
Preface to the Second Edition

In this second edition, we include new material on single-crystal plasticity and on various models of strain-gradient plasticity. The relevant mechanics and variational aspects of these models are presented, and attention is given to selected numerical analyses. We have also taken the opportunity to revise various parts of the first edition. For instance, Section 10.2 has been completely rewritten. The set of references has been updated and expanded, and a number of minor refinements have also been made.

We thank our many friends, colleagues and family members whose interest, guidance, and encouragement made this work possible. One of us (B.D.R.) is grateful to Morton Gurtin for his contributions, whether through collaborative work or the extensive discussions on the topic of gradient plasticity. We are grateful for the support from the Simons Foundation (to W.H.) and from the National Research Foundation through the South African Research Chair in Computational Mechanics (to B.D.R.). We thank Andrew McBride for his comments on drafts of the manuscript. Tim Povall prepared a number of new figures, and together with Andrew McBride also revised the figures in the first edition. This assistance is gratefully acknowledged. It is a pleasure to acknowledge the skillful assistance from the staff at Springer, especially Achi Dosanjh and Donna Chernyk, and members of the Springer TeX support team.

W.H.
Iowa City
August 2012

B.D.R.
Cape Town

Preface to the First Edition

The basis for the modern theory of elastoplasticity was laid in the nineteenth-century, by TRESCA, ST. VENANT, LÉVY, and BAUSCHINGER. Further major advances followed in the early part of this century, the chief contributors during this period being PRANDTL, VON MISES, and REUSS. This early phase in the history of elastoplasticity was characterized by the introduction and development of the concepts of irreversible behavior, yield criteria, hardening and perfect plasticity, and of rate or incremental constitutive equations for the plastic strain.

Greater clarity in the mathematical framework for elastoplasticity theory came with the contributions of PRAGER, DRUCKER, and HILL, during the period just after the Second World War. Convexity of yield surfaces, and all its ramifications, was a central theme in this phase of the development of the theory.

The mathematical community, meanwhile, witnessed a burst of progress in the theory of partial differential equations and variational inequalities from the early 1960s onwards. The timing of this set of developments was particularly fortuitous for plasticity, given the fairly mature state of the subject, and the realization that the natural framework for the study of initial boundary value problems in elastoplasticity was that of variational inequalities. This confluence of subjects emanating from mechanics and mathematics resulted in yet further theoretical developments, the outstanding examples being the articles by MOREAU, and the monographs by DUVAUT AND J.-L. LIONS, and TEMAM. In this manner the stage was set for comprehensive investigations of the well-posedness of problems in elastoplasticity, while the simultaneous rapid growth in interest in numerical methods ensured that equal attention was given to issues such as the development of solution algorithms, and their convergences.

The interaction between elastoplasticity and mathematics has spawned among many engineering scientists an interest in gaining a better understanding of the modern mathematical developments in the subject. In the same way, given the richness of plasticity in interesting and important mathemat-

ical problems, many mathematicians, either students or mature researchers, have developed an interest in understanding the mechanical and engineering basis of the subject, and its connections with the mathematical theory. While there are many textbooks and monographs on plasticity that deal with the mechanics of the subject, they are written mainly for a readership in the engineering sciences; there does not appear to us to have existed an extended account of elastoplasticity which would serve these dual needs of both engineering scientists and mathematicians. It is our hope that this monograph will go some way towards filling that gap.

We present in this work three logically connected aspects of the theory of elastic-plastic solids: the constitutive theory, the variational formulations of the related initial boundary value problems, and the numerical analysis of these problems. These three aspects determine the three parts into which the monograph is divided.

The constitutive theory, which is the subject of Part I, begins with a motivation grounded in physical experience, whereafter the constitutive theory of classical elastoplastic media is developed. This theory is then cast in a convex analytic setting, after some salient results from convex analysis have been reviewed. The term “classical” refers in this work to that theory of elastic-plastic material behavior which is based on the notion of convex yield surfaces, and the normality law. Furthermore, only the small strain, quasi-static theory is treated. Much of what is covered in Part I will be familiar to those working on plasticity, though the greater insights offered by exploiting the tools of convex analysis may be new to some researchers. On the other hand, mathematicians unfamiliar with plasticity theory will find in this first part an introduction that is self-contained and accessible.

Part II of the monograph is concerned with the variational problems in elastoplasticity. Two major problems are identified and treated: the primal problem, of which the displacement and internal variables are the primary unknowns; and the dual problem, of which the main unknowns are the generalized stresses.

Finally, Part III is devoted to a treatment of the approximation of the variational problems presented in the previous part. We focus on finite element approximations in space, and both semi- and fully discrete problems. In addition to deriving error estimates for these approximations, attention is given to the behavior of those solution algorithms that are in common use.

Wherever possible we provide background materials of sufficient depth to make this work as self-contained as possible. Thus, Part I contains a review of topics in continuum mechanics, thermodynamics, linear elasticity, and convex analytic setting of elastoplasticity. In Part II we include a treatment of those topics from functional analysis and function spaces that are relevant to a discussion of the well-posedness of variational problems. And Part III begins with an overview of the mathematics of finite elements.

In writing this work we have drawn heavily on the results of our joint collaboration in the past few years. We have also consulted, and made liberal use

of the works of many: we mention in particular the major contributions of G. DUVAUT AND J.-L. LIONS, C. JOHNSON, J.B. MARTIN, H. MATTHIES, AND J.C. SIMO. While we acknowledge this debt with gratitude, the responsibility for any inaccuracies or erroneous interpretations that might exist in this work, rests with its authors.

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W.H.
Iowa City

B.D.R.
Cape Town

Plasticity

Mathematical Theory and Numerical Analysis

Han, W.; Reddy, B.D.

2013, XVI, 424 p., Hardcover

ISBN: 978-1-4614-5939-2