Chapter 2
Signal Measurement

There is one *thing of which one can say neither that it is one metre long, nor that it is not one metre long, and that is the standard metre in Paris. But this is, of course, not to ascribe any extraordinary property to it, but only to mark its peculiar role in the language-game of measuring with a metre-rule.*

Ludwig Wittgenstein

In this chapter we look at the principles of recording physiological signals and subsequent analysis, including noise reduction. Signal measurement is predicated on a preliminary model of the system under observation. The quality of signal measurement has a profound impact on subsequent interpretation. A basic application of signal analysis is to use a measurement model to remove unwanted portions or "noise" from the measured signal. The two main purposes of signal analysis and systems modeling in physiology are (1) to reduce the contaminating noise in the observed signal and (2) to describe the process in terms of a few parameters. Modeling the system is critical to both these aspects. Before any manner of noise reduction is performed a conceptual model of the signal necessarily exists in the mind of the observer. It is this model that determines how effectively the "true" signal will be elucidated from the noisy recording. The selection of noise reduction techniques will depend on this conceptual model.

2.1 Physiological Measurement

The schematic block diagram of physiological measurement in Fig. 2.1 shows signal pickup followed by analogue processing and output. The signal is generated by the physiological process and is usually some physical quantity that varies in time (time signals) or space (images). The transducer converts this physical quantity into electrical signals amenable to subsequent processing by the instrument.

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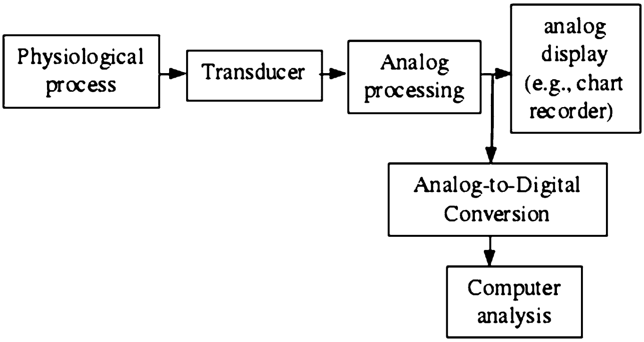


Fig. 2.1 General schematic of a measurement system

The analogue processing comprises amplifiers to magnify the desired signal, reduction of unwanted "noise", etc. The output device is a display or paper chart recorder to present the information to the user. Most modern instruments convert the analogue signal into digital form suitable for computer analysis. The digitized signal can be analyzed on a computer either immediately as the signal comes in (online processing) or stored in the computer for later more complex analysis (offline processing).

Cascading Systems

The block diagram in Fig. 2.1 presents a convenient pictorial representation of the measured signal being passed through several blocks in the system before final presentation to the user. Each of the blocks in the system modifies the signal in a manner characteristic of the block. The blocks must be chosen/designed such that the desired signal is obtained as clearly as possible while minimizing the effect of unwanted noise. A simple block will not change the shape of the signal, but might change its amplitude—either magnification or diminution. A cascade of such simple blocks is illustrated in Fig. 2.2. The scaling performed by each block (gain, G) is written inside. The final output is the cumulative amplification of all the blocks. For example, if the physiological signal is muscle force $p(t)$ Newtons, the transducer produces 0.2 V/N, the analogue processor is a simple amplifier with amplification of 1,000, and the display device produces a deflection of 3 mm/V, then, the final output is $600p(t)$ mm/N (Fig. 2.2).

Thus cascading systems (or subsystems) produce a cumulative effect on the input signal. The situation is somewhat more complex when the blocks do not just produce simple scaling of the input, but affect the shape of the signal as well. To understand such complex systems which are commonly encountered in real life we shall look at ways of dealing with them.

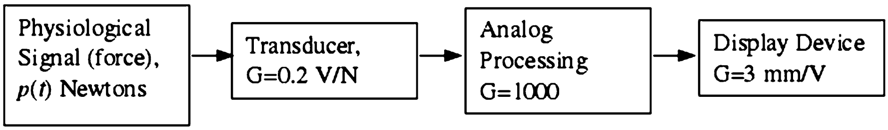


Fig. 2.2 A measurement system

Transduction is the conversion of one form of energy into another, or the conversion of one physical quantity to another. A simple example is a mercury thermometer which converts temperature into displacement of the mercury level. Transduction can involve several stages, for example: a force transducer may comprise (1) conversion of force to displacement and (2) conversion of the displacement to electrical resistance change. Usually, the final output is an electrical quantity so that electronic circuits can be used for further processing.

A system with several subsystems will have an overall behavior that is the cumulative effect of the subsystems. The set of subsystems includes everything from the transducer to the final output device. Some subsystems are expressly introduced to alter the measurement in specific ways, for example, frequency filtering, noise reduction circuits, etc. Other subsystems like the transducer and output device are intended to transfer the signal without any change as far as possible. However, in practice, these subsystems have imperfect characteristics and introduce undesirable changes in the signal. This degradation is different from the addition of "noise" or "interference" signals. The degradation is due to the inability of the subsystem to transfer the information perfectly. Knowledge of the characteristics of these subsystems will enable us to compensate for such deficiencies. In this chapter we'll look at some methods of characterizing subsystems like transducers used to pick up physiological signals. An important concept in such characterization is the ability to determine or predict the change effected in any signal by the characterized subsystem.

Static Calibration and Dynamic Calibration

The most basic calibration of a transducer is the static or steady-state calibration. The static calibration ignores the speed of response of the transducer. Therefore, the calibration procedure must ensure that all time-related factors are removed. For example, when calibrating a thermometer, the thermometer is subjected to different known temperatures and the corresponding reading is noted. When the applied or input temperature is changed, the change in reading takes a few seconds or even longer. The reading is noted only after it stabilizes or reaches a steady value (does not change further). Therefore, the transitory changes of temperature reading are ignored and only the steady-state reading is noted.

Dynamic calibration establishes the behavior of the system during transitory signals. In the case of the thermometer, how quickly the reading changes when the applied temperature changes is described by the dynamic characteristics.

2.2 Static Characteristics of Transducers: Linearity

The static characteristics of a system refer to its behavior when the input and output are steady and not varying with time. The static characteristics give the relation between the input and the output, also called the sensitivity of the system or its gain.

If the input quantity is x and the output quantity is y, the function that describes the steady-state relation between input and output, $y = f(x)$, is the static characteristic. If the function f is the equation of a straight line, then the system is said to be linear, otherwise it's a nonlinear transducer. In the case of transducers such a linear characteristic is desirable and the static input–output relation can be written as

$$y = g\,x + c. \tag{2.1}$$

In Eq. 2.1, g is the gain or sensitivity of the transducer and c is the offset. Most systems that are nominally linear will have a range of valid operation. Outside this range the system will not be linear. Saturation of electronic circuits which is due to inability of the signals to exceed the supply voltage is a common non-linearity. If the input signal falls outside the linear range of the system, the output will not be a good reproduction of the input—this manifestation of nonlinearity is called distortion. Figure 2.3 shows nonlinearity. The left side of the figure shows three signals over 1 s of time. On the top is the input signal in response to which, system A produces the output shown in the middle and system B produces the output shown at the bottom. The input–output graphs on the right side show nonlinear (or non-straight line) characteristics for the two systems.

In Fig. 2.3 the input–output characteristic of system A can be described as follows:

$$y = \begin{cases} 2x & -a < x < +a, \\ c & x > |a|, \end{cases}$$

where a is a constant and defines the linear limit of the system. Such a simple nonlinearity is seen in force transducers with physical limits built into the devices to prevent damage. In this nonlinearity more than one simple linear equation is required to describe the static characteristic of the system. In system B of Fig. 2.3, a more complex function is required to describe the static input–output characteristic and is a different kind of non-linearity. When such a nonlinearity exists with $y = f(x)$ it is possible to use the inverse function, $x = f^{-1}(y)$ to determine the input from the measured output. However, this inverse function may not be a

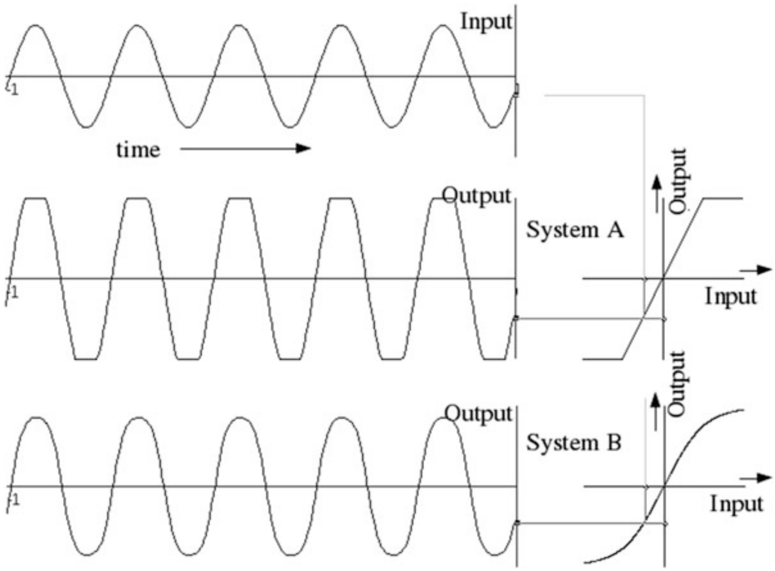


Fig. 2.3 Effect of nonlinearity. Two systems, A and B, with different types of nonlinear input–output relation and their effect on a sinusoidal input are shown.The sensitivity (or gain) of both systems is 2. The system in case A has simple saturation causing sharp truncation of the signal, while the system in case B has a gradual saturation and the effect of the nonlinearity is more subtle

simple function and immediate calculation of the inverse function may be difficult. Many nonlinear measurement systems use techniques to approximate the inverse function and these are called linearization techniques.

Linearization of Nonlinear Models

Most real systems are nonlinear. In order to submit any system to linear systems analysis it is necessary to use a linear model that adequately describes the system. The simplest linearization technique is to limit the use of the model in a region of operation where the properties are linear. For example, systems like thermometers and electronic amplifiers are linear in their normal range of operation. Extremely large signals, input to them will not produce a correspondingly large output; imagine a laboratory mercury thermometer being subjected to a temperature of few hundred degrees, it will certainly not be able to produce a corresponding reading. Therefore, these systems are nonlinear outside their specified range of operation, but are linear within a well-defined range. Some systems may exhibit more than one region that is linear within itself. Approximating such a system by several linear descriptions is called piecewise linearization. Often we may be interested in behavior of the system only within a small range of operation. If the nonlinear system behavior in this range of operation can be approximated by a linear function, then such a single piece linearization can be used.

Example

Consider a system that obtains the square of the input signal

$$y(t) = x^2(t).$$

This system fails the linearity test (input–output relation is not a straight line) and is nonlinear. If we know that the input is always within a small range x_1 to x_2, then the input–output sensitivity may be approximated as a linear function in this range of operation. If the output varies from y_1 to y_2 corresponding to the input variation x_1 to x_2, then we may treat the system as if it were linear about the center of this operating range, $x_m = (x_1 + x_2)/2$. The slope of the function or the sensitivity of the function at this midpoint of the range is

$$\frac{y_2 - y_2}{x_2 - x_1} = \frac{\Delta y}{\Delta x} \approx \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=x_m} = 2x_m.$$

Since x_m the midpoint of the selected input range is a constant (by definition), the resulting system is a simple multiplying factor

$$y(t) = 2x_m x(t).$$

Piecewise Linearization

This method of linearization can be extended to a larger range by breaking up the range into a number of small segments, (a) x_1 to x_2, (b) x_2 to x_3, (c) x_3 to x_4, etc., and linearizing the system about the center of each segment. This is piecewise linearization. However, not all systems are amenable to such linearization treatment. A system that exhibits hysteresis in its input–output behavior cannot be approximated by linear segments.

In modern digital measurement systems linearization is rather less of a problem as even a complex function for the input–output relation can be inverted empirically and a discrete form of the inverse function can be stored and used in the digital electronics. Such discretized inverse functions are called look-up tables of linearization.

2.3 Noise and Interference

Unwanted signals, interference, and disturbances are collectively termed noise. Usually noise is something that is added to the desired signal. Other disturbances are termed distortion and nonlinearity. Noise can be from a well-defined source with well-defined characteristics, or it can be from a mixture of sources and causes that change over time. Noise signals can have a pattern and even rhythm or they can vary unpredictably and be "random."

Figure 2.4 shows an example of a randomly varying noise signal that is added to the desired signal (ECG in this case). In the resulting signal the features of the ECG are difficult to discern. Such random noise commonly arise from thermal effects in electronic devices.

Figure 2.5 shows an example of a rhythmically varying noise signal, a sinusoidally varying signal that is added to the desired signal (ECG). Here too, the features of the ECG are difficult to discern. The source of sinusoidal noise in this case is the electromagnetic interference from the electrical powerline in the building. In all cases of signal contamination by noise, first and foremost, attempts should be made to reduce the noise pickup by improving the measurement setup. In the case of noise from extraneous electromagnetic sources, substantial noise reduction can be achieved by using a conductive shield around the signal lines. Physical methods of noise reduction are often addressed by empirical rules since detailed analysis of the noise sources is complex and difficult, as well as unnecessary if an empirical solution works.

Only if physical methods of noise reduction fail should we resort to post-acquisition noise removal. Once the noisy signal is acquired the signal and noise are mixed and signal processing methods of noise removal will involve a compromise of the amount of noise removed versus the amount of signal preserved.

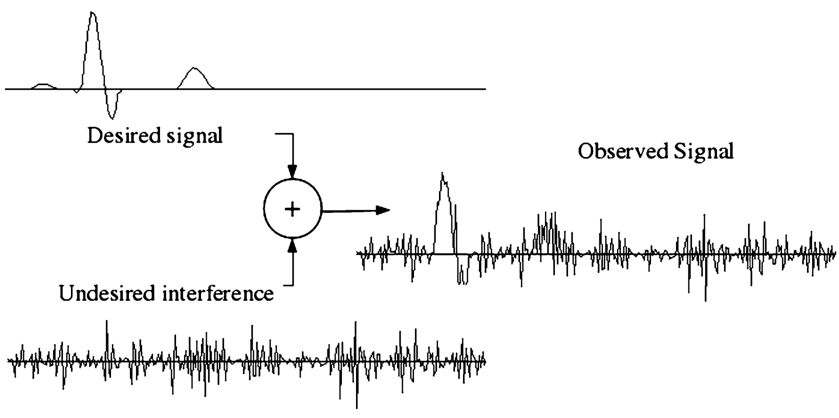


Fig. 2.4 Additive noise: random noise added to ECG

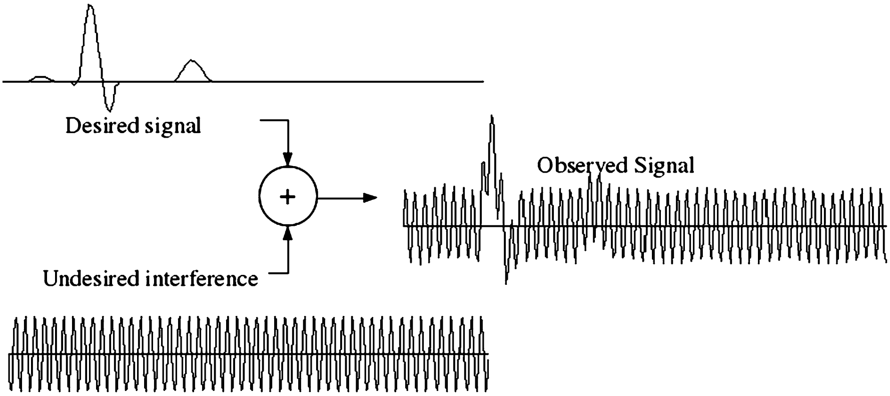


Fig. 2.5 Additive noise: 50Hz powerline interference added to ECG

2.4 Dynamic Characteristics of Transducers

The static characteristics of a transducer do not indicate the speed of response of the transducer. In many measurements the speed of response is important and dynamic calibration helps us to quantify the time-related behavior of the system. To obtain the dynamic characteristics standard time-varying functions are used as the input as discussed below.

Step Input: The Step Response

One of the simplest and most common methods of dynamic calibration of a transducer is to obtain its response to a step change in input.

Consider a simple mercury thermometer. When the thermometer is taken from room temperature and immersed in a glass of hot water, the thermometer reading will slowly rise and after a few seconds will show the correct temperature. The reason for the slow rise is due to the fact that the glass bulb and the mercury inside cannot undergo the change of temperature instantaneously. The temperature change of the thermometer is directly proportional to the difference in temperature between the water and thermometer. In other words, when the temperature difference is large, the temperature change is large. Therefore, the rate of change of temperature of the thermometer depends on the instantaneous difference in the temperature of the water and the thermometer itself. This can be written algebraically, using x to denote the temperature of the water (input) and y to denote the temperature of the thermometer which corresponds to the reading (output):

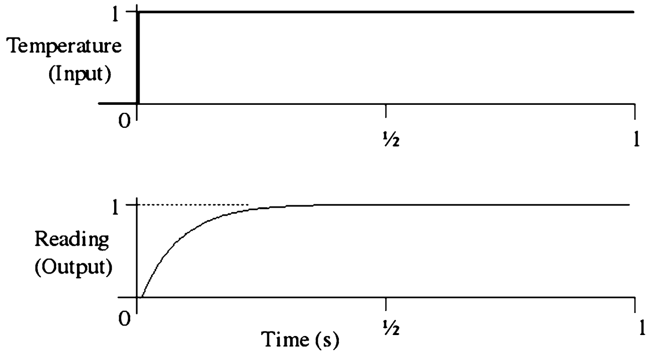


Fig. 2.6 Step response of a mercury thermometer (time constant = 0.1)

$$\frac{\Delta y}{\Delta t} = a\,[x(t) - y(t)]\,, \tag{2.2}$$

where a is a constant.

Writing Eq. 2.2 in differential form and rearranging the constants:

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} + ay(t) = ax(t). \tag{2.3}$$

In the step change of temperature described above, the thermometer is at room temperature T_o till time t_o and then suddenly the temperature is raised to T_1. Let us assume that $T_o = 0$ and $T_1 = 1$, and $t_o = 0$. Then the input $x(t)$ is a step function, $x(t) = 1$, for $t \geq 0$ and zero otherwise. Solving Eq. 2.3 for this value of x we get the step response (Fig. 2.6):

$$y_s(t) = \begin{cases} [1 - \mathrm{e}^{-at}] & t \geq 0, \\ 0 & t < 0. \end{cases} \tag{2.4}$$

A somewhat more complex response can be obtained from a weighing scale or pressure sensor. The increased complexity is due to the fact that the sensing entities in these have mass (m), frictional losses (B), and elasticity (K). The sensing depends on the displacement of part of the sensing element. If the applied force is designated by x and the resulting displacement in the sensing element is y, we can equate the applied force to the resisting forces:

$$x(t) = Ky(t) + B\frac{\mathrm{d}y(t)}{\mathrm{d}t} + m\frac{\mathrm{d}^2y(t)}{\mathrm{d}t^2}. \tag{2.5}$$

A step change in force or pressure can be produced by quickly applying or releasing an input force or pressure. The response to such a step change in input

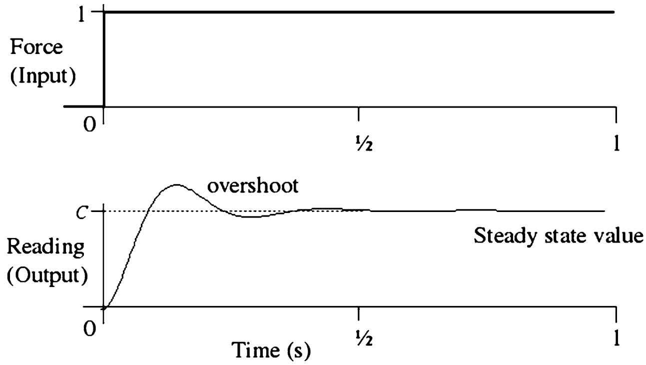


Fig. 2.7 Step response of a force transducer (natural freq = 3 Hz, damping = 0.4)

can be captured on an oscilloscope or computer and the transducer's characteristics determined.

Using such a step function for $x(t)$, we can solve Eq. 2.5:

$$y_s(t) = C\left[1 + \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin\left(\sqrt{1-\zeta^2}\,\omega_n t - \phi\right)\right]. \tag{2.6}$$

In Eq. 2.6 $\omega_n = \sqrt{K/m}$ is called the natural frequency $\zeta = B/\sqrt{4mK}$ is called the damping factor, and $C = m/K$, is a scaling constant. The phase shift $\phi = \tan^{-1}\left(-\sqrt{1-\zeta^2}/\zeta\right)$ is explained in later chapters. The step response of such a second-order system (underdamped, i.e., $\zeta < 1$) is shown in Fig. 2.7.

Sinusoidal Test Signals: The Frequency Response

Sinusoidal signals are eigenfunctions for linear time-invariant systems which we'll discuss later. An eigenfunction is one that preserves its shape when passed through the system. Unlike a step signal given to a system, when a sinusoidal signal is given to a linear system, the output will have the same sinusoidal shape but a different amplitude and a time shift. Most systems in general will respond to sinusoids of different frequencies with different gains (sensitivity) and different time shifts (or phase shift of each sinusoid). This frequency-dependent gain and phase shift is an alternative way of characterizing the system's dynamic properties. This so-called frequency response of the system is an important and commonly used characterization and will be discussed in Chap. 3.

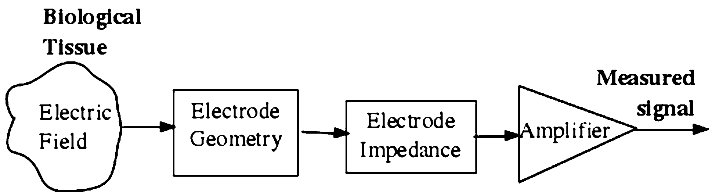


Fig. 2.8 Biopotential transduction and measurement

2.5 Transduction and Measurement Case Study

Biopotential signal recording shown schematically in Fig. 2.8 can be used to understand some of the issues discussed above. The biopotential signal originates in electric fields in biological tissue due to the movement of ions. The signal is picked up by conductive electrodes and transferred by wires to electronic amplifiers. The wires carrying these small potentials are easily affected by electromagnetic fields from the mains powerline, radio signals, etc., and noise is introduced. To reduce such noise, differential recording is usually used in biopotential recording. In differential recording, if the noise in both electrodes (and wires) is identical, then it is canceled electronically. In order for the noise in both electrodes and wires to be identical, the electrodes and wires should be spatially close together. The electrode spacing is set by various biological considerations, but placing the wires close together is often done easily. Next, the ability of the electronic amplifiers to subtract out the common signal and amplify only the differential signal is called the common mode rejection ratio (CMRR). If the CMRR is good (large) and the noise is similar in both wires, then most of the noise from electromagnetic interference can be avoided.

After the best measures are taken to reduce noise pickup, if noise still remains in the signal then other electronic ways of filtering the noise can be used. Electronic filters (or "hardware filters" or "analogue filters") have the same effect as digital filters (or "algorithmic filters" or "computational filters"), in that a compromise is involved in deciding between the amount of noise to remove and the amount of signal to preserve. The quality and nature of electronic filters and digital filters are different, since electronic filters are limited by physical components, while digital filters are usually limited by computational time.

2.6 Exercises

5. In each of the following transducers, what is the input and output? (a) Mercury manometer. (b) Weighing scale. (c) Accelerometer, (d) Gyroscope.

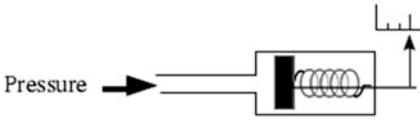
Table 2.1 Exercise 9

Time	5	10	15	20	25	30
Ice	−5	−8	−10	−11	−12	−12
Boiling water	12	18	23	25	28	28

Table 2.2 Exercise 10

Pressure (cm H_2O)	50	100	150	200
Output (V)	1.0	1.4	1.8	2.2

6. A piston in a cylinder with a spring as shown below can be used as a pressure sensor. The displacement read on the scale gives the pressure reading. Obtain an expression for the input–output relation of the sensor.



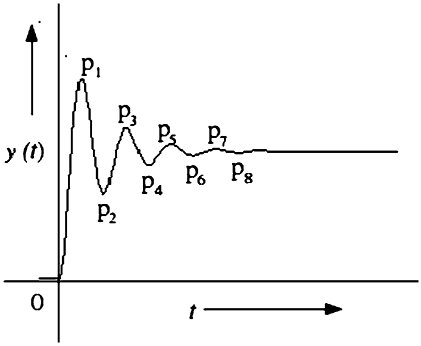
7. What is electrode impedance? How can it be measured? Why is it important?

8. When is an electrical pressure transducer better than a mercury manometer?

9. A lab thermometer is calibrated by immersing it into a beaker of controlled temperature—ice and boiling water are used for the purpose. The readings (of the length of the mercury column) are taken every 5 s after immersion (the time of immersion in each case is taken as $t = 0$) and tabulated in Table 2.1. The displacement of the mercury at room temperature, 30 °C, is taken as 0 mm. Plot time graphs of the two measurements. What is the sensitivity of the thermometer in mm/°C ?

10. A pressure transducer is calibrated using a column of water and the readings are given in Table 2.2. What is the calibration relation? Write the sensitivity and offset with units. What is the pressure if the output is 0.5 V?

11. What is the difference between static calibration and dynamic calibration? Discuss with respect to calibrating a blood pressure transducer. Why does an invasive catheter type transducer require more stringent dynamic specifications than a simple mercury manometer that can used for measuring systole and diastole noninvasively?



12. The step response of a transducer is given by Eq. 2.6. In the figure below, $P_1 = (0.028764\,\text{s}, 2.039\,\text{V})$ and $P_3 = (0.0812043\,\text{s}, 2.02306\,\text{V})$, where the units are seconds and volts. The final value of the signal is 1. Determine the damping coefficient and the natural frequency.

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