

Preface

When measures are considered, by means of integration, as functionals on spaces of functions, certain continuity properties of measures and of sets of measures often play a prominent role. This is the case in topological measure theory (smoothness of measures in duality with continuous functions), abstract harmonic analysis (conditions for well-behaved convolution of measures on topological groups) and probability theory (uniform tightness and its relationship with weak convergence of probability distributions on complete metric spaces).

The main thesis of this treatise is that a number of continuity properties commonly imposed on measures are instances of a general property defined in the language of uniform spaces and that several fundamental results about measures remain valid in such a setting. The general property singles out a class of functionals called *uniform measures* on the space of bounded uniformly continuous functions on a uniform space.

Although a uniform measure is a functional on a function space, not a genuine measure (a countably additive set function on a σ -algebra), in some respects uniform measures behave like measures, and for some purposes they may be used as a substitute for measures. Moreover, in many problems, the underlying space carries a natural uniform structure, which leads to questions about functionals on spaces of uniformly continuous functions; uniform measures often feature in answers to such questions.

The basic theory of uniform measures was developed by a number of researchers in the 1960s and 1970s, but an interested reader would need considerable patience to track down scattered sources, some unpublished, written using a variety of definitions and differing notations. Recently the need for an accessible exposition became more apparent in view of new results in abstract harmonic analysis. In this monograph I offer a unified treatment of the theory of uniform measures, with a view towards such applications and others that I expect still to come.

This is primarily a reference for the theory of uniform measures and related functionals on spaces of uniformly continuous functions. It is also suitable for graduate or advanced undergraduate courses on selected topics in topology and functional analysis.

Part I is a self-contained development of the necessary results about uniform spaces. Part II is a systematic presentation of the basic theory of uniform measures, concluding with applications in the study of convolution algebras. Part III complements the basic theory with results in several related areas.

Uniform measures may be, and have been, defined in several equivalent ways. Although it adds to their usefulness, it also means that anyone attempting to cover all their major facets in a linear text is faced with a number of choices about starting points and the order of presentation. In selecting and organizing the material, my main objective has been to assemble a foundation for applying the theory in functional analysis, in a way that is likely to appeal to those interested in such applications. I have omitted or deferred some developments that would distract from the main objective. Part III includes several such developments that are intrinsically interesting and supply a broader context for the theory in Part II.

Despite a distance in time and space, this book owes much to the late Zdeněk Frolík and to the supportive environment for young mathematicians that he created in Prague in the 1970s. He made major contributions to the theory described here, and uniform measures were often discussed in his seminars. I obtained my first results about uniform measures with his advice and encouragement.

I am grateful to Henri Buchwalter, David Fremlin, Ramon van Handel and Miloš Zahradník for allowing me to use their unpublished ideas and materials. I thank Anthony Hager, Petr Holický, Matthias Neufang and Juris Steprāns for their comments and valuable suggestions, Carl Riehm for shepherding the text through the publication process, and my wife Cynthia for her patience and support.

The content incorporates ideas and techniques that I learnt from many mathematicians over the years. I appreciate their implicit contributions, even if I cannot name them all here.

The monograph was written while I was a visitor at the Fields Institute in Toronto. Contacts with colleagues at the institute and access to its facilities helped me a great deal in assembling the material that follows.

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