

# Preface

In this book, we fully develop and compare two approaches for the numerical approximation of exact controls for wave propagation phenomena: the continuous one, based on a thorough analysis of the continuous model, and the discrete one, which relies upon the analysis of the discrete models under consideration. We do it in the abstract functional setting of conservative semigroups.

The main results of this book end up unifying, to a large extent, these two approaches yielding similar algorithms and convergence rates. The discrete approach, however, has the added advantage of yielding not only efficient numerical approximations of the continuous controls but also ensuring the partial controllability of the finite-dimensional approximated dynamics, i.e., the fact that a substantial projection of the approximate dynamics is controlled. It also leads to iterative approximation processes that converge without a limiting threshold in the number of iterations. Such a threshold has to be taken into account, necessarily, for methods derived by the continuous approach, and it is hard to compute and estimate in practice. This is a drawback of the methods emanating from the continuous approach that exhibit divergence phenomena when the number of iterations in the algorithms aimed to yield accurate approximations of the control goes beyond this threshold.

We shall also briefly explain how these ideas can be applied for data assimilation problems.

Though our results apply in a wide functional setting, our approach requires a fine analysis in the case of unbounded control operators, e.g., in the case of boundary controls. We will therefore show how this can be done in a simple case, namely the  $1 - d$  wave equation approximated by finite-difference methods. In particular, we present several new results on the rates of convergence for the solution of the wave equation with nonhomogeneous Dirichlet boundary data.

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