Chapter 2
Perspectives on Mathematics, Learning, and Teaching

This book is intended to provide some insights into the types of tasks that can assist students in learning mathematics. The insights though need to be interpreted in the context of our perspectives on some key issues. This chapter discusses what we see as the goals of teaching mathematics, it describes our perspectives on what it means to do mathematics, it presents a perspective on how students learn mathematics, and it lists some recommendations about effective mathematics teaching. The subsequent chapters use these perspectives as the basis of elaborations of the tasks that we discuss.

The Mathematics It Is Intended that Students Learn

Although sometimes seen as competing, there are two perspectives that inform the tasks that we present in this book.

The first perspective can be described as mathematical literacy or numeracy that emphasises the mathematics that students will need for work and for their lives generally. This perspective is represented in tasks that are connected to the experiences of students, that use contexts that are meaningful to them, or for which students can imagine the potential usefulness to them of learning to engage with the task. There is substantial numeracy in, for example, everyday experiences, in interpreting current events and priorities, in evaluating personal and civic priorities, in managing personal finances, in planning activities and projects, all of which all citizens are required to undertake for themselves. There is also considerable numeracy required in most workplaces, and school graduates who cannot cope with those numeracy demands have a restriction of work choices available to them (for more on this, see Bakker, Hoyles, Kent, & Noss, 2006; Human Capital Working Group, 2008; Zevenbergen & Zevenbergen, 2009).

The second perspective is mathematical. This perspective is represented in tasks that focus on definitions, principles, patterns, processes, and generalisations that have conventionally formed the basis of the mathematics curriculum, and which lay

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the basis for much later school and university mathematics study. It incorporates appreciation of the elegance of mathematical thinking, and exposure to the wonder of topics like "pattern, symmetry, structure, proof, paradox, recursion, randomness, chaos, and infinity" (Ernest, 2010, p. 24).

We argue that these two perspectives are not competing and indeed are complementary. We argue that not only is it important for teachers to induct students into the discipline of mathematics, but it is also essential that students have opportunity to use the mathematics they are learning to solve practical problems of some relevance to them. The tasks in the following chapters seek to address both perspectives.

Fostering a Breadth of Mathematical Actions

One of the often repeated criticisms of most mathematics teaching is that it represents a narrow view of mathematics and hardly addresses the numeracy perspective at all (e.g., Hollingsworth, Lokan, & McCrae, 2003). One of the purposes of using a variety of task types is to offer students a breadth of perspectives on mathematics and numeracy and a variety of ways to learn. To allow a multidimensional perspective on what it means to do mathematics, we draw on the five strands of mathematical learning described by Kilpatrick, Swafford, and Findell (2001). Watson and Sullivan (2008) described these strands as follows:

> conceptual understanding—comprehension of mathematical concepts, operations, and relations;
> mathematical fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately, and, in addition to these procedures, factual knowledge and concepts that come to mind readily;
> strategic competence—ability to formulate, represent, and solve mathematical problems;
> adaptive reasoning—capacity for logical thought, reflection, explanation, and justification; and
> productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. (p. 112)

These five sets of mathematical actions contribute to providing a rationale for our emphasis on tasks. We use the term actions since these represent ways that students do mathematics. These actions have been incorporated into the Australian Curriculum: Mathematics [Australian Curriculum and Assessment Agency (ACARA, 2011)] and are described as proficiencies.

This way of thinking about mathematical actions has direct implications for the choice of tasks. For example, if a teacher is keen to develop procedural or mathematical fluency, then this can be achieved by providing representative worked examples followed by repetitious practice. Of course, fluency is indeed a critical aspect of learning mathematics, but we do not focus on tasks that develop fluency in this book since such tasks are well represented in every school mathematics text we have seen.

If a teacher seeks to develop conceptual understanding, then it is possible to do this by using clear and interactive explanations, by encouraging communication between teacher and students and between students. This though can also be facilitated by choosing tasks that exemplify the underlying aspects of mathematics, such as in tasks that incorporate the use of models and representations. The tasks described and elaborated in Chap. 4 are examples of such tasks.

We see strategic competence and adaptive reasoning as key actions for students when learning and doing mathematics and argue that it is not possible to engage students in such actions through explanations and worked examples. It is necessary for students to work on tasks which require them, for example, to make and justify choices, to integrate ideas, to plan strategies, and to explain their thinking. Such tasks are the focus of Chaps. 5 and 6 in this book.

Considering Students' Perspectives on Mathematical Tasks

A further set of assumptions that underpin our approach to task choice and use is the ways that students might approach tasks.

One aspect is a perspective on what constitutes knowledge. We accept the social constructivist view, as summarised by Ernest (1994), that recognises knowing as active, "individual and personal, and that it is based on previously constructed knowledge" (p. 2). In this, knowledge is not fixed, rather it is socially negotiated, and is sought and expressed through language. In other words, students do not learn only by listening but also by engaging in experiences that contribute to their learning, and having the opportunity to share approaches to tasks with others.

A related aspect is the level at which tasks are pitched. A metaphor that is helpful for this is Vygotsky's (1978) zone of proximal development (ZPD) which he described as the "distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined by problem solving under adult guidance or in collaboration with more capable peers" (p. 86). In other words, tasks should go beyond the current level of students' knowledge, but be within that range within which students can reasonably be expected to engage with support.

A further aspect is the extent to which students choose to engage with the experiences associated with working on a task. Fredericks, Blumfield, and Paris (2004) categorise engagement as behavioural, affective, and cognitive. We are interested in tasks that promote cognitive engagement, but of course affective engagement is relevant in that students must see some point in choosing to work on a task. Connected to this are the key characteristics of motivation, described by Middleton (1995) as interest, control, and arousal. Another relevant perspective was proposed by Dweck (2000) who categorized students as either having a performance or mastery orientation. Students with a performance orientation seek social affirmation as the goal of their effort rather than understanding of the content, and avoid risk taking and challenging tasks due to fear of failure. In contrast, students with a mastery orientation seek to understand the content, and evaluate their success by whether they feel they can use and transfer their knowledge. They remain focused on mastering skills and knowledge even when challenged, they do not see failure as an indictment on themselves, and they believe that effort leads to success. In other words, it seems helpful if tasks allow students opportunity to have a sense of control by allowing them to make decisions, are interesting to the students, incorporate a rationale for them to engage, provide some challenge, reduce the risk of failure, and for which success provides the motivation for further engagement. The tasks described in Chaps. 4–6 are proposed as examples of such tasks.

Approaches to Teaching Mathematics

Our approach to task choice and use is connected to the pedagogies that are used to support the student engagement with the task. There are two sets of pedagogical approaches presented here that are then referred to in various places throughout the book.

The first set of pedagogical approaches was developed by Doug Clarke and Barbara Clarke (2004), arising from detailed case studies of teachers who had been identified as particularly effective in the Australian Early Numeracy Research Project. Their list is grouped under ten headings and 25 specific pedagogical actions. While their list was drawn from research with early years' mathematics teachers, we argue that the headings and actions are applicable at all levels. Their list is written in the form of advice to teachers:

Mathematical focus	Focus on important mathematical ideas
	Make the mathematical focus clear to the children
Features of tasks	Structure purposeful tasks that enable different possibilities, strategies, and products to emerge
	Choose tasks that engage children and maintain involvement
Materials, tools, and representations	Use a range of materials/representations/contexts for the same concept
Adaptations/ connections/links	Use teachable moments as they occur
	Make connections to mathematical ideas from previous lessons or experiences
Organisational style(s), teaching approaches	Engage and focus children's mathematical thinking through an introductory, whole group activity
	Choose from a variety of individual and group structures and teacher roles within the major part of the lesson
Learning community and classroom interaction	Use a range of question types to probe and challenge children's thinking and reasoning
	Hold back from telling children everything
	Encourage children to explain their mathematical thinking/ideas
	Encourage children to listen and evaluate others' mathematical thinking/ideas, and help with methods and understanding
	Listen attentively to individual children
	Build on children's mathematical ideas and strategies
Expectations	Have high but realistic mathematical expectations of all children
	Promote and value effort, persistence, and concentration

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Reflection	Draw out key mathematical ideas during and/or towards the end of the lesson
	After the lesson, reflect on children's responses and learning, together with activities and lesson content
Assessment methods	Collect data by observation and/or listening to children, taking notes as appropriate
	Use a variety of assessment methods
	Modify planning as a result of assessment
Personal attributes of the teacher	Believe that mathematics learning can and should be enjoyable
	Are confident in their own knowledge of mathematics at the level they are teaching
	Show pride and pleasure in individuals' success

A second set of pedagogical actions associated with mathematics teaching was developed as a prompt to school-based collaborative teacher learning, drawing on the Clarke and Clarke actions and similar recommendations such as Hattie and Timperley (2007) and Education Queensland (2010). The following presents six key principles of effective mathematics teaching. Sullivan (2011) had been working with teachers who had been presented with quite specific recommendations of ways to teach reading and literacy, and who had requested a similar specific list for mathematics teaching. These principles, also used in various discussions through this book to describe teacher thinking and actions, also written in the form of advice to teachers, are as follows:

Principle 1. Identify big ideas that underpin the concepts you are seeking to teach, and communicate to students that these are the goals of the teaching, including explaining how you hope they will learn.

Principle 2. Build on what the students know, mathematically and experientially, including creating and connecting students with stories that both contextualise and establish a rationale for the learning.

Principle 3. Engage students by utilising a variety of rich and challenging tasks that allow students time and opportunities to make decisions and which use a variety of forms of representation.

Principle 4. Interact with students while they engage in the experiences; encourage students to interact with each other including asking and answering questions, and specifically planning to support students who need it; and challenge those who are ready.

Principle 5. Adopt pedagogies that foster communication and mutual responsibilities by encouraging students to work in small groups, and using reporting to the class by students as a learning opportunity.

Principle 6. Fluency is important, and it can be developed in two ways: by short everyday practice of mental calculation or number manipulation; and by practice, reinforcement and prompting transfer of learnt skills.

These principles together with aspects from the above lists are used in interpreting some of the research results and recommendations for practice in the following chapters. Note that central to both these lists is the role and contribution of tasks.

Summary

The approaches to mathematics teaching described in this book are based on consideration of the goals of mathematics teaching, current approaches to teaching mathematics taken by many teachers, and recommendations about what actions seem likely to create experiences that are accessible by all students and which engage students in learning and creating mathematics.

We argue that an important component of understanding teaching and improving learning is to identify the types of tasks that prompt engagement, thinking, the making of cognitive connections, and the associated teacher actions that support the use of such tasks, including addressing the needs of individual learners. The challenge for mathematics teachers is to foster mathematical learning, and the key media for pedagogical interaction between teacher and students are the tasks in which the students engage.

The underlying argument, then, is that mathematical learning experiences are based on tasks; that the better the task (appropriately used), the better the opportunities for effective teaching; and the better the task, the better the learning.

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