Preface

This is the first of a two book series written for beginning graduate-level real analysis students and it is focused on essentials of set theory, topology, and measure theory. The word "essentials" is often associated with lecture notes, but this is exactly opposite to my intention. While there are quite a few fine abstract analysis books, they are difficult to read, and an enthusiastic reader needs more time and effort to master the subject, as opposed to elementary level books that only in part address the needs of forthcoming mathematics and advanced engineering courses.

That is why this book offers a thorough and yet rigorous treatment of key analysis subjects in abstract spaces by providing the reader with copious illustrations, examples, and exercises with selected solutions. Some difficult to understand topics are preceded by detailed discussions and blueprints, so that the reader will not get lost or intimidated in a long chain of proofs and notions. Furthermore, at the end of each section there is a summary of new terms and notation in the chronological order in which they appeared in the text. This should be helpful and relieve students of the potential overload of new words, definitions, and concepts. So, there is an increased likelihood of success by using this text in a course or by means of individual self-study.

My previous text, *Real Analysis*, published by Chapman and Hall in 2000, was the first effort to create this kind of book. However, this book only partially accomplished the goal I was striving to achieve. To fully realize that goal, it was necessary to write a new and expanded edition, including more topics and details, and it had to be produced as two books. The companion book, *Advances in Abstract Analysis and Applications*, includes further topics in topology and measure theory, which justifies and rewards the reader for investing the time spent on “essentials.”

As mathematical education has become increasingly more focused on applications and less on theory, and in order to save them from extinction, academics have repurposed courses in set theory, topology, abstract algebra, and measure and integration as a *real analysis* course. At the same time, mathematical research, driven by serious applications to other sciences, continued to require sound foundations. The pertinent precedents include physics, stochastic finance, mechanics, and now biology. There was even a time when some proponents called real analysis “the single most important graduate course in mathematics to prepare for a career in operations research.”

Today, real analysis is still very much alive, although it has undergone some significant modifications. One of these changes is that contemporary real analysis books include various, sometimes exotic, applications ranging from partial differentials equations to wavelet analysis, probability, and even physics. While such connections might be justified, one has to ensure that this propensity to connect analysis with remote disciplines does not relax its very substance. Consequently, facing a challenge of two alternatives to yield an oversized or abridged book, a broad-spectrum project (under strong encouragement from Springer) got bifurcated into two entirely differently focused texts.

Because real analysis, in its proper form, is likely to be the first abstract mathematics course that many students take, the associated topics should be taught in a strict order starting with basic set theory followed by point-set topology and then measure theory and integration. Throughout my book I follow these principles. I strongly advocate the idea of introducing measure and integration in abstract spaces wasting no valuable time on Euclidean spaces. Consequently, Lebesgue measure and Lebesgue integral are reduced to mere illustrations. The topology part, to be necessarily preceded by metric spaces, contains mostly fundamentals (such as bases, subbases, Hausdorff spaces, Tychonov product, and compactness). In particular, old good sequential convergence is enough to proceed with rigorous and comprehensive measure and integration (which accounts to over 55% of my book). Such topics as filters and nets, locally compact Hausdorff spaces, Radon measures, and Hilbert spaces (measure-theoretic version) I consider relatively advanced and therefore treat them in my forthcoming sequel to this book, along with various applications to stochastic analysis.

It is absolutely impossible to produce a sound text without building on the foundations of my predecessors' important scholarship. I am very grateful for the valuable remarks and suggestions, made to the present and earlier edition, by Gustave Choquet, Jerald Folland, Jordan Stoyanov, Jürgen Becker, Richard Syski, Jean Lasserre, Donald Konwinski, and Dean Spitzer. I am much indebted to Simon Smith, the creator of the EXP word processor, for his generous and timely support.

My deep appreciation also goes to Ms. Vaishali Damle, who is an extremely competent and efficient mathematics editor. Her constant help and patience was made available from the very beginning of the project and receiving her feedback was one of the key benefits of writing the book. Finally, I want to thank the Springer copyeditor, Ms. Valerie Greco, who did an excellent job in polishing the final draft of my manuscript. Because books are often written at home, I would be remiss in not ending with profound gratitude to my wife, Irina, who created an environment conducive to writing this book.

J.H. Dshalalow
Melbourne, Florida

http://www.springer.com/978-1-4614-5961-3

Foundations of Abstract Analysis
Dshalalow, E.
2013, XIII, 748 p. 30 illus., 22 illus. in color., Hardcover
ISBN: 978-1-4614-5961-3