Chapter 2
MotionCast: General Connectivity in Clustered Wireless Networks

Abstract We propose a novel concept of (k, m)-connectivity in mobile clustered wireless networks, in which there are n mobile cluster members and n^d static cluster heads. (k, m)-connectivity signifies that in each time period consisting of m time slots, there exist at least k time slots, during any one of which every cluster member can directly communicate with at least one cluster head. We investigate the critical transmission range of asymptotic (k, m)-connectivity when cluster members move according to random walk or i.i.d. mobility model. Under random walk model, we propose two general heterogeneous velocity models which characterize an inherent property of many applied wireless networks that cluster members move with different velocities. We define weak and strong parameters conditions under both mobility models and analyze the probability that the network is asymptotically (k, m)-connected, denoted as $P(\mathscr{C})$. For both mobilities, under weak parameters condition, we provide bounds on $P(\mathscr{C})$ and derive the critical transmission range for (k, m)-connectivity. For random walk mobility and i.i.d. mobility, under strong parameters condition, we present a precise asymptotic probability distribution of $P(\mathscr{C})$ in terms of the transmission radius. Our results provide fundamental insights and theoretical guidelines on design of large-scale wireless networks.

Keywords General connectivity · Critical transmission range · Mobility · Heterogeneous velocities · Precise probability distribution

2.1 System Model

2.1.1 Network Topology

Assume n cluster members and n^d cluster heads are both initially independently and uniformly placed in a unit square $\mathscr{S}$, where n is a positive integer and d is a positive constant. All cluster members have the same uniform transmission radius

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denoted as $r(n)$, where $r(n)$ is a function of n. Each cluster member is capable of communicating with a node (a cluster member or a cluster head) within $r(n)$. In some places of this chapter, we use r to stand for $r(n)$ for simplicity. The unit square $\mathscr{S}$ is assumed to be a torus.

2.1.2 *Mobility Models*

In both random walk and i.i.d. mobility models, all cluster heads remain static after the initial deployment. Each time slot has the same length T, and each time period consists of m time slots, where T and m are both positive constants.

2.1.2.1 Random Walk Mobility Model

At the beginning of each time period, each cluster member chooses a velocity. The selection of velocity by each cluster member is characterized by velocity models illustrated later. In the meantime, each cluster member independently and uniformly selects a random direction in $[0, 2\pi)$ and moves along this direction with its velocity during the time slot. As the unit square is assumed as a torus, cluster members do not bounce off the border. Note that in all the m time slots of a given time period, each cluster member only changes its direction and does not change its velocity. However, in different time periods, a cluster member can move with different velocities. Also, in a time slot, different cluster members may move with different velocities.

In our model, there are u groups of cluster members in the network denoted as $G_1, G_2, \ldots, G_u$, where u is a positive constant integer. For each $y = 1, 2, \ldots, u$, group G_y consists of M_y cluster members. We have $M_y \sim c_y n^{\alpha_y}$ and $\sum_{y=1}^{u} M_y = n$, where α_y, c_y are both positive constants and $\alpha_y < 1$. At the beginning of each time period, each cluster member in group G_y independently selects a velocity v_o according to a distribution $f_v^{(y)}(v)$ and then moves with v_o in *all the m time slots* of this time period, where $f_v^{(y)}(v)$ are different for different group G_y. We present two heterogeneous velocity models as follows.

- VELOCITY MODEL WITH CONSTANT NUMBER OF VALUES (SIMPLE V- MODEL)—for each $y = 1, 2, \ldots, u$, $f_v^{(y)}(v)$ is a single value distribution. Specifically, a random variable v_o following the distribution $f_v^{(y)}(v)$ is equal to $v^{(y)}$ with probability 1, where $v^{(y)}$ is a positive function of n and y.
- VELOCITY MODEL WITH CONSTANT NUMBER OF INTERVALS (GENERAL V- MODEL)—for each $y = 1, 2, \ldots, u$, $f_v^{(y)}(v)$ is a continuous uniform distribution in $\Delta_y = [v^{(y)}, v_a^{(y)}]$, where $v^{(y)}$ and $v_a^{(y)}$ are positive functions of n and y, and $v^{(y)} < v_a^{(y)}$. A simple illustration is given in Fig. 2.1.

Under both velocity models, we further define that $v_\star = \min\{\frac{v^{(y)}}{\alpha_y} | y = 1, 2, \ldots, u\}$ and assume

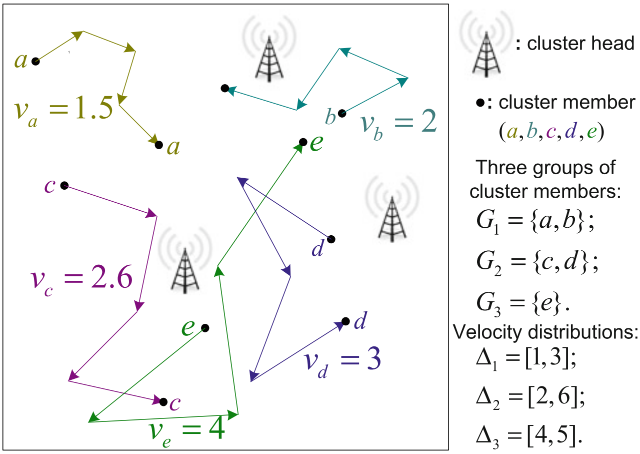

Fig. 2.1 Illustration of random walk mobility with *velocity model with constant number of intervals* in a simple network consisting of 5 cluster members and 3 cluster heads. A time period has 4 time slots here

$$\begin{cases} \frac{v^{(y)}}{\alpha_y} = v_\star, & \text{for } y = y_1, y_2, \ldots, y_z, \\ \frac{v^{(y)}}{\alpha_y} > v_\star, & \text{for other } y. \end{cases}$$

where $y_1, y_2, \ldots, y_z \in \{1, 2, \ldots, u\}$.

Under random walk mobility model, we define weak and strong parameters conditions as follows.

- Weak Parameters Condition—all velocities in the network are of the same order $w\left(\sqrt{\frac{\log n}{n^d}}\right)$ and less than $1/T$.
- Strong Parameters Condition—all velocities in the network are of the same order $w\left(\sqrt{\frac{\log n}{n^d}}\right)$ and $o(n^{-1})$; we require $d > 2$.

The condition that all velocities are $w\left(\sqrt{\frac{\log n}{n^d}}\right)$ ensures that the distance that a cluster member travels in a time slot is greater than its critical transmission range in the order sense while the condition that all velocities are less than $1/T$ constraints the above distance to be less than 1, the side length of the unit square.

2.1.2.2 i.i.d. Mobility Model

At the beginning of each time slot, each cluster member independently and uniformly chooses a point as its new position in the unit square $\mathscr{S}$ and remains static at the new position during the rest of the time slot. Note that the position of each cluster member in each time slot is uniformly distributed in the unit square. The position of each cluster member in a future time slot is independent with its position in a past time slot.

Similar to random walk mobility model, under i.i.d. mobility, we also define weak and strong parameters conditions as follows.

- Weak Parameters Condition—$d > \frac{1}{m-k+1}$.
- Strong Parameters Condition—$d > 2$.

2.1.3 Definition of (K, M)-Connectivity

Let $X_1, X_2, \ldots, X_n$ denote the n cluster members in the network. X_i can directly communicate with a cluster head if and only if the distance between X_i and a cluster head is no greater than X_i's transmission range r. For $i = 1, 2, \ldots, n$, We say the cluster member X_i is (k, m)-connected if in any given time period consisting of m time slots, there exist at least k time slots for X_i and in any one of these k time slots, X_i can directly communicate with at least one cluster head, where k and m are both positive constant integers and we have $k \leq m$. Otherwise, X_i is not (k, m)-connected and we use E_i to denote this event. Then[1] $P(E_i)$ is *the disconnected probability of cluster member* X_i. If the probability that n cluster members are all (k, m)-connected goes to 1 as $n \to \infty$, we say the network is asymptotic (k, m)-connected and let $\mathscr{C}$ denote this event. For simplicity, we refer asymptotic (k, m)-connectivity as (k, m)-connectivity.

2.1.4 Definition of Critical Transmission Range

The definition of critical transmission range is quite straightforward and is presented as follows.

Definition 1 *For clustered networks, $r_\star$ is the critical transmission range if the following two properties both hold, where c_1* and *c_2 are both constants.*

$$\lim_{n\to+\infty} P(\mathscr{C}) < 1, \ \text{if } r \leq c_1 r_\star, \ \text{for any } 0 < c_1 < 1; \tag{2.1}$$

$$\lim_{n\to+\infty} P(\mathscr{C}) = 1, \ \text{if } r \geq c_2 r_\star, \ \text{for any } c_2 > 1. \tag{2.2}$$

[1] For a event E, we use $P(E)$ to denote the probability that E happens, and use $\overline{E}$ to denote its complementary event.

2.2 Main Results

1. Under random walk mobility model:

   (1-a) with either simple or general V-model, in presence of the weak parameters condition, the critical transmission range is $r = \frac{\log n}{2(m-k+1)v_\star T n^d}$;

   (1-b) with simple V-model, in presence of the strong parameters condition, if the transmission range is $r = \frac{\log n + w}{2(m-k+1)v_\star T n^d}$, where w is a constant, we have, as $n \to +\infty$,

$$P(\mathscr{C}) \sim \exp\left[-\sum_{j=1}^{z} \binom{m}{k-1} c_{y_j} e^{-\frac{v^{(y_j)}}{v_\star} w} \right].$$

2. Under i.i.d. mobility model:

   (2-a) in presence of the weak parameters condition, the critical transmission range is $r = \sqrt{\frac{\log n}{(m-k+1)\pi n^d}}$;

   (2-b) in presence of the strong parameters condition, if the transmission range is $r = \sqrt{\frac{\log n + w}{(m-k+1)\pi n^d}}$, where w is a constant, we have, as $n \to +\infty$,

$$P(\mathscr{C}) \sim \exp\left[-\binom{m}{k-1} e^{-w} \right].$$

2.3 The Disconnected Probability of a Cluster Member

In this section, we present a general evaluation on $P(E_i)$, the disconnected probability of cluster member X_i. This general evaluation holds for both random walk and i.i.d. mobility models.

Let T_j denote the m time slots in a given time period, where $j = 1, 2, \ldots, m$. We define the indicator function

$$I_{ij} = \begin{cases} 1, & \text{if } X_i \text{ can directly communicate with at least} \\ & \text{one cluster head in time slot } T_j, \\ 0, & \text{otherwise.} \end{cases}$$

Let p_{ij} and q_{ij} be $P(I_{ij} = 0)$ and $P(I_{ij} = 1)$, respectively. The covered transmission area of cluster member X_i at a time instant is a circle centered at X_i with radius r. We use S_{ij} to denote the area covered by cluster member X_i within time slot T_j. Note that S_{ij} and $S_{i,j+1}$ may have overlapped areas, which will be discussed in the proof

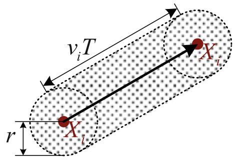

Fig. 2.2 The two *red solid points* represent positions of cluster member X_i at the beginning of and at the end of a time slot. The *arrowed line* between them is the moving track of X_i and the *arrow on the line* indicates the moving direction. X_i moves with velocity v_i in this time slot. The *dotted area* S_i is the area covered by X_i within this time slot. Clearly, $S_i = \pi r^2 + 2rv_iT$

of Lemma 14 in detail. Clearly, p_{ij} is the probability that none of the n^d cluster heads is in S_{ij}. Thus, $p_{ij} = (1 - S_{ij})^{n^d}$, $q_{ij} = 1 - (1 - S_{ij})^{n^d}$.

Under random walk mobility with simple or general V-model, if X_i moves with velocity v_i in all time slots of a time period, $S_{ij} = \pi r^2 + 2rv_iT$ for $1 \le j \le m$. An illustration is given in Fig. 2.2. Under random walk mobility, when X_i's velocity is v_i, for clarity, we use $P(E, v_i)$ instead of $P(E_i)$ to denote the disconnected probability of cluster member X_i. Under i.i.d. mobility, $S_{ij} = \pi r^2$. Thus, under both mobility models, for each particular cluster member X_i, S_{ij} are equal for $1 \le j \le m$. Therefore, for simplicity, we use S_i, p_i and q_i to denote S_{ij}, p_{ij} and q_{ij}, respectively. Then we have $p_i = (1 - S_i)^{n^d}$, $q_i = 1 - (1 - S_i)^{n^d}$.

Now we present three lemmas used to evaluate $P(E_i)$.

Lemma 13 *For any positive number H and any number h with $1 \le h \le H$, if*

$$\begin{aligned} &P(I_{i_1j_1} = 0, I_{i_2j_2} = 0, \ldots, I_{i_hj_h} = 0) \\ &\sim P(I_{i_1j_1} = 0)P(I_{i_2j_2} = 0)\cdots P(I_{i_hj_h} = 0), \end{aligned}$$

then we have

$$\begin{aligned} &P(I_{i_1j_1} = \beta_{i_1j_1}, I_{i_2j_2} = \beta_{i_2j_2}, \ldots, I_{i_hj_h} = \beta_{i_hj_h}) \\ &\sim P(I_{i_1j_1} = \beta_{i_1j_1})P(I_{i_2j_2} = \beta_{i_2j_2})\cdots P(I_{i_hj_h} = \beta_{i_hj_h}), \end{aligned}$$

where $\beta_{i_1j_1}, \beta_{i_2j_2}, \ldots, \beta_{i_hj_h} \in \{0, 1\}$.

Proof. The basic idea is that h-wise independence for the values 0 easily and inductively implies h-wise independence for values 0,1. Due to space limitation, we omit the details.

Lemma 14 *Under i.i.d. mobility model with $r \le c\sqrt{\frac{\log n}{(m-k+1)\pi n^d}}$ or under random walk mobility model with $r \le \frac{c \log n}{2(m-k+1)v_\star Tn^d}$, where c is a positive constant, we have, as $n \to +\infty$,*
(1) under weak parameters condition,

$$\begin{aligned} &(1\text{-}a)\ P(I_{ij_1} = \beta_{ij_1}, I_{ij_2} = \beta_{ij_2}, \ldots, I_{ij_h} = \beta_{ij_h}) \\ &\quad \sim P(I_{ij_1} = \beta_{ij_1})P(I_{ij_2} = \beta_{ij_2})\cdots P(I_{ij_h} = \beta_{ij_h}), \end{aligned}$$

where $1 \le i \le n$, $1 \le h \le m$, $1 \le j_1 < j_2 < \cdots < j_h \le m$, *and* $\beta_{ij_1}, \beta_{ij_2}, \ldots, \beta_{ij_h} \in \{0, 1\}$.

$$\begin{aligned}(1\text{-}b)\ P(I_{i_1 1} = \beta_{i_1 1}, I_{i_1 2} = \beta_{i_1 2}, \ldots, I_{i_1 h} = \beta_{i_1 h},\\ I_{i_2 1} = \beta_{i_2 1}, I_{i_2 2} = \beta_{i_2 2}, \ldots, I_{i_2 h} = \beta_{i_2 h})\\ \sim P(I_{i_1 1} = \beta_{i_1 1})P(I_{i_1 2} = \beta_{i_1 2}) \cdots P(I_{i_1 h} = \beta_{i_1 h})\\ P(I_{i_2 1} = \beta_{i_2 1})P(I_{i_2 2} = \beta_{i_2 2}) \cdots P(I_{i_2 h} = \beta_{i_2 h}),\end{aligned}$$

where $1 \le i_1 < i_2 \le n$, $1 \le h \le m$, *and* $\beta_{i_1 1}, \beta_{i_1 2}, \ldots, \beta_{i_1 h}, \beta_{i_2 1}, \beta_{i_2 2}, \ldots, \beta_{i_2 h} \in \{0, 1\}$.

$$(1\text{-}c) P(E_{i_1}, E_{i_2}) \sim P(E_{i_1})P(E_{i_2}),\ 1 \le i_1 < i_2 \le n.$$

$$(1\text{-}d)\ \sum_{i=1}^{n} P(E_i) - \Big(\sum_{i=1}^{n} P(E_i)\Big)^2 \le P(\overline{\mathscr{C}}) \le \sum_{i=1}^{n} P(E_i).$$

$$(1\text{-}e)\ P(\overline{\mathscr{C}}) \ge \sum_{i=1}^{n} P(E_i) \Big/ \Big(1 + 2\sum_{i=1}^{n} P(E_i)\Big).$$

(2) under strong parameters condition,

$$P(\mathscr{C}) \sim \prod_{i=1}^{n} P(\overline{E_i}).$$

Proof. Refer to appendix.

Lemma 15 x and y are both positive functions of n. If $x, x^2 y \to 0$ as $n \to +\infty$, then $(1 - x)^y \sim e^{-xy}$.

Proof. Proof is provided in [42].

Then, we have the following proposition.

Proposition 1 *Given that* $S_i, S_i^2 n^d, e^{-S_i n^d} \to 0$ *as* $n \to +\infty$, *then under both mobility models, in presence of the weak parameters condition, we have*

$$P(E_i) \sim \binom{m}{k-1} e^{-S_i n^d (m-k+1)}.$$

Proof. If cluster member X_i is not (k, m)-connected in a time period, this means that the number of time slots that X_i can directly communicate with at least one cluster head can be $0, 1, 2, \ldots, k-1$. Therefore,

$$P(E_i) = \sum_{x=0}^{k-1} P\left(\sum_{j=1}^{m} I_{ij} = x\right) \tag{2.3}$$
$$= \sum_{x=0}^{k-1} \sum_{\sum_{j=1}^{m} \beta_{ij}=x} P(I_{i1} = \beta_{i1}, I_{i2} = \beta_{i2}, \ldots, I_{im} = \beta_{im}).$$

In $P(E_i)$, the number of items $P(I_{i1} = \beta_{i1}, I_{i2} = \beta_{i2}, \cdots, I_{im} = \beta_{im})$ is $\sum_{x=0}^{k-1} \binom{m}{x} \le 2^m = \Theta(1)$. According to property (1-a) of Lemma 14, we obtain

$$P(E_i) \sim \sum_{x=0}^{k-1} \binom{m}{x} {p_i}^{m-x} {q_i}^{x}. \tag{2.4}$$

Since $S_i, S_i^2 n^d \to 0$ as $n \to +\infty$, from Lemma 15, $p_i = (1-S_i)^{n^d} \sim e^{-S_i n^d} \to 0$ as $n \to +\infty$. Therefore, $q_i \to 1$ as $n \to +\infty$. Applying $p_i \sim e^{-S_i n^d}$ and $q_i \to 1$ into (2.4), we acquire

$$P(E_i) \sim \binom{m}{k-1} {p_i}^{m-(k-1)} {q_i}^{k-1}. \tag{2.5}$$

Using $p_i \sim e^{-S_i n^d}$ and $q_i \to 1$ again in (2.5), the result follows.

Remark 1 *This proposition shows that if $S_i, S_i^2 n^d$ and $e^{-S_i n^d}$ all go to 0 as $n \to +\infty$, the dominant part of $P(E_i)$, which is the disconnected probability of cluster member X_i, is that in a time period consisting of m time slots, there exist exactly $k-1$ time slots for X_i and in any one of these $k-1$ time slots X_i can directly communicate with at least one cluster head while in the other $m-k+1$ time slots, X_i can not directly communicate with any cluster head.*

2.4 (K, M)-Connectivity Under Random Walk Mobility Model

2.4.1 Disconnected Probability of a Cluster Member Under Random Walk Mobility Model

Under random walk mobility, we use Proposition 1 given before to evaluate $P(E, v_i)$. Afterwards, under weak parameters condition, we provide bounds on $P(\mathscr{C})$ and derive the critical transmission range. Under strong parameters condition, we present precise asymptotic evaluation on $P(\mathscr{C})$.

Proposition 2 *Under random walk mobility, if cluster member X_i moves with velocity v_i in all time slots of a time period, under weak parameters condition,*

(a) if $r = \frac{\log n + w}{2(m-k+1)v_\star T n^d}$, *where w is a constant, then*

$$P(E, v_i) \sim \binom{m}{k-1} n^{-\frac{v_i}{v_\star}} e^{-\frac{v_i}{v_\star} w},$$

(b) if $r = \frac{c \log n}{2(m-k+1)v_\star T n^d}$, *where c is a constant, then*

$$P(E, v_i) \sim \binom{m}{k-1} n^{-\frac{c v_i}{v_\star}},$$

where $v_\star = \min\{\frac{v^{(y)}}{\alpha_y} | y = 1, 2, \ldots, u\}$.

Proof. (a) If $r = \frac{\log n + w}{2(m-k+1)v_\star T n^d}$, we obtain

$$S_i = 2 r v_i T + \pi r^2 \sim \frac{(\log n + w) v_i}{(m-k+1) v_\star n^d}. \tag{2.6}$$

Considering that all velocities in the network are of the same order and the definition of $v_\star$, we have $\frac{v_i}{v_\star} = \Theta(1)$. Using this and $v_\star = w\left(\sqrt{\frac{\log n}{n^d}}\right)$, we can derive S_i, $S_i^2 n^d$ and $e^{-S_i n^d}$ all go to 0 as $n \to +\infty$. Thus, using (2.6) in Proposition 1, the result follows.

(b) If $r = \frac{c \log n}{2(m-k+1)v_\star T n^d}$, we acquire

$$S_i = 2 r v_i T + \pi r^2 \sim \frac{c v_i \log n}{(m-k+1) v_\star n^d}. \tag{2.7}$$

Using $\frac{v_i}{v_\star} = \Theta(1)$ and $v_\star = w\left(\sqrt{\frac{\log n}{n^d}}\right)$, we can derive $S_i, S_i^2 n^d, e^{-S_i n^d} \to 0$ as $n \to +\infty$. Therefore, using (2.7) in Proposition 1, the result follows.

2.4.2 *The Critical Transmission Range Under Random Walk Mobility Model with Simple V-Model*

We have a theorem on the critical transmission range.

Theorem 11 *Under random walk mobility model with simple V-model, in presence of the weak parameters condition, for (k, m)-connectivity in clustered wireless networks, the critical transmission range is*

$$r_\star = \frac{\log n}{2(m-k+1)v_\star T n^d},$$

where $v_\star = \min\{\frac{v^{(y)}}{\alpha_y} | y = 1, 2, \ldots, u\}$.

To derive the critical transmission range, from its definition, we prove the necessary and sufficient conditions, respectively.

2.4.2.1 Necessary Condition of Theorem 11

Proposition 3 *Under random walk mobility with simple V-model, if* $r(n) = \frac{\log n + w}{2(m-k+1)v_\star T n^d}$, *where w is a constant, defining* Φ *as*

$$\Phi = \sum_{j=1}^{z} \binom{m}{k-1} c_{y_j} e^{-\frac{v^{(y_j)}}{v_\star} w},$$

then we have, as $n \to +\infty$,

(a) in presence of the weak parameters condition,

$$\Phi - \Phi^2 \ \le \ P(\overline{\mathscr{C}}) \ \le \ \Phi;$$

(b) in presence of the strong parameters condition,

$$P(\mathscr{C}) \ \sim \ e^{-\Phi}.$$

Proof. By Proposition 2 (a), we obtain that

$$M_y P(E, v^{(y)}) \sim c_y \binom{m}{k-1} n^{\alpha_y - \frac{v^{(y)}}{v_\star}} e^{-\frac{v^{(y)}}{v_\star} w}.$$

Due to $\frac{v^{(y)}}{v_\star} = \alpha_y$ for $y = y_1, y_2, \ldots, y_z$ and $\frac{v^{(y)}}{v_\star} > \alpha_y$ for other y, then we obtain $M_y P(E, v^{(y)}) = O(1)$ for $1 \le y \le u$, and as $n \to +\infty$,

$$\sum_{i=1}^{n} P(E_i) = \sum_{y=1}^{u} M_y P(E, v^{(y)}) \sim \Phi.$$

Using the above result in Lemma 14 (1-d), we acquire **property (a)**.

For $1 \le y \le u$, owing to $M_y P(E, v^{(y)}) = O(1)$, then $P(E, v^{(y)})$ and $P^2(E^{(y)})M_y$ both go to 0 as $n \to +\infty$. Thus, according to Lemma 15,

$$\big(1 - P(E, v^{(y)})\big)^{M_y} \sim \exp\big(-M_y P(E, v^{(y)})\big). \tag{2.8}$$

Under strong parameters condition, from Lemma 14 (2), $P(\mathscr{C}) \sim \prod_{y=1}^{u} \big(1 - P(E, v^{(y)})\big)^{M_y}$. Then

$$P(\mathscr{C}) \sim \exp\Big(-\sum_{y=1}^{u} M_y P(E, v^{(y)})\Big) \sim e^{-\Phi}.$$

Hence, we have also proved **property (b)**.

2.4.2.2 Sufficient Condition of Theorem 11

Proposition 4 *Under random walk mobility model with simple V-model, if* $r(n) = \frac{c\log n}{2(m-k+1)v_\star T n^d}$, *where c is a constant and* $c > 1$, *then in presence of the weak parameters condition, we have*

$$P(\mathscr{C}) \to 1, \text{ as } n \to +\infty.$$

Proof. From Proposition 2 (b), we have

$$M_y P(E, v^{(y)}) \sim c_y C_m^{k-1} n^{\alpha_y - \frac{cv^{(y)}}{v_\star}}.$$

Due to $c > 1$, $\frac{v^{(y)}}{v_\star} = \alpha_y$ for $y = y_1, y_2, \ldots, y_z$ and $\frac{v^{(y)}}{v_\star} > \alpha_y$ for other y, we get as $n \to +\infty$, for $1 \le y \le u$, $M_y P(E, v^{(y)}) \to 0$. Thus

$$\sum_{i=1}^{n} P(E_i) = \sum_{y=1}^{u} M_y P(E, v^{(y)}) \to 0, \text{ as } n \to +\infty. \tag{2.9}$$

Using (2.9) in Lemma 14 (1-d), the result follows.

2.4.3 The Critical Transmission Range Under Random Walk Mobility Model with General V-Model

We have a theorem on the critical transmission range.

Theorem 12 *Under random walk mobility model with general V-model, in presence of the weak parameters condition, for* (k, m)*-connectivity in clustered wireless networks, the critical transmission range is*

$$r_\star = \frac{\log n}{2(m-k+1)v_\star T n^d},$$

where $v_\star = \min\left\{\frac{v^{(y)}}{\alpha_y} | y = 1, 2, \ldots, u\right\}$.

2.4.3.1 Necessary Condition of Theorem 12

Proposition 5 *Under random walk mobility model with general V-model, if* $r(n) = \frac{c \log n}{2(m-k+1)v_* T n^d}$, *where c is a constant and* $0 < c < 1$, *then in presence of the weak parameters condition,*

$$P(\mathscr{C}) \leq 1/2 \text{ , as } n \to +\infty.$$

Proof. We define $v_\star^{(y)} = v^{(y)}(1 + \frac{1}{\log n})$. For each cluster member in G_y, we denote the probability that its velocity lies in $[v^{(y)}, v_\star^{(y)}]$ as $p_\star$. Then considering $v^{(y)}$ and $v_a^{(y)}$ are of the same order, we acquire

$$p_\star = \frac{v_\star^{(y)} - v^{(y)}}{v_a^{(y)} - v^{(y)}} = \frac{v^{(y)}}{(v_a^{(y)} - v^{(y)}) \log n} = \Theta\Big(\frac{1}{\log n}\Big).$$

We assume the number of cluster members in G_y with velocities in $[v^{(y)}, v_\star^{(y)}]$ is N_y. Then we can obtain for any number A with $1 \leq A \leq M_y$,

$$P(N_y \leq A) = \sum_{j=0}^{A} \binom{M_y}{j} p_\star^j (1 - p_\star)^{(M_y - j)}.$$

From Hoeffding's inequality[43], when $A \leq M_y p_\star$,

$$P(N_y \leq A) \leq \exp\Bigg[- \frac{2(M_y p_\star - A)^2}{M_y} \Bigg].$$

Let $A = M_y p_\star/2$. Due to $p_\star = \Theta\big(\frac{1}{\log n}\big)$, we get

$$P\Big(N_y \leq \frac{M_y p_\star}{2}\Big) \ \leq \ \exp\Big(- \frac{c_y n^{\alpha_y} p_\star^2}{2}\Big) \ \to \ 0.$$

Therefore, $N_y \geq \frac{M_y p_\star}{2}$ almost surely.

From Proposition 2 (b), we have

$$P(E, v_i) \sim \binom{m}{k-1} n^{-\frac{c v_i}{v_\star}}. \tag{2.10}$$

Hence, $P(E, v_i)$ is monotonically decreasing for v_i. Then we can further obtain

$$\sum_{i \in G_y} P(E_i) \geq P(E, v_\star^{(y)}) M_y p_\star/2. \tag{2.11}$$

Considering that all velocities in the network are of the same order, we acquire that $\frac{v^{(y)}}{v_\star} = \Theta(1)$. From $\frac{v^{(y)}}{v_\star} = \Theta(1)$, $v_\star^{(y)} = v^{(y)}(1 + \frac{1}{\log n})$ and (2.10),

$$\frac{P(E, v_\star^{(y)})}{P(E, v^{(y)})} \sim n^{-\frac{c(v_\star^{(y)} - v^{(y)})}{v_\star}} = e^{-\frac{cv^{(y)}}{v_\star}} = \Theta(1). \tag{2.12}$$

Since $\frac{v^{(y)}}{v_\star} = \alpha_y (y = y_1, y_2, \ldots, y_z)$, we have $\alpha_y - \frac{cv^{(y)}}{v_\star} > 0$ for $0 < c < 1$. Note that $p_\star = \Theta\left(\frac{1}{\log n}\right)$, we have for $y = y_1, y_2, \ldots, y_z$, as $n \to +\infty$,

$$\begin{aligned} P(E, v^{(y)}) M_y p_\star &\sim \binom{m}{k-1} n^{-\frac{cv^{(y)}}{v_\star}} \cdot c_y n^{\alpha_y} p_\star \\ &\sim c_y \binom{m}{k-1} n^{\alpha_y - \frac{cv^{(y)}}{v_\star}} p_\star \\ &\to +\infty. \end{aligned} \tag{2.13}$$

Combining (2.12) and (2.13), for $y = y_1, y_2, \ldots, y_z$,

$$P(E, v_\star^{(y)}) M_y p_* \to +\infty, \text{ as } n \to +\infty. \tag{2.14}$$

Applying (2.14) in (2.11), for $y = y_1, y_2, \ldots, y_z$, we obtain that $\sum_{i \in G_y} P(E_i) \to +\infty$ as $n \to +\infty$. Thus,

$$\sum_{i=1}^{n} P(E_i) = \sum_{y=1}^{u} \sum_{i \in G_y} P(E_i) \to +\infty, \text{ as } n \to +\infty. \tag{2.15}$$

Using (2.15) in Lemma 14 (1-e), the result follows.

2.4.3.2 Sufficient Condition of Theorem 12

Proposition 6 *Under random walk mobility model with general V-model, if* $r(n) = \frac{\log n + w}{2(m-k+1)v_\star T n^d}$, *where w is a constant, then in presence of the weak parameters condition,*

$$P(\mathscr{C}) \to 1, \text{ as } n \to +\infty.$$

Proof. We define $v_\star^{(y)} = v^{(y)}\left(1 + \frac{1}{\sqrt{\log n}}\right)$. For each cluster member in G_y, we denote the probability that its velocity lies in $[v^{(y)}, v_\star^{(y)}]$ as $p_\star$. Similar to the proof of necessary condition, we acquire $p_\star = \Theta\left(\frac{1}{\sqrt{\log n}}\right)$.

We also assume the number of cluster members in G_y with velocities in $[v^{(y)}, v_\star^{(y)}]$ is N_y. For any number A with $1 \le A \le M_y$,

$$P(N_y \geq A) = \sum_{j=0}^{M_y - A} \binom{M_y}{j} p_\star^{(M_y - j)} (1 - p_\star)^j.$$

Following Hoeffding's inequality [43] and several similar steps, we can obtain that $N_y \leq 2M_y p_\star$ almost surely.

From Proposition 2 (a), we have

$$P(E, v_i) \sim \binom{m}{k-1} n^{-\frac{v_i}{v_\star}} e^{-\frac{v_i}{v_\star} w}. \tag{2.16}$$

Hence, $P(E, v_i)$ is monotonically decreasing for v_i. Then, we can further obtain

$$\begin{aligned} \sum_{i \in G_y} P(E_i) &\leq P(E, v^{(y)}) N_y + P(E, v_\star^{(y)})(M_y - N_y) \\ &\leq 2P(E, v^{(y)}) M_y p_\star + P(E, v_\star^{(y)}) M_y. \end{aligned} \tag{2.17}$$

From $\dfrac{v^{(y)}}{v_\star} = \Theta(1)$, $v_\star^{(y)} = v^{(y)}(1 + \frac{1}{\sqrt{\log n}})$ and (2.16),

$$\begin{aligned} P(E, v_\star^{(y)})/P(E, v^{(y)}) &\sim n^{-\frac{v_\star^{(y)} - v^{(y)}}{v_\star}} e^{-\frac{v_\star^{(y)} - v^{(y)}}{v_\star} w} \\ &\sim e^{-\frac{v^{(y)}}{v_\star \sqrt{\log n}} (\log n + w)} \to 0. \end{aligned} \tag{2.18}$$

From (2.16), we have

$$P(E, v^{(y)}) M_y \sim c_y \binom{m}{k-1} n^{\alpha_y - \frac{v^{(y)}}{v_\star}} e^{-\frac{v^{(y)}}{v_\star} w}. \tag{2.19}$$

Due to $\frac{v^{(y)}}{v_\star} = \alpha_y$ for $y = y_1, y_2, \ldots, y_z$ and $\frac{v^{(y)}}{v_\star} > \alpha_y$ for other y, we obtain $P(E, v^{(y)}) M_y = O(1)$ for $1 \leq y \leq u$. Using this and (2.18), we obtain for $1 \leq y \leq u$,

$$P(E, v_\star^{(y)}) M_y \to 0, \text{ as } n \to +\infty. \tag{2.20}$$

From $P(E, v^{(y)}) M_y = O(1)$ and $p_\star = \Theta(\frac{1}{\sqrt{\log n}})$,

$$P(E, v^{(y)}) M_y p_\star \to 0, \text{ as } n \to +\infty. \tag{2.21}$$

Applying (2.20) and (2.21) into (2.17), then we acquire for $1 \leq y \leq u$, $\sum_{i \in G_y} P(E_i) \to 0$ as $n \to +\infty$. Thus,

$$\sum_{i=1}^{n} P(E_i) = \sum_{y=1}^{u} \sum_{i \in G_y} P(E_i) \to 0, \text{ as } n \to +\infty. \tag{2.22}$$

Using (2.22) in (1-d) of Lemma 14, the result follows.

2.4.4 The Critical Transmission Range Under Random Walk Mobility Model with Homogeneous Velocity Model

By the term of homogeneous velocity model, we mean all cluster members have a same velocity at any time and in addition, any cluster member doesn't change its velocity in all the m time slots of any particular time period. Therefore, there is only one value of velocity in the network and we denote it as v.

Under random walk mobility model, we can regard homogeneous velocity model as a special case $u = 1$ of *velocity model with constant number of values*. Consequently, now the values of the parameters defined in *velocity model with constant number of values* are as follows: $M_1 = n, c_1 = 1, \alpha_1 = 1, z = 1, y_1 = 1, v_\star = v$. Hence, we obtain

1. the critical transmission range is $r = \frac{\log n}{2(m-k+1)vTn^d}$;
2. if $r = \frac{\log n + w}{2(m-k+1)vTn^d}$, where w is a constant, we have, as $n \to +\infty$,
   (2-a) in presence of the weak parameters condition,
   $\binom{m}{k-1}e^{-w} - \left[\binom{m}{k-1}e^{-w}\right]^2 \ \le\ P(\overline{\mathscr{C}}) \ \le\ \binom{m}{k-1}e^{-w}$;
   (2-b) in presence of the strong parameters condition,
   $P(\mathscr{C}) \ \to\ \exp\left[-\binom{m}{k-1}e^{-w}\right]$;
3. if $r = \frac{c \log n}{2(m-k+1)vTn^d}$, where c is a constant and $c > 1$, under weak parameters condition, as $n \to +\infty$,
   $P(\mathscr{C}) \ \to\ 1.$

2.5 (K, M)-Connectivity Under i.i.d. Mobility Model

Theorem 13 *Under i.i.d. mobility model with weak parameters condition, for (k, m)-connectivity in clustered wireless networks, the critical transmission range is* $r(n) = \sqrt{\frac{\log n}{(m-k+1)\pi n^d}}$.

2.5.1 Necessary Condition of Theorem 13

Proposition 7 *Under i.i.d. mobility model, if* $r = \sqrt{\frac{\log n + w}{(m-k+1)\pi n^d}}$, *where* w *is a constant, as* $n \to +\infty$,

(a) under weak parameters condition,
$\binom{m}{k-1}e^{-w} - \left[\binom{m}{k-1}e^{-w}\right]^2 \ \leq \ P(\overline{\mathscr{C}}) \ \leq \ \binom{m}{k-1}e^{-w}$
(b) under strong parameters condition,
$P(\mathscr{C}) \ \rightarrow \ \exp\left[-\binom{m}{k-1}e^{-w}\right]$

Proof. We have that

$$S_i = \pi r^2 = \frac{\log n + w}{(m-k+1)n^d}.$$

Then we can derive S_i, $S_i^2 n^d$, $e^{-S_i n^d} \rightarrow 0$ as $n \rightarrow +\infty$. $P(E_i)$ are equal for $i = 1, 2, \ldots, n$, so we use $P(E)$ to denote $P(E_i)$. From Proposition 1, we have

$$P(E) \sim \binom{m}{k-1} n^{-1} e^{-w}.$$

Using this in Lemma 14 (1-d), **property (a)** follows.

Under strong parameters condition, from Lemma 14 (2), we acquire $P(\mathscr{C}) \sim \left[1 - P(E)\right]^n$. Since $P(E), nP^2(E) \rightarrow 0$ as $n \rightarrow +\infty$, then from Lemma 15,

$$P(\mathscr{C}) \sim e^{-nP(E)} \rightarrow \exp\left(-\binom{m}{k-1}e^{-w}\right), \text{as } n \rightarrow +\infty.$$

Thus, we have proved **property (b)**.

2.5.2 Sufficient Condition of Theorem 13

Proposition 8 *Under i.i.d. mobility model, if*
$r = c\sqrt{\frac{\log n}{(m-k+1)\pi n^d}}$, *where* c *is a constant and* $c > 1$, *then under weak parameters condition,*

$$P(\mathscr{C}) \rightarrow 1, \ \textit{as } n \rightarrow +\infty.$$

Proof. $P(E_i)$ are equal for $i = 1, 2, \ldots, n$, so we use $P(E)$ to denote $P(E_i)$. We obtain $P(E) \sim \binom{m}{k-1} n^{-c^2}$ by similar steps in Proposition 7. Using this in property (1-d) of Lemma 14, the result follows.

2.6 Discussion

In this section, we discuss our results in some aspects.

2.6.1 Explanation on the Expression of the Critical Transmission Range

When r is equal to the critical transmission range, for $1 \leq i \leq n$, $1 \leq j \leq m$, the covered transmission area of cluster member X_i in time slot T_j, i.e., S_{ij}, is $\frac{v_i \log n}{(m-k+1)v_\star n^d}$ under random walk mobility and is $\frac{\log n}{(m-k+1)n^d}$ under i.i.d. mobility model. Note as shown in Sect. 2.4, under both mobility models, for each particular cluster member X_i, S_{ij} are equal for $1 \leq j \leq m$. Therefore, for simplicity, we use S_i to denote S_{ij}. Below we explain the common part in the expression of S_i under both mobility models. Since the density of cluster head is n^d, an item n^d exists in the denominator of S_i. The $\log n$ in the nominator of S_i is due to the randomness caused by the distribution of the n^d cluster heads in the unit square. From Proposition 1, we have $P(E_i) \sim \binom{m}{k-1} e^{-S_i n^d (m-k+1)}$, the intuition of which has already been discussed in Remark 3 and is now explained again for clarity. The above result shows that the dominant part of $P(E_i)$, which is the disconnected probability of cluster member X_i, is that in a period consisting of m time slots, there exist exactly $k-1$ time slots for X_i and in any one of these $k-1$ time slots X_i can directly communicate with at least one cluster head while in the other $m-k+1$ time slots, X_i can not directly communicate with any cluster head. Therefore, after derivation, there is an item $(m-k+1)$ in the denominator of the critical transmission range.

Note that we have $S_i = \Theta(\frac{\log n}{n^d})$ under both mobility models. Owing to $S_i = \pi r^2 + 2rv_iT \sim 2rv_iT$ under random walk mobility and $S_i = \pi r^2$ under i.i.d. mobility, therefore, if all velocities are constants, the critical transmission range is $\Theta\left(\frac{\log n}{n^d}\right)$ under random walk mobility and is $\Theta\left(\sqrt{\frac{\log n}{n^d}}\right)$ under i.i.d. mobility.

2.6.2 Random Walk Mobility Model with Different Velocity Models

Under random walk mobility with general V-model, only the lower boundary $v^{(y)}$ of the interval $[v^{(y)}, v_a^{(y)}]$ affects the critical transmission range and the upper boundary $v_a^{(y)}$ has no impact on it, which we call as *dominant phenomenon of minimum velocity in a group*. If the item $\log n$ in the critical transmission range is replaced with $\log n + w$, where w is a constant, the probability of (k, m)-connectivity goes to 1 as $n \to +\infty$.

However, under random walk mobility with simple V-model, when the item $\log n$ in the critical transmission range is replaced with $\log n + w$, the probability that the network does *not* have (k, m)-connectivity is bounded away from zero as $n \to +\infty$. The reason of different results for the two velocity models is that for general V-model, velocities are continuously distributed in intervals. The intuition is that in this case, most cluster members in the network travel with velocities greater than the lower boundary of the velocities interval, i.e., $v^{(y)}$, so they are (k, m)-connected when the item $\log n$ in the critical transmission range is replaced with $\log n + w$.

Under random walk mobility model, from the expression of the critical transmission range, the impact on (k, m)-connectivity by groups and velocities in the network is embodied in the form of $v_\star$, where $v_\star$ is defined as $\min\{\frac{v^{(y)}}{\alpha_y} | y = 1, 2, \ldots, u\}$. For each y, we call $\frac{v^{(y)}}{\alpha_y}$ as the *velocity-number index* of cluster member group G_y. Then we know that only groups with minimum values of velocity-number indexes exert impact on the critical transmission range. The effect of group G_y on the (k, m)-connectivity of the network is decided by both α_y and $v^{(y)}$. The factor α_y corresponds to the number of nodes M_y in the group, where $M_y \sim c_y n^{\alpha_y}$. The constant c_y has no influence on the critical transmission range. The factor $v^{(y)}$ is related with the velocities in group G_y. Smaller α_y and greater $v^{(y)}$ mean less impact of the group G_y on (k, m)-connectivity of the network. Clearly, if the velocities of groups with minimum values of velocity-number indexes all increase, then v_* increases. If so, the critical transmission range decreases, so we can reduce the energy and power for communication. Thus, in some sense, mobility increases (k, m)-connectivity in clustered wireless networks.

2.7 Conclusion

We investigate (k, m)-connectivity in mobile clustered wireless networks which means that in a time period consisting of m time slots, there exist at least k time slots for each cluster member and in each of these k time slots the cluster member can directly communicate with at least one cluster head. For random walk mobility model with simple V-model and i.i.d. mobility model, under strong parameters condition, we present a precise asymptotic distribution of the probability that the network has (k, m)-connectivity in terms of the transmission radius. For both mobility models, under weak parameters condition, we provide bounds on the probability that the network has (k, m)-connectivity and derive the critical transmission range for (k, m)-connectivity.

Appendix: Proof of Lemma 14

Proof of property (1-a):

We define $\mathscr{P}_{a1}$ and $\mathscr{P}_{a2}$ as follows:

$$\mathscr{P}_{a1} = P(I_{ij_1} = 0, I_{ij_2} = 0, \dots, I_{ij_h} = 0)$$
$$\mathscr{P}_{a2} = P(I_{ij_1} = 0)P(I_{ij_2} = 0) \cdots P(I_{ij_h} = 0)$$

From Lemma 13, to prove property (1-a), we only have to show $\mathscr{P}_{a1} \sim \mathscr{P}_{a2}$.

Under both mobility models, for $1 \le j \le m$, we have $P(I_{ij} = 0) = (1 - S_i)^{n^d}$, so we get $\mathscr{P}_{a2} = (1 - S_i)^{n^d h}$.

Recall that S_{ij} is the covered transmission area of cluster member X_i in time slot T_j. We use S_i^* to denote the union of the areas that $S_{ij_1}, S_{ij_2}, \dots, S_{ij_h}$ cover. Note that $S_{ij_1}, S_{ij_2}, \dots, S_{ij_h}$ may have overlapped areas, under random walk mobility model due to the change of direction and the intersection of the segments of the track, and under i.i.d. mobility model due to the factor of randomness.

(1^o) **under random walk mobility with weak parameters condition**

From the result in [37], we get $S_i^* \sim hS_i$. Then as S_i and $S_i n^d h$ both go to 0 as $n \to +\infty$, from Lemma 15,

$$\frac{\mathscr{P}_{a1}}{\mathscr{P}_{a2}} = \frac{(1 - S_i^*)^{n^d}}{(1 - S_i)^{n^d h}} \sim \frac{e^{-S_i^* n^d}}{e^{-hS_i n^d}} \to 1, \text{as } n \to +\infty.$$

(2^o) **under i.i.d. mobility with weak parameters condition**

Clearly, property (1-a) holds for $h = 1$. Now we prove property (1-a) for $h = 2$. As S_{ij} is a circle with radius r, we use O_{ij_1} and O_{ij_2} to denote the centers of S_{ij_1} and S_{ij_2}, respectively. Let δ stand for the distance between O_{ij_1} and O_{ij_2}. As given in [37], it is easy to prove that $P(\delta > 2r) \le 4\pi r^2$. Then from Lemma 15,

$$\mathscr{P}_{a1} \le (1 - \pi r^2)^{n^d} \pi r^2 + (1 - 2\pi r^2)^{n^d} \sim e^{-2\pi r^2 n^d}$$
$$\mathscr{P}_{a1} \ge (1 - 2\pi r^2)^{n^d} (1 - 4\pi r^2) \sim e^{-2\pi r^2 n^d}$$

Thus, $\mathscr{P}_{a1} \sim e^{-2\pi r^2 n^d}$. Due to $\mathscr{P}_{a2} \sim e^{-2S_i n^d} = e^{-2\pi r^2 n^d}$, therefore, we have $\mathscr{P}_{a1} \sim \mathscr{P}_{a2}$ for $h = 2$. Using similar technique, we can also easily show the result for $3 \le h \le m$. Due to space limitation, we omit the details. Finally, under both mobility models, we obtain property (1-a).

Proof of property (1-b):

We define $\mathscr{P}_{b1}$ and $\mathscr{P}_{b2}$ as follows:

$$\begin{aligned}\mathscr{P}_{b1} =& P(I_{i_1 1} = 0, I_{i_1 2} = 0, \ldots, I_{i_1 h} = 0, \\ & I_{i_2 1} = 0, I_{i_2 2} = 0, \ldots, I_{i_2 h} = 0) \\ \mathscr{P}_{b2} =& P(I_{i_1 1} = 0)P(I_{i_1 2} = 0) \cdots P(I_{i_1 h} = 0) \\ & \times P(I_{i_2 1} = 0)P(I_{i_2 2} = 0) \cdots P(I_{i_2 h} = 0)\end{aligned}$$

From Lemma 13, to prove property (1-b), we only have to present that $\mathscr{P}_{b1} \sim \mathscr{P}_{b2}$.

Note that $S_{i_1 1}, S_{i_1 2}, \ldots, S_{i_1 h}, S_{i_2 1}, S_{i_2 2}, \ldots, S_{i_2 h}$ may have overlap areas, under random walk mobility model due to the change of direction and the intersection of the segments of the track, and under i.i.d. mobility model due to the factor of randomness.

Below we will prove that for $1 \le i_1 < i_2 \le i_2 \le n$, $1 \le j_1 \le h$, $1 \le j_2 \le h$, we have

$$P(I_{i_1 j_1} = 0, I_{i_2 j_2} = 0) \sim P(I_{i_1 j_1} = 0)P(I_{i_2 j_2} = 0) \tag{2.23}$$

Clearly, setting $j_1 = j_2 = 1$ in (2.23), then property (1-b) holds for $h = 1$. Since property (1-a) holds and m is a constant, using (2.23), we can also easily prove property (1-b) for $2 \le h \le m$. Due to space limitation, we omit the details. Now we focus on proving (2.23).

(1^{o}) **under random walk mobility with weak parameters condition**

Due to $v = w\left(\sqrt{\frac{\log n}{n^d}}\right)$ and $r = O(\frac{\log n}{vn^d}) = o(\sqrt{\frac{\log n}{n^d}})$, we have $vT = \Omega(r)$. As shown in Fig. 2.3, for simplicity, we regard $S_{i_x j_x} (x = 1, 2)$ as the rectangle with length $v_{i_x j_x} T$ and width $2r$ in it, denoted as $R_{i_x j_x}$. Let $O_{i_2 j_2}$ be the center of $R_{i_2 j_2}$ and S_φ be the area that $O_{i_2 j_2}$ covers when $R_{i_1 j_1}$ and $R_{i_2 j_2}$ have overlapped areas. Assume $v_{i_1 j_1} \ge v_{i_2 j_2}$ and then $S_{i_1 j_1} \ge S_{i_2 j_2}$. Let $\xi = \arcsin \frac{2r}{v_{i_2 j_2} T}$ and $\mathscr{P}_b = e^{-2r(v_{i_1 j_1} + v_{i_2 j_2})Tn^d}$. We can obtain that

$$\begin{aligned}S_\varphi =& (v_{i_1 j_1} T + 2r \cot(\varphi/2))(v_{i_2 j_2} T + 2r \cot(\varphi/2)) \sin \varphi \\ & - 4r^2(1 + \cos \varphi)^2 \cot \varphi - 4r^2 \cos \varphi \sin \varphi\end{aligned}$$

The expression of $\mathscr{P}_{b1}$ in terms of S_φ is

$$\begin{aligned}\mathscr{P}_{b1} =& \frac{4}{2\pi} \Big\{ \int_0^\xi (1 - 2rv_{i_1 j_1} T)^{n^d} S_\varphi \mathrm{d}\varphi \\ & + \int_\xi^{\frac{\pi}{2}} \Big[1 - \Big(2r(v_{i_1 j_1} + v_{i_2 j_2})T - \frac{4r^2}{\sin \varphi}\Big)\Big]^{n^d} S_\varphi \mathrm{d}\varphi \\ & + \int_0^{\frac{\pi}{2}} (1 - 2r(v_{i_1 j_1} + v_{i_2 j_2})T)^{n^d} (1 - S_\varphi) \mathrm{d}\varphi \Big\}\end{aligned}$$

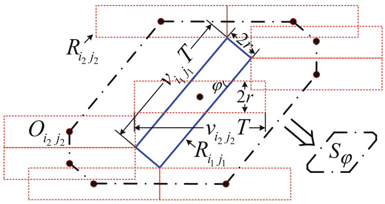

Fig. 2.3 Overlap of covered transmission areas $S_{i_1 j_1}$ and $S_{i_2 j_2}$ under random walk mobility model. For simplicity, we regard $S_{i_x j_x}(x = 1, 2)$ as the *rectangle* with length $v_{i_x j_x}T$ and width $2r$ in it, denoted as $R_{i_x j_x}$. $O_{i_2 j_2}$ is the center of $R_{i_2 j_2}$ and S_φ is the area that $O_{i_2 j_2}$ covers when $R_{i_1 j_1}$ and $R_{i_2 j_2}$ have overlapped areas

Then we evaluate $\mathscr{P}_{b1}$ as below.

$$\mathscr{P}_{b1} \geq \frac{2}{\pi}(1 - 2r(v_{i_1 j_1} + v_{i_2 j_2})T)^{n^d} \sim \mathscr{P}_b \tag{2.24}$$

$$\begin{aligned}\frac{\mathscr{P}_{b1}}{\mathscr{P}_b} \leq \frac{2}{\pi}\Big(\int_\xi^{\frac{\pi}{2}} e^{\frac{4r^2}{\sin\varphi}n^d} S_\varphi \mathrm{d}\varphi - \int_0^{\frac{\pi}{2}} S_\varphi \mathrm{d}\varphi\Big) \\ + \frac{2}{\pi} e^{2rv_{i_1 j_1}Tn^d} \int_0^\xi S_\varphi \mathrm{d}\varphi + 1\end{aligned} \tag{2.25}$$

Let $\mathscr{X}$ and $\mathscr{Y}$ denote the first and second item in the right hand side (R.H.S.) of (2.25), respectively. For sufficiently large n,

$$S_\varphi \leq 2v_{i_1 j_1} v_{i_2 j_2} T^2 \sin\varphi \tag{2.26}$$

Below for simplicity, we place (2.26) above operator symbols when (2.26) is used in the derivation. We acquire

$$\mathscr{X} \geq -\frac{2}{\pi}\int_0^\xi S_\varphi \mathrm{d}\varphi \overset{(2.26)}{\to} 0, \text{ as } n \to +\infty. \tag{2.27}$$

Consider the function $f(x) = \left(e^{4r^2 n^d x} - 1\right) \big/ x$. We can show that $f(x)$ is a monotonically increasing function for $x > 0$. Thus, as $n \to +\infty$

$$\begin{aligned}\mathscr{X} &\overset{(2.26)}{\leq} \frac{2}{\pi}\int_\xi^{\frac{\pi}{2}} \big(e^{\frac{4r^2}{\sin\varphi}n^d} - 1\big) 2v_{i_1 j_1} v_{i_2 j_2} T^2 \sin\varphi \mathrm{d}\varphi \\ &\leq 2v_{i_1 j_1} v_{i_2 j_2} T^2 f(1/\sin\xi) \to 0.\end{aligned} \tag{2.28}$$

From (2.27) and (2.28), we get $\mathscr{X} \to 0$, as $n \to +\infty$. Also $\mathscr{Y} \overset{(2.26)}{\to} 0$, as $n \to +\infty$. Due to space limitation, we omit the details. As $\mathscr{X}$ and $\mathscr{Y}$ both go to 0 as $n \to +\infty$, then R.H.S. of (2.25) $\to$ 1. Combining this with (2.24), $\mathscr{P}_{b1} \sim$

$e^{-2r(v_{i_1j_1}+v_{i_2j_2})T}$. Finally, as shown below, $\mathscr{P}_{b1} \sim \mathscr{P}_{b2}$.

$$\begin{aligned}\mathscr{P}_{b2} &= \prod_{k=1}^{2}(1-(2rv_{i_kj_k}T+\pi r^2))^{n^d}\\ &\sim e^{-2r(v_{i_1j_1}+v_{i_2j_2})T} = \mathscr{P}_b \sim \mathscr{P}_{b1}\end{aligned}$$

(2°) **under i.i.d. mobility with weak parameters condition**

The proof is similar to that of (2°) in property (1-a).

Proof of property (1-c/d/e):

Firstly, using properties (1-a) and (1-b), we acquire property (1-c). Due to space limitation, we omit the details. For simplicity, we define $\mathscr{U} = \sum_{i=1}^{n} P(E_i)$ and $\mathscr{V} = \sum_{1\le i<j\le n} P(E_iE_j)$. Then from property (1-c),

$$\mathscr{V} \sim \sum_{1\le i<j\le n} P(E_i)P(E_j) \le \mathscr{U}^2.$$

Since $\overline{\mathscr{C}} = \bigcup_{i=1}^{n} E_i$, we have the following two inequalities. First, from Bonferroni inequality [44], $\mathscr{U} - \mathscr{V} \le P(\overline{\mathscr{C}}) \le \mathscr{U}$ (the right part of this inequality is simply the union bound). In addition, from K. Chung-P. Erdös inequality [45], $P(\overline{\mathscr{C}}) \ge \frac{\mathscr{U}^2}{\mathscr{U}+2\mathscr{V}}$. Applying $\mathscr{V} \le \mathscr{U}^2$ as $n \to \infty$ into the above two inequalities, properties (1-d) and (1-e) directly follow, respectively.

Proof of property (2):

(1°) **under i.i.d. mobility with strong parameters condition**

For each i, j, the area S_{ij} is a circle with radius r. Let P^* be the probability that any two of the mn circles do not overlap. For any given point X, its distance away from another point Y no less than $2r$ means that Y falls outside the area of the circle centered at X with radius $2r$. Thus, for any given point, the probability that all other $u-1$ points are away from it no less than $2r$ is $(1-(1-4\pi r^2)^{u-1})$. Then using Lemma 15, the union bound, $r \le c\sqrt{\frac{\log n}{(m-k+1)\pi n^d}}$ and $u \le mn$, we have

$$\begin{aligned}P^* &\ge 1-u(1-(1-4\pi r^2)^{u-1})\\ &\ge 1-u\left\{1-e^{-4\pi r^2u-40\pi^2r^4u/3}\right\}\\ &\ge 1-u\left[4\pi r^2u+40\pi^2r^4u/3\right] \to 1,\ \text{as } n\to+\infty.\end{aligned}$$

Thus, $\lim_{n\to+\infty} P^* = 1$.

(2°) **under random walk mobility with strong parameters condition**

Let the center of the rectangle $(R_{ij} : 2r \times v_{ij}T)$ in S_{ij} be P_{ij}. Use L_{ij} to denote the length of the diagonal of R_{ij}. The circle with center P_{ij} and radius L_{ij} covers S_{ij} for sufficiently large n. Let P^* be the probability that any two of the mn circles don't overlap. Using similar method in (1^o) above, we obtain $P^* \to 1$, as $n \to +\infty$.

Finally, under both mobility models, in presence of the strong parameters condition, $P^* \to 1$ means the mn covered transmission areas do not overlap almost surely, therefore property (2) holds.

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