Preface

These lecture notes are based on a course entitled "Topics in Group Theory"[1] given by Gilbert Baumslag at the Graduate Center of CUNY during the Spring 2007 semester that focused on finitely generated solvable groups from the perspective of combinatorial group theory.

Among the major results highlighted by the course were the following:

- There are continuously many isomorphism classes of finitely generated 3-solvable groups, and the same is true for the (proper) subclass comprised of all the (finitely generated) center-by-metabelian groups (P. Hall, Theorem 1.1).
- Any extension G of a group A by a group H can be embedded in an unrestricted wreath product of A by H (Krasner and Kaloujnine, Theorem 4.3).
- If a finitely presented group G has an infinite cyclic quotient then either G is an ascending HNN-extension or else G contains a free group of rank 2 (Bieri and Strebel, Theorem 5.1).
- A finitely presented solvable group is either finite or else virtually an ascending HNN-extension of a finitely generated group (Bieri and Strebel, Theorem 5.2).
- Finitely generated metabelian groups satisfy the maximal condition for normal subgroups (P. Hall, Theorem 6.1).
- There are only countably many isomorphism classes of finitely generated metabelian groups (P. Hall, Theorem 6.2).
- Any finitely generated metabelian group can be embedded in a finitely presented metabelian group (G. Baumslag, Theorem 7.1).

Much of the course was devoted to building foundations and before studying techniques later utilized in proving the important structure and embedding theorems by Krasner and Kaloujnine [12], Bieri and Strebel [7], and Baumslag [3]. Among the tools included were semi-direct products, wreath products, HNN-extensions, representations; among the techniques employed in various settings were Cayley's

[1] http://math.gc.cuny.edu/courses/CourseDescriptions/CourseDescriptionMath87200.pdf

and Frobenius' permutation representations and Hall's module-over-group-ring approach to the normal subgroup structure of metabelian groups (see [8–10]).

In these notes, background material that goes beyond a first year graduate algebra course and some basic elements of combinatorial group theory (such as basis and rank of a free group) is developed, reviewed, and discussed as needed. Augmenting the more compact classroom lecture notes we include details of proofs, comments, and examples to a greater extent; and yet a number of proofs are still only sketched or hinted at, if not absent. The reader is encouraged to fully complete them.

Preparatory material takes up a sizable portion of the course. Deviating from the "live" version here, for clarity's sake, we decided to relegate some of the proof details to the appendices.

Several chapters and sections include concrete examples. More assistance from the relevant literature can also be found in the bibliography at the end of the document.

The authors claim full responsibility for any errors appearing in these notes.

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