

Chapter 2

Heat Transfer Through a Porous Medium

2.1 Energy Equation: Simple Case

In this chapter we focus on the equation that expresses the first law of thermodynamics in a porous medium. We start with a simple situation in which the medium is isotropic and where radiative effects, viscous dissipation, and the work done by pressure changes are negligible. Very shortly we shall assume that there is local thermal equilibrium so that $T_s = T_f = T$, where T_s and T_f are the temperatures of the solid and fluid phases, respectively. Here we also assume that heat conduction in the solid and fluid phases takes place in parallel so that there is no net heat transfer from one phase to the other. More complex situations will be considered in Sect. 6.5. The fundamentals of heat transfer in porous media are also presented in Bejan et al. (2004) and Bejan (2004a).

Taking averages over an elemental volume of the medium we have, for the solid phase,

$$(1 - \varphi)(\rho c)_s \frac{\partial T_s}{\partial t} = (1 - \varphi) \nabla \cdot (k_s \nabla T_s) + (1 - \varphi) q_s''' \quad (2.1)$$

and, for the fluid phase,

$$\varphi (\rho c_P)_f \frac{\partial T_f}{\partial t} + (\rho c_P)_f \mathbf{v} \cdot \nabla T_f = \varphi \nabla \cdot (k_f \nabla T_f) + \varphi q_f''' \quad (2.2)$$

Here the subscripts s and f refer to the solid and fluid phases, respectively, c is the specific heat of the solid, c_P is the specific heat at constant pressure of the fluid, k is the thermal conductivity, and $q'''(\text{W/m}^3)$ is the heat production per unit volume.

In writing Eqs. (2.1) and (2.2) we have assumed that the surface porosity is equal to the porosity. This is pertinent to the conduction terms. For example, $-k_s \nabla T_s$ is the conductive heat flux through the solid, and thus $\nabla \cdot (k_s \nabla T_s)$ is the net rate of heat conduction into a unit volume of the solid. In Eq. (2.1) this appears multiplied by

the factor $(1 - \varphi)$, which is the ratio of the cross-sectional area occupied by solid to the total cross-sectional area of the medium. The other two terms in Eq. (2.1) also contain the factor $(1 - \varphi)$ because this is the ratio of volume occupied by solid to the total volume of the element. In Eq. (2.2) there also appears a convective term, due to the seepage velocity. We recognize that $\mathbf{V} \cdot \nabla T_f$ is the rate of change of temperature in the elemental volume due to the convection of fluid into it, so this, multiplied by $(\rho c_p)_f$, must be the rate of change of thermal energy, per unit volume of fluid, due to the convection. Note further that in writing Eq. (2.2) use has been made of the Dupuit-Forchheimer relationship $\mathbf{v} = \varphi \mathbf{V}$.

Setting $T_s = T_f = T$ and adding Eqs. (2.1) and (2.2) we have

$$(\rho c)_m \frac{\partial T}{\partial t} + (\rho c)_f \mathbf{v} \cdot \nabla T = \nabla \cdot (k_m \nabla T) + q_m''', \quad (2.3)$$

where

$$(\rho c)_m = (1 - \varphi) (\rho c)_s + \varphi (\rho c_p)_f, \quad (2.4)$$

$$k_m = (1 - \varphi) k_s + \varphi k_f, \quad (2.5)$$

$$q_m''' = (1 - \varphi) q_s''' + \varphi q_f''' \quad (2.6)$$

are, respectively, the overall heat capacity per unit volume, overall thermal conductivity, and overall heat production per unit volume of the medium.

2.2 Energy Equation: Extensions to More Complex Situations

2.2.1 Overall Thermal Conductivity of a Porous Medium

In general, the overall thermal conductivity of a porous medium depends in a complex fashion on the geometry of the medium. As we have just seen, if the heat conduction in the solid and fluid phases occurs in parallel, then the overall conductivity k_A is the weighted arithmetic mean of the conductivities k_s and k_f :

$$k_A = (1 - \varphi) k_s + \varphi k_f. \quad (2.7)$$

On the other hand, if the structure and orientation of the porous medium is such that the heat conduction takes place in series, with all of the heat flux passing through both solid and fluid, then the overall conductivity k_H is the weighted harmonic mean of k_s and k_f :

$$\frac{1}{k_H} = \frac{1 - \varphi}{k_s} + \frac{\varphi}{k_f}. \quad (2.8)$$

In general, k_A and k_H will provide upper and lower bounds, respectively, on the actual overall conductivity k_m . We always have $k_H \leq k_A$, with equality if and only if $k_s = k_f$. For practical purposes, a rough and ready estimate for k_m is provided by k_G , the weighted geometric mean of k_s and k_f , defined by

$$k_G = k_s^{1-\varphi} k_f^{\varphi}. \quad (2.9)$$

This provides a good estimate so long as k_s and k_f are not too different from each other (Nield 1991b). More complicated correlation formulas for the conductivity of packed beds have been proposed. Experiments by Prasad et al. (1989b) showed that these formulas gave reasonably good results provided that k_f was not significantly greater than k_s . The agreement when $k_f \gg k_s$ was not good, the observed conductivity being greater than that predicted. This discrepancy may be due to porosity variation near the walls. Since k_m depends on φ there is an effect analogous to the hydrodynamic effect already noted in Sect. 1.7. Some of the discrepancy may be due to the difficulty of measuring a truly stagnant thermal conductivity in this case (Nield 1991b).

In the case when the fluid is a rarefied gas and the Knudsen number has a large value, temperature slip occurs in the fluid at the pore boundaries. In these circumstances one could expect that the fluid conductivity would tend to zero as the Knudsen number increases. Then in the case of external heating the heat would be conducted almost entirely through the solid matrix. In the case of just internal heating in the fluid, the situation is reversed as the fluid phase becomes thermally isolated from the solid phase.

Further models for stagnant thermal conductivity have been put forward by Hsu et al. (1994, 1995), Cheng et al. (1999), and Cheng and Hsu (1998, 1999). In particular, Cheng et al. (1999), and also Hsu (2000), contain comprehensive reviews of the subject. Volume averaging was used by Buonanno and Carotenuto (1997) to calculate the effective conductivity taking into account particle-to-particle contact. Experimental studies have been made by Imadojemu and Porter (1995) and Tavman (1996). The former concluded that the thermal diffusivity and conductivity of the fluid played the major role in determining the effective conductivity of the medium. Hsu (1999) presented a closure model for transient heat conduction, while Hsiao and Advani (1999) included the effect of heat dispersion. Hu et al. (2001) discussed unconsolidated porous media, Paek et al. (2000) dealt with aluminum foam materials, and Fu et al. (1998) studied cellular ceramics. Boomsma and Poulikakos studied the effective thermal conductivity of a three-dimensionally structured fluid-saturated metal foam. Carson et al. (2005) obtained thermal conductivity bounds for isotropic porous materials.

A unified closure model for convective heat and mass transfer has been presented by Hsu (2005). He notes that r.e.v. averaging leads to the introduction of new unknowns (dispersion, interfacial tortuosity, and interfacial transfer) whose determination constitutes the closure problem. More experiments are needed to determine some of the coefficients that are involved. His closure relation for the

interfacial force contains all the components due to drag, lift, and transient inertia to the first-order approximation. He concludes that the macroscopic energy equations are expected to be valid for all values of the time scale and Reynolds number, for the case of steady flows. Further investigations are needed for unsteady flows.

So far we have been discussing the case of an isotropic medium, for which the conductivity is a scalar. For an anisotropic medium k_m will be a second-order tensor. Lee and Yang (1998) modeled a heterogeneous anisotropic porous medium.

A fundamental issue has been raised by Merrikh et al. (2002, 2005a, b) and Merrikh and Lage (2005). This is the question of how the internal regularity of a solid/fluid physical domain affects global flow and heat transfer. These authors have considered a situation (a regular distribution of rectangular solid obstacles in a rectangular box) that is sufficiently simple for a comparison to be made between the results of numerical modeling involving a treatment of the fluid and solid phases considered separately (“continuum model”) and a standard r.e.v.-averaged porous medium (“porous continuum model”). The results for the two models can be substantially different. In other words, the internal regularity can have an important effect. The authors considered situations where the obstacles were separated from the boundary walls, and thus some of the difference is due to a channeling effect. Further contributions have been made by Braga and de Lemos (2005a, b).

The effective thermal conductivity of rough spherical packed beds was studied by Bahrami et al. (2006). Two effective conductivity models for porous media composed of hollow spherical agglomerates were proposed by Yu et al. (2006a). A collocated parameter model was employed by Reddy and Karhikeyan (2009) to estimate the effective thermal conductivity of two-phase materials, a subject also studied by Samantray et al. (2006).

Works on the effective thermal conductivity of saturated porous media have been surveyed by Aichlmayr and Kulacki (2006).

The analogy between dual-phase-lagging and porous-medium conduction was discussed by Wang et al. (2008d). The analogy permits existence, uniqueness, and structural stability results established for the former to be applied to the latter.

A comprehensive review of various models for the effective conductivity was made by Singh (2011), who pointed out that this quantity was dependent not only on the conductivities and volume fractions of the constituents, the morphology of the constituent particles, and the structure of the material but also on interphase interactions. Qu et al. (2012a) introduced an octet-truss lattice unit cell model.

2.2.2 *Effects of Pressure Changes and Viscous Dissipation*

If the work done by pressure changes is not negligible (i.e., the condition $\beta T(g\beta/c_{Pf})L \ll 1$ is not met), then a term $-\beta T(\partial P/\partial t + \mathbf{v} \cdot \nabla P)$ needs to be added to the left-hand side of Eq. (2.3). Here β is the coefficient of volumetric thermal expansion, defined by

$$\beta = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p. \quad (2.10)$$

Viscous dissipation is negligible in natural convection if $(g\beta/c_p)L \ll 1$, which is usually the case. If it is not negligible, another term must be added to the right-hand side of Eq. (2.3), as noted first by Ene and Sanchez-Palencia (1982). If Darcy's law holds, that term is $(\mu/K)\mathbf{v} \cdot \mathbf{v}$ in the case of an isotropic medium, and $\mu\mathbf{v} \cdot \mathbf{K}^{-1} \cdot \mathbf{v}$ if the medium is anisotropic. To see this, note that the average of the rate of doing work by the pressure, on a unit volume of an r.e.v., is given by the negative of $\text{div}(P\varphi\mathbf{V}) = \text{div}(P\mathbf{v}) = \mathbf{v} \cdot \text{grad } P$, since $\text{div } \mathbf{v} = 0$. The Forchheimer drag term, dotted with the velocity vector, contributes to the dissipation, despite the fact that the viscosity does not enter explicitly. This apparent paradox was resolved by Nield (2000). The contribution of the Brinkman drag term is currently a controversial topic. Nield (2004b) proposed that the Brinkman term be treated in the same way as the Darcy and Forchheimer terms, so that the total viscous dissipation remains equal to the power of the total drag force. Thus the viscous dissipation φ would then be modeled by

$$\varphi = \frac{\mu}{K} \mathbf{v} \cdot \mathbf{v} + \frac{c_p}{K^{1/2}} |\mathbf{v}|_p \mathbf{v} \cdot \mathbf{v} - \tilde{\mu} \mathbf{v} \cdot \nabla^2 \cdot \mathbf{v}. \quad (2.11)$$

Al-Hadhrani et al. (2003) prefer a form that remains positive and reduces to that for a fluid clear of solid material in the case where the Darcy number tends to infinity. Accordingly, they would add the usual clear fluid term to the Darcy and Forchheimer terms. Nield (2004b) suggested that the Brinkman equation may break down in this limit. In most practical situations the Brinkman term will be small compared with the Darcy term, and so the form of the Brinkman term is then not important. Additional discussion of viscous dissipation in porous media and the validity of the Brinkman equation can be found in Salama (2011a), who included an additional term involving the gradient of the porosity. Salama et al. (2012) compared the effects of various terms on boundary layer flow on a vertical wall.

Nield (2000) noted that scale analysis, involving the comparison of the magnitude of the viscous dissipation term to the thermal diffusion term, shows that viscous dissipation is negligible if $N \ll 1$, where $N = \mu U^2 L^2 / K k_m \Delta T = Br/Da$, where the Brinkman number is defined by $Br = \mu U^2 / k_m \Delta T = EcPr$, where the Eckert number Ec is defined by $Ec = U^2 / c_p \Delta T$. For most situations the Darcy number K/L^2 is small, so viscous dissipation is important at even modest values of the Brinkman number. For forced convection the choice of the characteristic velocity is obvious. For natural convection, scale analysis leads to the estimate $U \sim (k_m / \rho c_p L) Ra^{1/2}$ and the condition that viscous dissipation is negligible becomes $Ge = 1$, where Ge is the Gebhart number defined by $Ge = g\beta L / c_p$. The above comments on forced convection are made on the assumption that the Péclet number $Pe = \rho c_p UL / k_m$ is not large. If it is large, then the proper comparison is one between the magnitudes of the viscous dissipation term and the convective transport term. This ratio is of order $Ec/DaRe$, where the Reynolds number

$Re = \rho UL/\mu$. Further aspects of the effects of viscous dissipation on the flow in porous media are discussed in the survey by Magyari et al. (2005b).

The question of how the viscous dissipation relates to the pressure work and other non-Boussinesq effects has been the subject of considerable discussion by Costa (2009, 2010), Nield (2007b, 2009a), Barletta (2008), and Nield and Barletta (2010a). Costa argued that the first law of thermodynamics required that the contributions of viscous dissipation and pressure work had to be in balance. Nield and Barletta argued that Costa had misapplied the first law to an unsteady problem which he treated as a steady-state one, and that there are physical situations where the viscous dissipation is significant and the pressure work is not significant.

2.2.3 Absence of Local Thermal Equilibrium

Usually it is a good approximation to assume that the solid and fluid phases are in thermal equilibrium but there are situations, such as highly transient problems and some steady-state problems (Nield 1998a), where this is not so. Now this is commonly referred to as local thermal nonequilibrium (LTNE), though Vadasz (2005a, b) prefers the expression lack of thermal equilibrium.

If one wishes to allow for heat transfer between solid and fluid (that is, one no longer has local thermal equilibrium), then one can, following Combarous (1972) and Bories (1987), replace Eqs. (2.1) and (2.2) by

$$(1 - \varphi) (\rho c)_s \frac{\partial T_s}{\partial t} = (1 - \varphi) \nabla \cdot (k_s \nabla T_s) + (1 - \varphi) q_s''' + h (T_f - T_s), \quad (2.12)$$

$$\varphi (\rho c_p)_f \frac{\partial T_f}{\partial t} + (\rho c_p) \mathbf{v} \cdot \nabla T_f = \varphi \nabla \cdot (k_f \nabla T_f) + \varphi q_f''' + h (T_s - T_f), \quad (2.13)$$

where h is a heat transfer coefficient. See also Eqs. (2.12a) and (2.13a) later in this section. A critical aspect of using this approach lies in the determination of the appropriate value of h . Experimental values of h are found in an indirect manner; see, e.g., Polyakov et al. (1996). According to correlations for a porous bed of particle established in Dixon and Cresswell (1979),

$$h = a_{fs} h^*, \quad (2.14)$$

where the specific surface area (surface per unit volume) a_{fs} is given by

$$a_{fs} = 6 (1 - \varphi) / d_p, \quad (2.15)$$

and

$$\frac{1}{h^*} = \frac{d_p}{Nu_{fs} k_f} + \frac{d_p}{\beta k_s} \quad (2.16)$$

where d_p is the particle diameter and $\beta = 10$ if the porous bed particles are of spherical form. The fluid-to-solid Nusselt number Nu_{fs} is, for Reynolds numbers (based on d_p) $Re_p > 100$, well correlated by the expression presented in Handley and Heggis (1968):

$$Nu_{fs} = (0.255/\varphi) Pr^{1/3} Re_p^{2/3}, \quad (2.17)$$

while for low values of Re_p the estimates of Nu_{fs} vary between 0.1 and 12.4, these being based on Miyauchi et al. (1976) and Wakao et al. (1976, 1979). As an alternative to Eq. (2.17), Wakao and Kaguei (1982) proposed the correlation

$$Nu_{fs} = 2.0 + 1.1 Pr^{1/3} Re_p^{0.6} (\varphi d_p / d_h)^{0.6} \quad (2.17a)$$

Here d is the pore-scale hydraulic diameter.

Other authors have used alternative expressions for h^* and a_{fs} and some of these were considered by Alazmi and Vafai (2000), who found that the various models give closely similar results for forced convection channel flow when the porosity is high or the pore Reynolds number is large or the particle diameters are small. Theoretical and experimental results reported by Grangeot et al. (1994) indicate that h^* depends weakly on the Péclet number of the flow. This subject is discussed further in Sects. 6.5 and 6.9.2. The topic in the context of turbulence has been discussed by Saito and de Lemos (2005). An experimental study for a metallic packed bed was reported by Carrillo (2005). The effect of different packings was investigated experimentally by Yang et al. (2012b). They found that the formula in Eq. (2.17a) over-predicted their results unless the coefficients 2.0 and 1.1 were replaced by smaller values.

A discussion of further aspects of the two-medium approach to heat transfer in porous media is given by Quintard et al. (1997) and Quintard and Whitaker (2000). Nield (2002a) noted that Eqs. (2.12) and (2.13) are based on the implicit assumption that the thermal resistances of the fluid and solid phases are in series. For the case of a layered medium in a parallel plate channel with fluid/solid interfaces parallel to the x -direction, he suggested that the appropriate equations in the absence of internal heating are

$$(1 - \varphi) (\rho c)_s \frac{\partial T_s}{\partial t} = (1 - \varphi) \left[\frac{\partial}{\partial x} \left(k'_s \frac{\partial T_s}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_s \frac{\partial T_s}{\partial y} \right) \right] + h (T_f - T_s), \quad (2.12a)$$

$$\varphi(\rho c_P)_f \frac{\partial T_f}{\partial t} + (\rho c_P)_f \mathbf{v} \cdot \nabla T_f = \varphi \left[\frac{\partial}{\partial x} \left(k'_f \frac{\partial T_f}{\partial x} \right) + \frac{\partial}{\partial y} \left(k'_f \frac{\partial T_f}{\partial y} \right) \right] + h(T_s - T_f), \quad (2.13a)$$

where $k'_f = k'_s = k_H$ with k_H given by Eq. (2.8). Equations (2.12) and (2.13) have to be solved subject to certain applied thermal boundary conditions. If a boundary is at uniform temperature, then one has $T_f = T_s$ on the boundary. If uniform heat flux is imposed on the boundary, then there is some ambiguity about the distribution of flux between the two phases. Nield and Kuznetsov (1999) argued that if the flux is truly uniform, then it has to be uniform with respect to the two phases, and hence the flux on the r.e.v. scale has to be distributed between the fluid and solid phases in the ratio of the surface fractions; for a homogeneous medium that means in the ratio of the volume fractions, that is in the ratio $\varphi: (1 - \varphi)$. This distribution allows the conjugate problem considered by them to be treated in a consistent manner. The consequences of other choices for the distribution were explored by Kim and Kim (2001) and Alazmi and Vafai (2002). The Nield and Kuznetsov (1999) approach is equivalent to Model 1D in Alazmi and Vafai (2002) and is not equivalent to either approach used in Kim and Kim (2001).

The particular case of LTNE in a steady process is discussed by Nield (1998a). Petit et al. (1999) have proposed an LTNE model for two-phase flow. A numerical study of the interfacial convective heat transfer coefficient was reported by Kuwahara et al. (2001). Their results were modified by Pallares and Grau (2010) to produce agreement between the theoretical results for the Nusselt number and experimental data. An application of the method of volume averaging to the analysis of heat and mass transfer in tubes was made by Golfier et al. (2002). An alternative two-equation model for conduction only was presented by Fourie and Du Plessis (2003a, b). Vadasz (2005a) demonstrated that, for heat conduction problems, local thermal equilibrium applies for any conditions that are a combination of constant temperature and insulation. He also questioned whether a linear relationship between the average temperature difference between the phases and the heat transferred over the fluid–solid surface was appropriate in connection with conditions of LTNE. The exclusion of oscillations in the context of conduction with LTNE and an associated paradox were discussed by Vadasz (2005b, 2006b, 2007). (The apparent paradox arises in trying to reconcile the results from two alternative mathematical approaches to modeling the problem.) This work is surveyed by Vadasz (2008b), who also shows the relevance of LTNE to the study of nanofluids and bi-composite media, as well as to the experimental measurement of the effective thermal conductivity of a porous medium via the transient hot wire method.

Rees and Pop (2005) surveyed studies of LTNE with special attention to natural and forced convection boundary layers and on internal natural convection. Their survey complements that by Kuznetsov (1998e) for internal forced convection. The effect of LTNE on conduction in channels with a uniform heat source was investigated by Nouri-Borujerdi et al. (2007b). Several causes of LTNE were

discussed by Virtol et al. (2009). Some microscopic modeling of conduction with LTNE was carried out by Rees (2010).

The topic of LTNE was reviewed by Haji-Sheikh and Minkowycz (2008). They cite references to a number of engineering applications, such as nuclear devices, fuel cells, electronic systems, and micro devices, in the context of rapid transport of heat. They include a discussion of the development of the thermal field with a moving fluid. They summarize experimental results obtained by Nnanna et al. (2004, 2005) which conform to the observation by Vadasz (2005b, 2006b, 2007) that the physical conditions for thermal waves to materialize are not obtainable in a porous slab subject to a combination of constant heat flux and temperature boundary conditions.

When one examines LTNE at the boundary of a porous medium, or at an interface with a fluid clear of solid material, the solution of the differential equation system that arises is undetermined until further information is available to determine how the total heat flux is split between the two phases. Two second order differential equations are involved and so at an interface one needs four boundary conditions, two involving the temperature and two involving the heat flux. The conservation of energy imposes just one heat flux condition, and hence another condition must be sought. For this Yang and Vafai (2010, 2011a, b, c) and Vafai and Yang (2012) introduced five models for what they called “heat flux bifurcation,” but they did not clearly distinguish between them. Nield (2012) argued that this approach was not satisfactory. Rather, one should distinguish between the heat transfer in the bulk of the porous medium (which depends on the interphase heat transfer coefficient) and the heat transfer across the interface (which is affected by what happens on the other side of the interface, i.e., outside the porous medium). For example, if the porous medium is bounded by a solid with high thermal conductivity (effectively a constant-temperature boundary), then one has LTNE at the boundary and one can use the formulation employed by Nield and Kuznetsov (2012c). Much the same is true if the neighboring region is a fluid of high conductivity. If the region is a solid of very low conductivity (an insulating boundary) then there is essentially no boundary flux to be divided between the two phases. More generally, if the solid boundary is controlled by an imposed constant flux, then the natural assumption is that just across the interface in the porous medium the flux is also constant. Thus the splitting occurs so that the flux in the fluid phase is the same as in the solid phase. This means that the interfacial heat transport is divided between the fluid and solid phases in the ratio of φ to $(1-\varphi)$. This was the model employed by Nield and Kuznetsov (1999).

2.2.4 Thermal Dispersion

A further complication arises in forced convection or in vigorous natural convection in a porous medium. There may be significant thermal dispersion, i.e., heat transfer due to hydrodynamic mixing of the interstitial fluid at the pore scale. In

addition to the molecular diffusion of heat, there is mixing due to the nature of the porous medium. Some mixing is due to the obstructions; the fact that the flow channels are tortuous means that fluid elements starting a given distance from each other and proceeding at the same velocity will not remain at the same distance apart. Further mixing can arise from the fact that all pores in a porous medium may not be accessible to a fluid element after it has entered a particular flow path.

Mixing can also be caused by recirculation caused by local regions of reduced pressure arising from flow restrictions. Within a flow channel mixing occurs because fluid particles at different distances from a wall move relative to one another. Mixing also results from the eddies that form if the flow becomes turbulent. Diffusion in and out of dead-end pores modifies the nature of molecular diffusion. For details, see Greenkorn (1983, p. 190).

Dispersion is thus a complex phenomenon. Rubin (1974) took dispersion into account by generalizing Eq. (2.3) so that the term $\nabla \cdot (\alpha_m \nabla T)$, where $\alpha_m = k_m/(\rho c)_m$ is the thermal diffusivity of the medium, is replaced by $\nabla \cdot \mathbf{E} \cdot \nabla T$ where \mathbf{E} is a second-order tensor (the dispersion tensor). In an isotropic medium the dispersion tensor is axisymmetric and its components can be expressed in the form

$$E_{ij} = F_1 \cdot \delta_{ij} + F_2 V_i V_j, \quad (2.18)$$

where $V_i (= v_i/\varphi)$ is the i^{th} component of the barycentric (intrinsic) velocity vector, and F_1 and F_2 are functions of the pore size and the Péclet and Reynolds numbers of the flow.

At any point in the flow field it is possible to express \mathbf{E} with reference to a coordinate system in which the first axis coincides with the flow direction; when this is done we have

$$\begin{aligned} E_{11} &= \eta_1 U + \alpha_m, \\ E_{22} &= E_{33} = \eta_2 U + \alpha_m, \\ E_{ij} &= 0 \quad \text{for } i \neq j, \end{aligned} \quad (2.19)$$

where E_{11} is the longitudinal dispersion coefficient, E_{22} and E_{33} are the lateral dispersion coefficients, and U is the absolute magnitude of the velocity vector.

If the Péclet number of the flow is small, then η_1 and η_2 are small and the molecular thermal diffusivity α_m is dominant. If the Péclet number of the flow is large, then η_1 and η_2 are large and almost constant. It is found experimentally that $\eta_2 = \eta_1/30$, approximately.

For an account of the treatment of dispersion in anisotropic media in the context of convection, the reader is referred to Tyvand (1977). In the particular case when heat conduction is in parallel, Catton et al. (1988) conclude on the basis of their statistical analysis that the effective thermal conductivity k_{zz}^* , for mass and thermal transport in the z -direction through a bed of uniform spherical beads, is given by

$$k_{zz}^* = (1 - \varphi) k_s + \varphi \left(\frac{2B}{\pi} \right) Pe k_f \quad (2.20)$$

In this expression B is a constant introduced by Ergun (empirically, $B = 1.75$) and Pe is the Péclet number defined by $Pe = vd_p/\alpha_f(1 - \varphi)$, where d_p is the spherical particle diameter and α_f is the thermal diffusivity of the fluid, defined by $\alpha_f = k_f/(\rho c_p)_f$.

Thermal dispersion plays a particularly important role in forced convection in packed columns. The steep radial temperature gradients that exist near the heated or cooled wall were formerly attributed to channeling effects, but more recent work has indicated that thermal dispersion is also involved. For a nearly parallel flow at high Reynolds numbers, the thermal dispersivity tensor reduces to a scalar, the transverse thermal dispersivity. Cheng and his colleagues [see Hsu and Cheng (1990) and the references given in Sect. 4.9] assumed that the local transverse thermal dispersion conductivity k'_T is given by

$$\frac{k'_T}{k_f} = D_T Pe_d \lambda \frac{u}{u_m}. \quad (2.21)$$

In this equation Pe_d is a Péclet number defined by $Pe_d = u_m d_p/\alpha_f$, in terms of the mean seepage velocity u_m , the particle diameter d_p , and fluid thermal diffusivity α_f , while D_T is a constant and λ is a dimensionless dispersive length normalized with respect to d_p . In recent work the dispersive length is modeled by a wall function of the Van Driest type:

$$\lambda = 1 - \exp(-y/\omega d_p). \quad (2.22)$$

The empirical constants ω and D_T depend on the coefficients N and C in the wall porosity variation formula [Eq. (1.28)]. The best match with experiments is given by $D_T = 0.12$ and $\omega = 1$, if $N = 5$ and $C = 1.4$. The theoretical results based on this *ad hoc* approach agree with a number of experimental results.

A theoretical backing for this approach has been given by Hsu and Cheng (1990). This is based on volume averaging of the velocity and temperature deviations in the pores in a dilute array of spheres, together with a scale analysis. The thermal diffusivity tensor \mathbf{D} is introduced as a multiplying constant which accounts for the interaction of spheres. For the case of high pore Reynolds number flow, Hsu and Cheng (1990) found the thermal dispersion conductivity tensor \mathbf{k}' to be given by

$$\mathbf{k}' = \mathbf{D} k_f \frac{1 - \varphi}{\varphi} Pe_d \quad (2.23)$$

The linear variation with Pe_d is consistent with most of the existing experimental correlations for high pore Reynolds number flow. At low pore Reynolds number flow, they found

$$\mathbf{k}' = \mathbf{D}^* k_f \frac{1 - \varphi}{\varphi^2} Pe_d^2 \quad (2.24)$$

where \mathbf{D}^* is another constant tensor. The quadratic dependence on Pe_d has not yet been confirmed by experiment.

Kuwahara et al. (1996) and Kuwahara and Nakayama (1999) have studied numerically thermal diffusion for a two-dimensional periodic model. A limitation of their correlation formulas as the porosity tends to unity was discussed by Yu (2004) and Nakayama and Kuwahara (2004). A similar model was examined by Souto and Moyne (1997a, b). The frequency response model was employed by Muralidhar and Misra (1997) in an experimental study of dispersion coefficients. The role of thermal dispersion in the thermally developing region of a channel with a sintered porous metal was studied by Hsieh and Lu (2000). Kuwahara and Nakayama (2005) have extended their earlier numerical studies to the case of three-dimensional flow in highly anisotropic porous media. Niu et al. (2006) reported direct measurements of eddy transport and thermal dispersion in a high-porosity matrix. An equation for thermal dispersion-flux transport was introduced by Nakayama et al. (2006).

For further information about dispersion in porous media, the reader is referred to the review by Liu and Masliyah (2005), which deals with the dispersion of mass, heat, and momentum. Rudraiah and Ng (2007) have reviewed dispersion in porous media with and without reaction.

2.2.5 Cellular Porous Media

Cellular porous media have the property that to a good approximation the effect of radiation can be modeled using a temperature-dependent thermal conductivity (Viskanta 2009). For a few situations an analytical solution can be obtained. This was done by Nield and Kuznetsov (2010a, c) and Nield and Kuznetsov (2010b) for paradigmatic forced convection, external natural convection, and internal natural convection problems.

2.3 Oberbeck-Boussinesq Approximation

In studies of natural convection we add the gravitational term $\rho_f \mathbf{g}$ to the right-hand side of the Darcy equation (1.4) or its appropriate extension. [Note that in Eq. (1.4) the term ∇P denotes an *intrinsic* quantity, so we add the gravitational force per unit

volume of the *fluid*]. For thermal convection to occur, the density of the fluid must be a function of the temperature, and hence we need an equation of state to complement the equations of mass, momentum, and energy. The simplest equation of state is

$$\rho_f = \rho_0[1 - \beta(T - T_0)], \quad (2.25)$$

where ρ_0 is the fluid density at some reference temperature T_0 and β is the coefficient of thermal expansion.

In order to simplify the subsequent analysis, one employs the Boussinesq approximation whenever it is valid. Strictly speaking, one should call this the *Oberbeck-Boussinesq approximation*, since Oberbeck (1879) has priority over Boussinesq (1903), as documented by Joseph (1976). The approximation consists of setting constant all the properties of the medium, except that the vital buoyancy term involving β is retained in the momentum equation. As a consequence the equation of continuity reduces to $\nabla \cdot \mathbf{v} = 0$, just as for an incompressible fluid. The Boussinesq approximation is valid provided that density changes $\Delta\rho$ remain small in comparison with ρ_0 throughout the flow region and provided that temperature variations are insufficient to cause the various properties of the medium (fluid and solid) to vary significantly from their mean values. Johannsen (2003) discussed the validity of the Boussinesq approximation in the case of a bench mark problem known as the Elder problem.

2.4 Thermal Boundary Conditions

Once the thermal conductivity in the porous medium has been determined, the application of thermal boundary conditions is usually straightforward. At the interface between two porous media, or between a porous medium and a clear fluid, we can impose continuity of the temperature (on the assumption that we have local thermodynamic equilibrium) and continuity of the normal component of the heat flux. We note that two conditions are required because the equation of energy (2.3) contains second-order derivatives.

The heat flux vector is the sum of two terms: a convective term $(\rho c_p)_f T \mathbf{v}$ and a conductive term $-k \nabla T$. The normal component of the former is continuous because both T and the normal component of $\rho_f \mathbf{v}$ are continuous. It follows that the normal component of $k \nabla T$ also must be continuous. At an impermeable boundary the usual thermal condition appropriate to the external environment can be applied, e.g., one can prescribe either the temperature or the heat flux, or one can prescribe a heat transfer coefficient.

Sahraoui and Kaviani (1993, 1994) have discussed the errors arising from the use of approximations of the effective conductivity near a boundary, due to nonuniformity of the distributions of the solid and fluid phases there. They have

introduced a slip coefficient into the thermal boundary condition to adjust for this, for the case of two-dimensional media.

Ochoa-Tapia and Whitaker (1997, 1998) have developed flux jump conditions applicable at the boundary of a porous medium and a clear fluid. These are based on a nonlocal form of the volume-averaged thermal energy equations for fluid and solid. The conditions involve excess surface thermal energy and an excess nonequilibrium thermal source. Min and Kim (2005) have used the special two-dimensional model of Richardson (1971) in order to obtain estimates of the coefficients that occur in the thermal and hydrodynamic jump conditions. The jump conditions were further analyzed by d'Hueppe et al. (2011). Valdes-Parada et al. (2009b) included the effects of adsorption and a chemical reaction. Betchen et al. (2006) considered a nonequilibrium model. d'Hueppe et al. (2012) discussed the coupling of a two-temperature model with a one-temperature model at a fluid-porous interface.

An analogous mass transfer jump condition was formulated by Valencia-López et al. (2003). The thermal interaction at the interface between a porous medium and an impermeable wall was studied by Kim and Kim (2001). The role of particle-particle contact on effective thermal properties in the interfacial region was examined by Aguilar-Madera et al. (2011b).

2.5 Hele-Shaw Analogy

The space between two plane walls a small distance apart constitutes a Hele-Shaw cell. If the gap is of thickness h and the walls each of thickness d , then the governing equations for gap-averaged velocity components (parallel to the plane walls) are identical with those for two-dimensional flow in a porous medium whose permeability K is equal to $h^3/[12(h + 2d)]$, for the case where the heat flow is parallel to the plane walls (Hartline and Lister 1977). The Hele-Shaw cell thus provides a means of modeling thermal convection in a porous medium, as in the experiments by Elder (1967a).

For the analogy to hold, the three quantities h/δ , $Uh^2/\nu\delta$, and $Uh^2/\alpha_f\delta$ must all be small compared with unity. Here U is the velocity scale and δ the smallest length scale of the motion being modeled, while ν and α_f are the kinematic viscosity and thermal diffusivity of the fluid. These conditions ensure that there is negligible advection of vorticity and rapid diffusion of vorticity and heat across the flow.

The experimental temperature profiles found by Vorontsov et al. (1991) were in good agreement with the theory. Schöpf (1992) extended the comparison to the case of a binary mixture. Specific studies of convection in a Hele-Shaw cell were reported by Cooper et al. (1997), Goldstein et al. (1998), and Gorin et al. (1998).

The Hele-Shaw cell experiments are especially useful for revealing streamline patterns when the walls are made of transparent material. The analogy has obvious limitations. For example, it cannot deal with the effects of lateral dispersion or instabilities associated with three-dimensional disturbances. The discrepancies

associated with these effects have been examined by Kvernfold (1979) and Kvernfold and Tyvand (1981).

Hsu (2005) has compared the governing equations for the averaged flows and heat transfer in Hele-Shaw cells with those of porous media and he observed the following differences: (a) the averaged Hele-Shaw cell is two-dimensional, (b) the interfacial force in the averaged Hele-Shaw flows is contributed entirely from the shear force, and (c) there exists no thermal tortuosity for the averaged Hele-Shaw flows. Thus the Hele-Shaw analogy is good for viscous dominated two-dimensional flow with negligible thermal tortuosity. However, these simplifications help in the verification of closure modeling. Furthermore, a three-dimensional numerical simulation of the convection heat transfer in Hele-Shaw cells may reveal some detailed physics of heat transfer in porous media that are impossible to tackle due to the randomness and the complexity of the microscopic solid geometry. Hsu (2005) illustrates this with results for the case of oscillating flows past a heated circular cylinder. Babuskin and Demin (2006) reported an experimental and theoretical investigation of transient convective regimes. Backhaus et al. (2011) investigated the convective instability and mass transport of diffusion layers. Abdelkareem et al. (2009) performed an experimental study on oscillatory convection in a Hele-Shaw cell due to an unstably heated side.

2.6 Bioheat Transfer and Other Approaches

Convective heat transfer in biological tissues involves a special situation. In some cases applications of porous media theory are appropriate (see for example, the surveys by Khanafer et al. (2008a) and Khanafer and Vafai (2009).) Some aspects relevant to biological tissues were discussed by Khanafer et al. (2003), Khaled and Vafai (2003), Yao and Gu (2007), Wood et al. (2007), Mahjoob and Vafai (2009, 2010, 2011), and Wang and Fan (2011). A feature of bioheat transfer is that in many situations there is counterflow. For example, blood flows in adjacent arteries and veins in opposite directions. Nield and Kuznetsov (2008a, 2009a, 2010b) and Kuznetsov and Nield (2009a, b) have modeled forced convection in a porous medium with counterflow. A general set of bioheat transfer equations based on volume averaging theory has been obtained by Nakayama et al. (2011), who applied the bioheat equation to cryoablation therapy for the treatment of malignant cancers.

Direct numerical simulation of heat and fluid flow, using the full Navier–Stokes equations at the pore scale, for regularly spaced square or circular rods or spheres has been conducted by Kuwahara et al. (1994). A direct numerical simulation was applied by He and Georgiadis (1992) to the study of the effect of randomness on one-dimensional heat conduction. Direct numerical simulation has also been employed by Rahimian and Poushaghaghy (2002), Yu et al. (2006b), Pourshaghaghy et al. (2007), Narasimhan and Raju (2007), Gamrat et al. (2008), and Ma and Zabarar (2008). Lattice gas cellular automata simulations were performed by McCarthy (1994) for flow through arrays of cylinders, and by Yoshino and Inamura (2003)

for flow in a three-dimensional structure. Buikis and Ulanova (1996) have modeled nonisothermal gas flow through a heterogeneous medium using a two-media approach. A diffuse approximation has been applied by Prax et al. (1996) to natural convection. Martins-Costa et al. (1992, 1994), Martins-Costa et al. (1994), and Martins-Costa (1996) have applied the continuous theory of mixtures to the modeling and simulation of heat transfer in various contexts. Modeling of convection in reservoirs having fractal geometry has been conducted by Fomin et al. (2002). Spaid and Phelan (1997) applied lattice Boltzmann methods to model microscale flow in fibrous porous media.

A general discussion of the dynamic modeling of convective heat transfer in porous media was provided by Hsu (2005). Further simulation studies with a lattice Boltzmann model have been reported by Guo and Zhao (2005a, b) (with the viscosity independent or dependent on the temperature), Zhou et al. (2010b) (a problem involving double diffusion), Seta et al. (2006), Rong et al. (2010a), Shokouhmand et al. (2009), Xu et al. (2005, 2008), Wang et al. (2007a), Yan et al. (2006), Zhao et al. (2010a, b), Roussellet et al. (2011), and Vishnampet Ramanathan et al. (2011). Visser et al. (2008a, b) have introduced an artificial compressibility method for buoyancy-driven flow.

Petrasch et al. (2008) described a tomography-based determination of the interfacial heat transfer coefficient in reticulate porous dynamics.

Radiative heat transfer is beyond the scope of this book, but we mention that a review of this subject was made by Howell (2000) and a combined radiation and convection problem was studied by Talukdar et al. (2004).



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