $\Delta M = 0.01M_{2008}$ and $\Delta G = 0.01G_{2008}(\%)$

$100\Delta Y^*/Y^*$	$100\Delta P^*/P^*$	$100\Delta e^*/e^*$	$100\Delta i^*/i^*$
0.5477	-0.0780	0.5675	-0.6434

increase in the interest rate, whereas increasing M_{2008} by ΔM while keeping the value G_{2007} also results in growth of the common economic equilibrium of the GNI, but it also results in a decrease in the interest rate. Also, from the tables it follows that the growth in public expenses has a stronger influence on the national income growth, whereas the money supply growth affects the currency exchange rate more strongly.

Here Y^* , P^* , e^* , i^* are the equilibrium solutions for the year 2008, $\Delta Y^* = Y_M^* - Y^*$, $\Delta P^* = P_M^* - P^*$, $\Delta e^* = e_M^* - e^*$, $\Delta i^* = i_M^* - i^*$, where Y_M^* , P_M^* , e_M^* , i_M^* are the equilibrium solutions corresponding to $M = M_{2008} + \Delta M$.

According to the macroeconomic theory, the money supply growth shows the following influence on the equilibrium solutions of system (2.41): The national income, level of prices, and national currency exchange must increase, whereas the interest rate must decrease. The results of the influence of the money supply instrument on the equilibrium state of the national economy in 2008 presented in Table 2.9 coincide with the theoretical assumptions except the price-level index, which in this case decreases.

Here $\Delta Y^* = Y_G^* - Y^*$, $\Delta P^* = P_G^* - P^*$, $\Delta e^* = e_G^* - e^*$, $\Delta i^* = i_G^* - i^*$, where Y_G^* , P_G^* , e_G^* , i_G^* are the equilibrium solutions corresponding to $G = G_{2008} + \Delta G$.

According to macroeconomic theory, the public expenses growth exerts the following influence on the equilibrium solutions of system (2.41): The national income, level of prices, national currency exchange rate, and interest rate must grow. The results of the money supply instrument influence on the equilibrium state of the national economy in 2008 presented in Table 2.9 completely coincide with these theoretical assumptions.

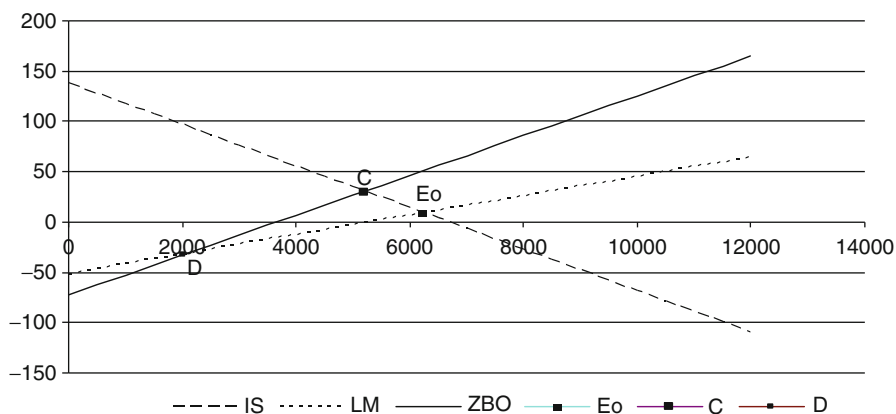


Fig. 2.6 Plots IS–LM–ZBO by actual values of P , e for 2008

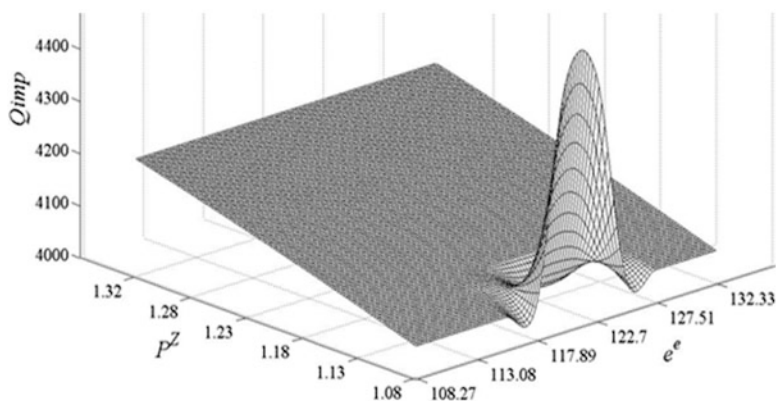


Fig. 2.7 Plot of the dependence of optimal values of criterion Q_{imp} on pair P^Z , e^e

Here $\Delta Y^* = Y_{MG}^* - Y^*$, $\Delta P^* = P_{MG}^* - P^*$, $\Delta e^* = e_{MG}^* - e^*$, $\Delta i^* = i_{MG}^* - i^*$, where Y_{MG}^* , P_{MG}^* , e_{MG}^* , i_{MG}^* are the equilibrium solutions corresponding to $M = M_{2008} + \Delta M$ and $G = G_{2008} + \Delta G$.

Figures 2.7 and 2.8 present the plots of the IS, LM, and ZBO curves from the derived econometric models for the actual statistical information for 2008.

As stated above (Fig. 2.5), the country has cyclical unemployment and a deficit in balance of payments from the constructed models. In Fig. 2.6, such a situation is represented by point E_0 . According to the macroeconomic theory, the balance of payments deficit can be eliminated by applying a restrictive monetary policy by means of shifting the LM curve to the left up to its intersection with the IS curve at point C , or the counteractive fiscal policy by means of the IS curve to the left up to its intersection with the LM curve at point D .

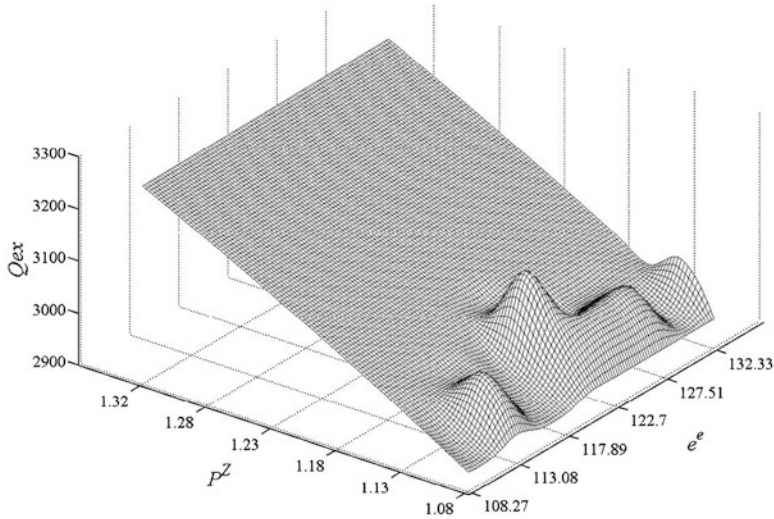


Fig. 2.8 Plot of the dependence of optimal values of criterion Q_{ex} on pair P^Z , e^e

2.2.3 Parametric Control of an Open Economy State Based on a Small Country Model

Estimate the optimal values of instruments M and G given the external exogenous parameters e^e , i^Z , P^Z on the basis of model (2.41) for the year 2008 in the sense of the criteria

$$Q_{\text{ex}} = aeP^Z/P \rightarrow \max \quad (2.42)$$

and

$$Q_{\text{imp}} = bY^S + ceP^Z/P \rightarrow \min. \quad (2.43)$$

Such an estimate can be obtained by solving the following problems of mathematical programming:

Problem 1

On the basis of mathematical model (2.41), find the values (M, G) maximizing criterion (2.42) under the constraints

$$\left\{ \begin{array}{l} |M - M^*| \leq 0.1M^*, \\ |G - G^*| \leq 0.1G^*, \\ |P - P^*| \leq 0.1P^*, \\ |e - e^*| \leq 0.1e^*, \\ |i - i^*| \leq 0.1i^*, \\ |Y - Y^*| \leq 0.1Y^*. \end{array} \right. \quad (2.44)$$

Here M^* and G^* are the actual values of the money supply and public expenses in the year 2008.

Problem 2

On the basis of mathematical model (2.41), find the values (M, G) minimizing criterion (2.42) under constraints (2.44).

Solving Problems 1 and 2 by the iterative technique [66] given the values $e^e = 120.3$, $i^Z = 1.32$, $P^Z = 1.2002$, the following results are obtained:

For Problem 1, the optimal values of the parameters are $M = 5877.96$, $G = 4246$, providing the attainment of the maximum value $Q_{\text{ex}} = 3122.74$. The value of this criterion without control is 3023.01.

For Problem 2, the optimal values of the parameters are $M = 4809.234$, $G = 3474$, providing the attainment of the minimum value $Q_{\text{imp}} = 4010.64$. The value of this criterion without control is 4183.73.

On the basis of Problems 1 and 2, we carried out the analysis of the dependencies of the optimal values of the criteria Q_{ex} and Q_{imp} on the one pair and one set of three of the parameters from the set of the external parameters $\{e^e, i^Z, P^Z\}$ given within the respective regions. The plots of the dependencies of the optimal values of criteria (2.42) and (2.43) for the single cases including that on the pair of the parameters (P^Z, e^e) and (i^Z, e^e) are shown in Figs. 2.7 and 2.8.

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