

Preface

This book contains an account of results obtained by the author and his collaborators on billiards in the complement of bounded domains and their applications in aerodynamics and geometrical optics.

We consider several problems related to the aerodynamics of bodies in highly rarefied media. It is assumed that the medium particles do not interact with each other and are elastically reflected when colliding with the body boundary; these assumptions drastically simplify the aerodynamics and make it possible to reduce it to a number of purely mathematical problems.

First we examine problems of minimal resistance in the case of translational motion of bodies. These problems generalize the Newton problem of least resistance; the difference is that the bodies are generally nonconvex in our case, and therefore the particles can make multiple reflections from the body surface. It is proved that typically the infimum of resistance equals zero; thus, there exist “almost perfectly streamlined” bodies.

Next we consider the generalization of Newton’s problem on the minimal resistance of convex axisymmetric bodies to the case of media with a thermal motion of particles. Two kinds of solution are found: first, Newton-like bodies and, second, shapes obtained by gluing together two Newton-like bodies along their rear ends.

Further, we present results on the characterization of billiard scattering by nonconvex and rough bodies; next we solve some special problems of optimal mass transportation. These two groups of results are applied to problems of minimal and maximal resistance for bodies that move forward and at the same time slowly rotate. It is found, in particular, that the resistance of a three-dimensional convex body can be increased at most twice and decreased at most by 3.05 % by roughening its surface.

Next, we consider a rapidly rotating rough disc moving in a rarefied medium on a plane. It is shown that the force acting on the disc is not generally parallel to the direction of the disc motion, that is, it has a nonzero transversal component. This phenomenon is called the Magnus effect (proper or inverse, depending on the direction of the transversal component). We show that the kind of Magnus effect

depends on the kind of disc roughness, and we study this dependence. The problem of finding all admissible values of the force acting on the disc is formulated in terms of a vector-valued problem of optimal mass transportation.

Finally, we describe bodies that have zero resistance when being translated through a medium and present results on the existence or nonexistence of bodies with a mirror surface invisible in one or several directions. We also consider the problem of constructing retroreflectors: bodies with a specular surface that reverses the direction of any incident beam of light.

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Exterior Billiards

Systems with Impacts Outside Bounded Domains

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