Preface

This book is written on the occasion of the centenary of the birth of Kurt Gödel (1906–1978), the most exciting logician of all time, whose discoveries shook the foundations of mathematics. His beautiful technique of examining the whole edifice of mathematics within mathematics itself has been likened, not only figuratively but also in precise technical terms, to the music of Bach and drawings of Escher [6]. It has had a deep impact on philosophers and linguists. In a way, it ushered in the era of computers. His idea of arithmetization of formal systems led to the discovery of a universal computer program that simulates all programs. Based on his incompleteness theorems, physicists have propounded theories concerning artificial intelligence and the mind–body problem [13].

The main goal of this book is to state and prove Gödel's completeness and incompleteness theorems in precise mathematical terms. This has enabled us to present a short, distinctive, modern, and motivated introduction to mathematical logic for graduate and advanced undergraduate students of logic, set theory, recursion theory, and computer science. Any mathematician interested in knowing what mathematical logic is concerned with and who would like to learn the famous completeness and incompleteness theorems of Gödel should also find this book particularly convenient. The treatment is thoroughly mathematical, and the entire subject has been approached like any other branch of mathematics. Serious pains have been taken to make the book suitable for both classroom and self-instructional purposes. The book does not strive to be a comprehensive encyclopedia of logic, nor does it broaden its audience to linguists and philosophers. Still, it gives essentially all the basic concepts and results in mathematical logic.

The main prerequisite for this book is the willingness to work at a reasonable level of mathematical rigor and generality. However, a working knowledge of elementary mathematics, particularly naive set theory and algebra, is required. We suggest [17, pp. 1–15] for the necessary prerequisites in set theory. A good source for the algebra needed to understand some examples and applications would be [10].

Students who wish to specialize in foundational subjects should read the entire book, preferably in the order in which it is presented, and work out all the problems. Sometimes we have only sketched the proof and left out the routine arguments for readers to complete. Students of computer science may leave out sections on model theory and arithmetical sets. Mathematicians working in other areas who wish to know about the completeness and incompleteness theorems alone may also omit these sections. However, sections on model theory give applications of logic to mathematics. Chapters 1–4, except for Sect. 2.4 and Sects. 5.1 and 5.4, should constitute a satisfactory course in mathematical logic for undergraduate students.

The book prepares students to branch out in several areas of mathematics related to foundations and computability such as logic, model theory, axiomatic set theory, definability, recursion theory, and computability. Hinman's recent book [5] is the most comprehensive one, with representation in all these areas. Shoenfield's [16] is still a very satisfactory book on logic. For axiomatic set theory, we particularly recommend Kunen [9] and Jech [8]. For model theory, readers should also consult Chang and Keisler [3] and Marker [11]. For recursion theory we suggest [12].

Acknowledgments. I thank M. G. Nadkarni, Franco Parlamento, Ravi A. Rao, B. V. Rao, and H. Sarbadhikari for very carefully reading the entire manuscript and for their numerous suggestions and corrections. Thanks are also due to my colleagues and research fellows at the Stat-Math Unit, Indian Statistical Institute, for their encouragement and help. I fondly acknowledge my daughter Rosy, my son Ravi, and my grandsons Pikku and Chikku for keeping my spirits up while I was writing this book. Last but not least, I shall ever be grateful to my wife, H. Sarbadhikari, for cheerfully putting up with me at home as well as at the office for the duration of my work on the book.

Preface to the Second Edition. In the second edition, we have given a fairly respectable introduction to model theory. It shows that logic is a lively subject with surprising connections elsewhere in mathematics. The work of Tarski, Julia Robinson, Vaught, Morley, Shelah, and others is a testimony to the fact that model theory in its own right is a beautiful and deep subject. Hrushovski's proof of the function-field Mordell–Lang conjecture [7], the proof of the Manin–Mumford conjecture by Pila and Zannier [15], and Pila's proof of the André–Oort conjecture [14] are some of the spectacular applications of model theory to geometry and number theory.

Our second edition aims at giving a first introduction to the easier parts of model theory. However, this is in no way a complete book on model theory. Still, we hope it will motivate and prepare readers to embark on a more serious study of the subject. In Chap. 2 we have added a section on homogeneous structures and a section on definability. The first three sections (including the proof of the completeness theorem for first-order logic) and the last section of Chap. 5 of the first edition have been transplanted to Chap. 4. Chapter 5 of the second edition is largely new and is devoted exclusively to model theory.

Significant new additions are ultraproduct of models, elimination of quantifiers, types, and atomic, saturated, and stable models. We study a large number of examples from algebra to illustrate our methods. A substantial study of real closed fields is an important new example from algebra. Several applications in algebraically closed fields and real closed fields such as Chevalley's theorem, Hilbert's Nullstellensatz, A. Robinson's proof of Hilbert's 17th problem, and others are given. Apart from cosmetic changes and the additional examples, the rest of the material from the first edition has remained unchanged.

I have been greatly helped by my wife and colleague H. Sarbadhikari for her review of the first edition. I remain grateful to her. I fondly acknowledge the contribution of my daughter-in-law Deepali to keep me and my wife free of any domestic worry and my new grandson Rhishant (Totu) for providing sufficient entertainment as a diversion from this work. Help on LaTeX-related problems provided by my colleagues Pradipta Bandyopadhyay and Asish Mondal is gratefully acknowledged.

Springer
http://www.springer.com/978-1-4614-5746-6

A Course on Mathematical Logic
Srivastava, S.M.
2013, XII, 198 p.,
ISBN: 978-1-4614-5746-6