

Chapter 2

Physical Principle of Optical Tweezers

The radiation pressure of light was first deduced theoretically by James C. Maxwell in 1873 based on his electromagnetic theory [1, 2], and measured experimentally by Lebedev [3], and Nichols and Hull in 1901 [4]. The radiation pressure force exerted on a totally reflecting mirror by an incident beam of light perpendicular to the mirror is $F_{\text{mirror}} = 2P/c$, where P is the power of the light and c is the speed of light in vacuum [5]. The factor of 2 in the formula is due to reflection. The force is about 7 nN for 1 W of light, which is tiny and had almost no application before the invention of the laser. In contrast to classical light sources, a laser beam can be strongly focused onto a small particle with a diameter on the order of 1 μm . Due to the small mass of the particle, the radiation force of a 1 W laser can be 10^5 times larger than the gravitational force on the particle, and can therefore have huge effects on the motion of the particle.

In 1970, Arthur Ashkin published a seminal paper [6] demonstrating that one could use focused laser beams to accelerate and trap micrometer-sized transparent particles. Optical levitation of oil droplets and glass microspheres in air [7] and vacuum was demonstrated several years later [8]. The laser radiation pressure was soon used to cool and trap atoms [9–12], leading to dramatic breakthroughs in atomic, molecular and optical physics, including a new generation of atomic clocks, and realization of Bose–Einstein condensation and degenerate Fermi gas. In 1986, Ashkin et al. [13] observed stable trapping of dielectric particles with the gradient force of a strongly focused laser beam. This technique was soon used to trap and manipulate viruses and bacteria [14, 15], and became a standard tool in biophysics [5].

In this chapter, we will first explain the principle of optical trapping of microspheres with ray optics, which is valid when the size of the microspheres is much larger than the wavelength of the trapping laser. This will be followed by theoretical calculations of the optical forces on a particle with the Rayleigh approximation, and numerical results of Lorentz-Mie theory. The differences between trapping microspheres in air and in water will be discussed.

2.1 Ray Optics Approximation

When the size of a microsphere is much larger than the wavelength of the trapping laser (usually $R > 10\lambda_0$, where R is the radius of the microsphere and λ_0 is the wavelength of the laser in vacuum), the optical forces on the microsphere can be calculated by ray optics [16].

A qualitative view of optical trapping of microspheres in the ray optics regime is shown in Fig. 2.1 [13, 16]. If we neglect surface reflection from the microsphere, then the microsphere will be trapped at the focus of the laser beam as shown in Fig. 2.1b. If the microsphere moves to the left of the focus (Fig. 2.1a), it will deflect the laser beam to the left and thus increase the momentum of photons to the left. The counter force from the deflected photons will push the microsphere to the right, i.e. back to the focus of the laser beam. If the microsphere moves along the propagation direction of the laser beam (Fig. 2.1c), it will focus the laser more strongly and thus increase the momentum of photons along the propagation direction. The counter force from the deflected photons will push the microsphere back to the focus of the laser beam. The same thing will happen if the microsphere moves away from the focus in other directions. Thus a focused laser beam forms a stable optical trap in 3D.

The above discussion neglected surface reflection from the microsphere. In reality, we have to consider the effect of this surface reflection. The photons reflected back by the surface of a microsphere will push the microsphere forward. If this force is larger than the restoring force due to refraction (Fig. 2.1c), the microsphere will be pushed away from the focus, and thus cannot be trapped. The surface reflection depends on the relative refractive index of the microsphere and the medium $m = n_p/n_{md}$, where n_p is the refractive index of the microsphere and n_{md} is the refractive index of the medium. Larger m implies more surface reflection, and thus greater difficulty in trapping the microsphere with an optical tweezer [17]. m is about 1.10 for a silica microsphere ($n_{silica} = 1.46$) in water ($n_{water} = 1.33$), and is about 1.46 for a silica microsphere in air ($n_{air} = 1.00$) (see Table A.1 for more information). Thus it is more difficult to trap microspheres in air than in water.

To increase the restoring force, the laser beam should be strongly focused by a high numerical aperture (NA) objective lens. The typical NA of objective lenses used for creating optical tweezers is about 1.2 and 0.95 in water [13, 17] and air [18], respectively.

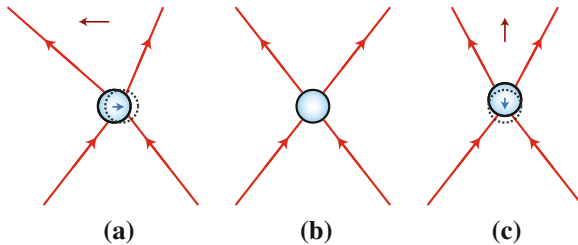


Fig. 2.1 Qualitative view of optical trapping of dielectric spheres. **a** Displays the force on the particle when the particle is displaced laterally from the focus. **b** Shows that there is no net force on the particle when the particle is trapped at the focus. **c** Displays the force on the particle when the particle is positioned above the focus

2.2 Rayleigh Approximation

If the size of a nanosphere (microsphere) is much smaller than the wavelength of the trapping laser (usually $R < \lambda_0/10$), the nanosphere can be approximated as a dipole. In this regime, the optical force on the nanosphere can be calculated analytically with the Rayleigh scattering theory [13, 19]. Here we will calculate the optical forces in the Rayleigh regime following the formulas of Ref. [19].

We consider a nanosphere with radius R and a refractive index n_p being illuminated by a laser beam propagating along the z axis in the positive direction, as shown in Fig. 2.2. The power of the laser beam is P . The refractive index of the medium in which the nanosphere is suspended is n_{md} . The laser beam is a linearly polarized Gaussian beam (TEM_{00}) with beam waist radius ω_0 at the focus. The polarization direction of the electric field of the laser is parallel to the x axis. The center of the laser beam is located at the origin, and the center of the nanosphere is at $\vec{r} = (x, y, z)$.

The wavefront of a Gaussian beam is flat at the focus, and its waist ($1/e^2$ radius) spreads in accordance with [20]:

$$\omega(z) = \omega_0 \left[1 + \left(\frac{\lambda_{md} z}{\pi \omega_0^2} \right)^2 \right]^{1/2}, \quad (2.1)$$

where $\lambda_{md} = \lambda_0/n_{md}$ is the wavelength of the laser in the medium.

The Rayleigh range (z_R), defined as the distance over which the beam radius spreads by a factor of $\sqrt{2}$, is given by

$$z_R = \frac{\pi \omega_0^2}{\lambda_{md}}. \quad (2.2)$$

The intensity distribution of the Gaussian beam is

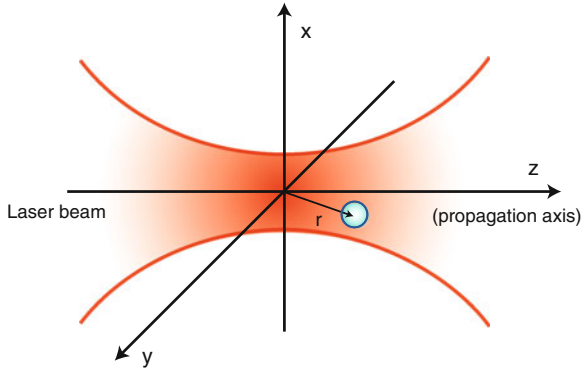


Fig. 2.2 Schematic of a nanosphere near the focus of a laser beam

$$I(x, y) = I_0 e^{-2(x^2+y^2)/\omega^2} = \frac{2P}{\pi\omega^2} e^{-2(x^2+y^2)/\omega^2}, \quad (2.3)$$

where $\omega = \omega(z)$ and P is the power of the laser beam.

The numerical aperture ($1/e^2$ points in k-space) of a Gaussian beam is

$$NA = \frac{\lambda_0}{\pi\omega_0} = n_{md} \frac{\lambda_{md}}{\pi\omega_0}. \quad (2.4)$$

The optical force of the focused laser beam on the nanosphere can be separated into two components: the scattering force $\vec{F}_{scat}(\vec{r})$ which is proportional to the intensity of the laser, and the gradient force $\vec{F}_{grad}(\vec{r})$ which is proportional to the gradient of the intensity of the laser. The scattering force is a nonconservative force and the gradient force is a conservative force. The gradient force forms a trapping potential for the nanosphere, and the scattering force tends to push the nanosphere out of the trap. In order to form a stable trap, the gradient force should be larger than the scattering force.

The scattering force of the laser on a nanosphere is [19]:

$$\vec{F}_{scat}(\vec{r}) = \hat{z} \left(\frac{n_{md}}{c} \right) C_{scat} I(\vec{r}) = \hat{z} \frac{128\pi^5 R^6}{3c\lambda_0^4} \left(\frac{m^2 - 1}{m^2 + 2} \right)^2 n_{md}^5 I(\vec{r}), \quad (2.5)$$

where c is the speed of light in vacuum and C_{scat} is the scattering cross section. Because of the larger relative refractive index m , the scattering force on a nanosphere in air is about 4.2 times greater than the scattering force on the same nanosphere in water with the same laser intensity. The number of scattered photons per second is

$$N_{scat} = \frac{\lambda_{md}}{h} |\vec{F}_{scat}|, \quad (2.6)$$

where h is the Planck constant.

The gradient force on the nanosphere is [19]:

$$\vec{F}_{grad}(\vec{r}) = [\vec{p}(\vec{r}, t) \cdot \nabla] \vec{E}(\vec{r}, t) = \frac{2\pi n_{md} R^3}{c} \left(\frac{m^2 - 1}{m^2 + 2} \right) \nabla I(\vec{r}), \quad (2.7)$$

where $\vec{p}(\vec{r}, t)$ is the induced dipole of the nanosphere due to the instantaneous electric field ($\vec{E}(\vec{r}, t)$) of the laser. The gradient force forms a trapping potential:

$$V(\vec{r}) = -\frac{2\pi n_{md} R^3}{c} \left(\frac{m^2 - 1}{m^2 + 2} \right) I(\vec{r}). \quad (2.8)$$

The total force on the nanosphere is $\vec{F}(\vec{r}) = \vec{F}_{scat}(\vec{r}) + \vec{F}_{grad}(\vec{r})$. The minimum force along the z axis $F_z^{min} = \min(F_z(\vec{r}))$ must be negative in order to form a stable trap. Otherwise the force of the laser will always push the nanosphere forward and

there will be no trap. Because the scattering force is proportional to R^6 while the gradient force is proportional to R^3 , the scattering force decreases much faster than the gradient force when the size of the nanosphere decreases. Thus it is easier to achieve a negative F_z^{min} for a small nanosphere than a large particle.

In order to trap a nanosphere stably, the well depth should be at least 10 times larger than the average kinetic energy of the nanosphere. This is due to the fact that the kinetic energy of a nanosphere follows the Maxwell–Boltzmann distribution at thermal equilibrium. The nanosphere has a significant probability for its instantaneous kinetic energy to be much larger than its average kinetic energy. According to the energy equipartition theorem, the average kinetic energy of a nanosphere is $k_B T/2$ in each direction, where k_B is the Boltzmann constant and T is the temperature of the medium. While the average kinetic energy is independent of the size of the nanosphere, the well depth of the trapping potential decreases as the size of the particle decreases. Thus it is difficult to trap a nanosphere if its size is too small.

Fig. 2.3 shows the calculated potentials and forces on a silica nanosphere in air with a focused laser beam. Since the refractive index of air is very close to the refractive index of vacuum, the potential and force on a nanosphere in air is practically the same as that in vacuum. For the calculations yielding Fig. 2.3, the wavelength of the laser is 1064 nm, the power of the laser is 200 mW, and the waist of the laser at the focus

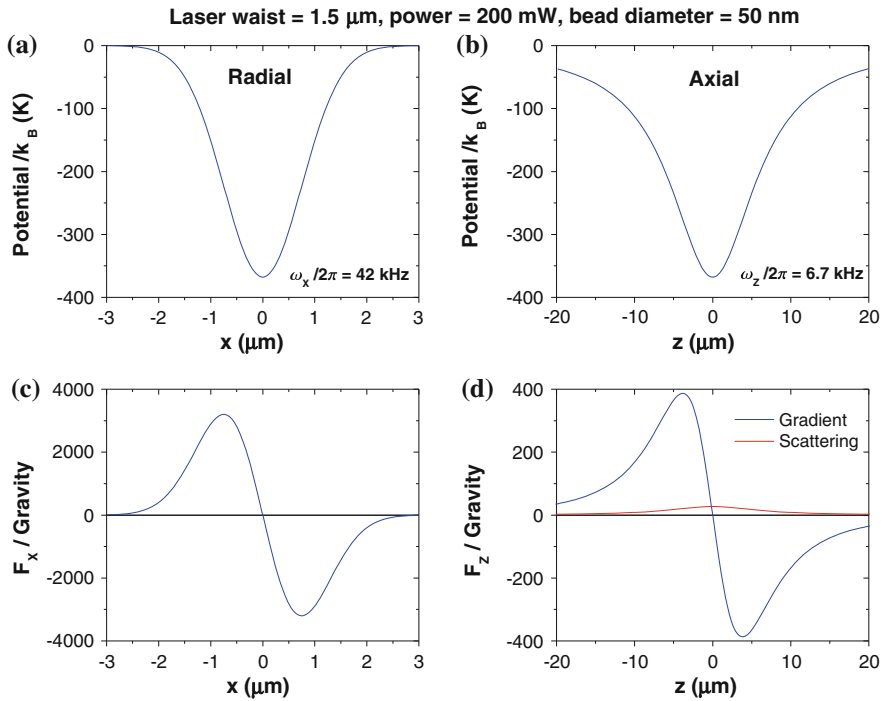


Fig. 2.3 Optical potentials and forces on a nanosphere in air when the waist of the trapping laser is 1.5 μm . The power of the trapping laser is 200 mW and the diameter of the nanosphere is 50 nm

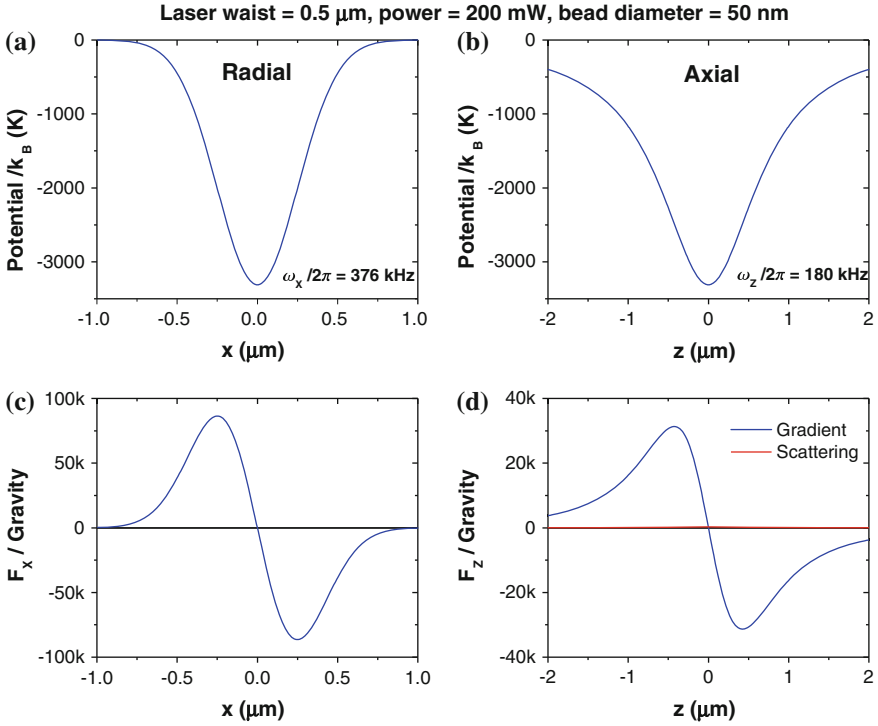


Fig. 2.4 Optical potentials and forces on a nanosphere in air when the waist of the trapping laser is 0.5 μm . The power of the trapping laser is 200 mW and the diameter of the nanosphere is 50 nm

is 1.5 μm , corresponding to $\text{NA} = 0.22$. The Rayleigh range of the laser is 6.6 μm . The diameter ($D = 2R$) of the nanosphere is 50 nm. The calculated well depth of the trap is 367 K. The laser will therefore only be able to trap a 50 nm nanosphere at a temperature much lower than room temperature. The potential is approximately harmonic near the bottom of the trap. The oscillation frequency is about 42 kHz in the radial direction and 6.7 kHz in the axial direction for a 50 nm nanosphere trapped near the bottom of the potential. The scattering force is zero in the radial direction (Fig. 2.3c) and is positive along the axial direction (Fig. 2.3d). The gradient force is negative at positive coordinates, and positive at negative coordinates. Thus it will always pull back the nanosphere to the center of the trap.

Fig. 2.4 shows the calculated potentials and forces on a silica nanosphere in a laser beam with a much smaller waist. The waist of the laser beam is 0.5 μm , which corresponds to $\text{NA} = 0.68$. Other conditions are the same as in Fig. 2.3. Because of the smaller waist, the well depth becomes large enough (3310 K) to trap a 50 nm nanosphere at room temperature. The trapping frequency is about 376 kHz in the radial direction and 180 kHz in the axial direction. The scattering force is negligible compared to the gradient force. Comparing Figs. 2.4 and 2.3, it is clear that a laser

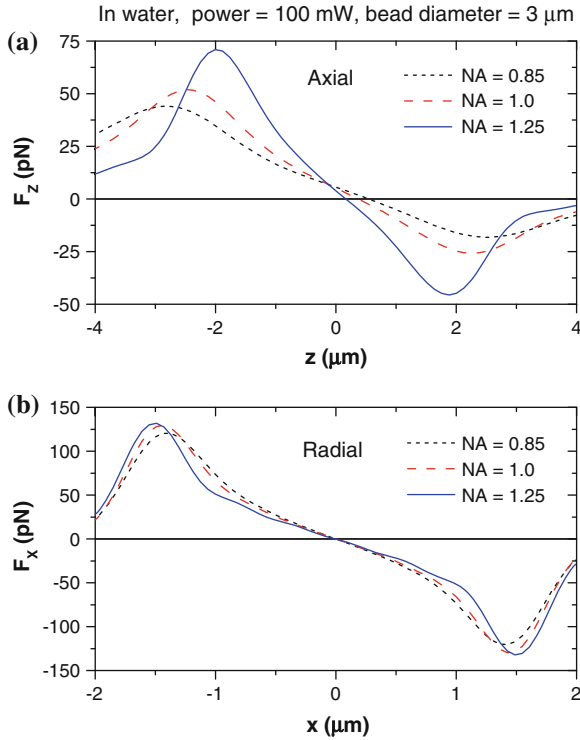


Fig. 2.5 Optical forces on a microsphere in water along axial (a) and radial (b) directions of a laser beam focused by objective lenses with different numerical apertures (NA's). The power of the laser is 100 mW, and the diameter of the microsphere is 3 μm

beam focused by an objective lens with a larger NA is much better for trapping nanospheres.

2.3 Generalized Lorentz-Mie Theory

In most experiments with optical tweezers, the sizes of the dielectric particles are comparable with the wavelength of the trapping laser ($R \sim \lambda_0$). In this case, neither ray optics nor the Rayleigh approximation is appropriate. Instead the electromagnetic theory of light has to be used. For optical trapping of homogeneous and isotropic microspheres, one can use the generalized Lorentz-Mie theory. The mathematical calculation of the generalized Lorentz-Mie theory is quite complex. Here we will only introduce this method briefly, and use the computational toolbox developed by Nieminen et al. [21] to obtain some numerical results of the optical force on a microsphere.

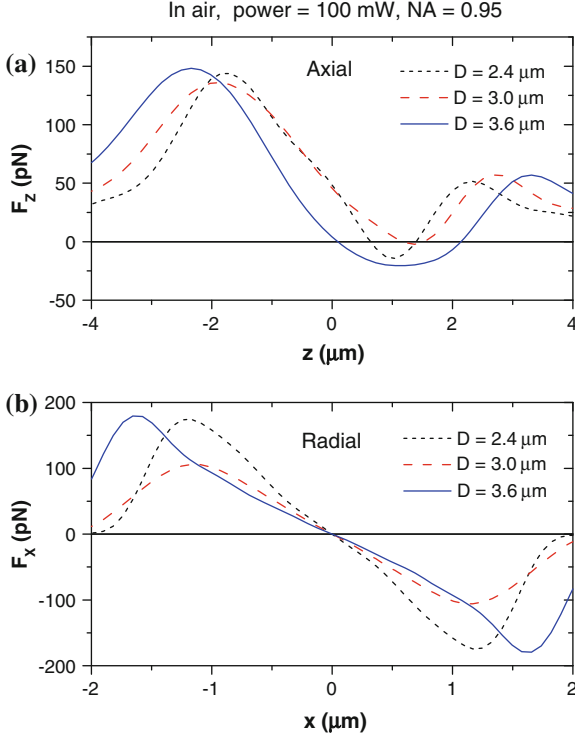


Fig. 2.6 Optical forces on microspheres in air along axial (a) and radial (b) directions of a laser beam focused by an objective lens with NA = 0.95. The power of the laser is 100 mW. The diameter of the microsphere is 2.4 μm for the dotted lines, 3.0 μm for the dashed lines, and 3.6 μm for the solid lines

The optical force on a microsphere comes from the momentum of photons (electromagnetic field) from a laser. It can be obtained by calculating the change of the momentum of the electromagnetic field scattered by the microsphere. A natural choice of coordinate system for calculating the light scattering by a microsphere is spherical coordinates (r, θ, ϕ) centered on the trapped microsphere. The incoming and outgoing fields can be expanded in terms of incoming and outgoing vector spherical wavefunctions [21]:

$$\mathbf{E}_{in} = \sum_{i=1}^{\infty} \sum_{j=-i}^i a_{ij} \mathbf{M}_{ij}^{(2)}(k\mathbf{r}) + b_{ij} \mathbf{N}_{ij}^{(2)}(k\mathbf{r}), \quad (2.9)$$

$$\mathbf{E}_{out} = \sum_{i=1}^{\infty} \sum_{j=-i}^i p_{ij} \mathbf{M}_{ij}^{(1)}(k\mathbf{r}) + q_{ij} \mathbf{N}_{ij}^{(1)}(k\mathbf{r}), \quad (2.10)$$

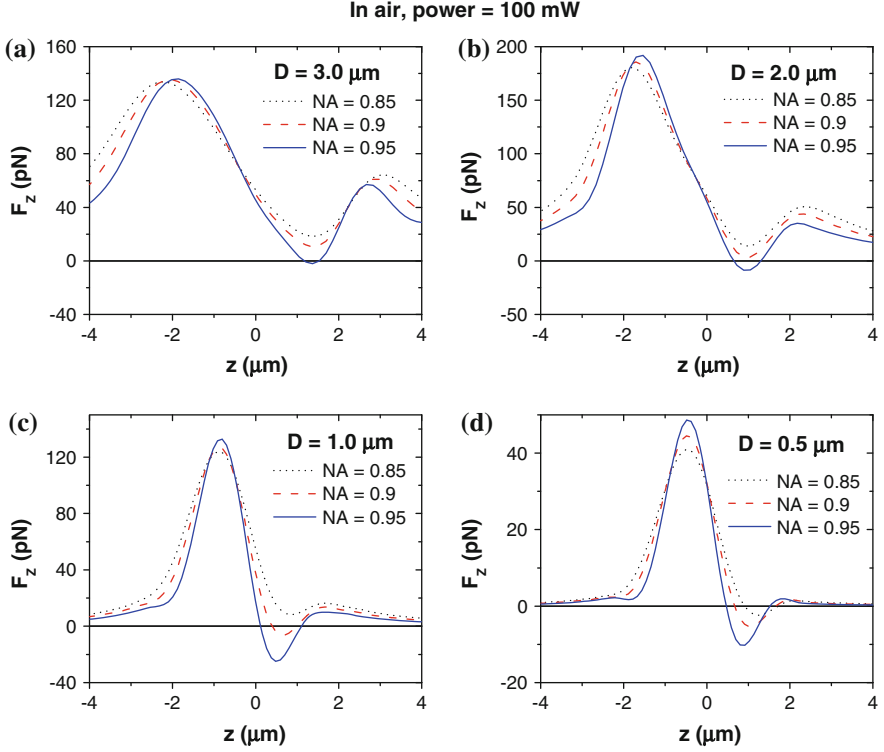


Fig. 2.7 Optical forces on microspheres in air as a function of the diameter of the microspheres and the NA of laser beams

where $\mathbf{M}_{ij}^{(1)}$ and $\mathbf{N}_{ij}^{(1)}$ are outward-propagating TE and TM multipole fields, and $\mathbf{M}_{ij}^{(2)}$ and $\mathbf{N}_{ij}^{(2)}$ are the corresponding inward-propagating multipole fields.

The optical force on the microsphere along the axial direction is [21]:

$$\begin{aligned}
 F_z = & \frac{2n_{md}P}{cS} \sum_{i=1}^{\infty} \sum_{j=-i}^i \frac{j}{i(i+1)} \text{Re}(a_{ij}^* b_{ij} - p_{ij}^* q_{ij}) \\
 & - \frac{1}{i+1} \left[\frac{i(i+2)(i-j+1)(i+j+1)}{(2i+1)(2i+3)} \right]^{1/2} \\
 & \times \text{Re}(a_{ij} a_{i+1,j}^* + b_{ij} b_{i+1,j}^* - p_{ij} p_{i+1,j}^* - q_{ij} q_{i+1,j}^*), \quad (2.11)
 \end{aligned}$$

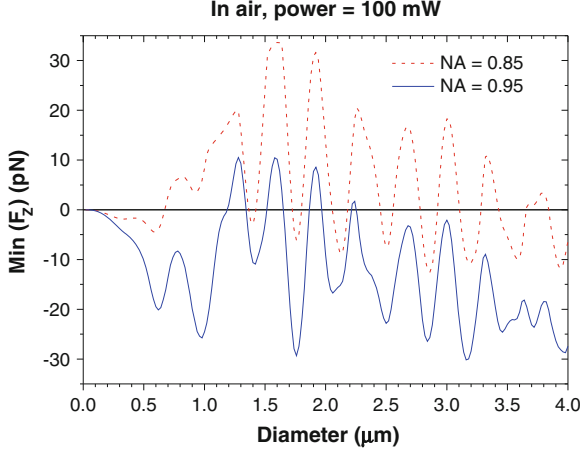


Fig. 2.8 Minimum optical forces on a microsphere in air along the axial direction as a function of the diameter of the microspheres. The power of the laser is 100 mW

where

$$S = \sum_{i=1}^{\infty} \sum_{j=-i}^i (|a_{ij}|^2 + |b_{ij}|^2). \quad (2.12)$$

Figure 2.5 shows the calculated optical forces on a microsphere in water from a laser beam focused by objective lenses with three different NA's. The wavelength of the laser is 1064 nm, the power of the laser is 100 mW, and the diameter of the microsphere is 3 μm . The optical forces along the radial direction are similar for all three NA's (NA = 0.85, 1.0, 1.25) as shown in Fig. 2.5b. On the other hand, the optical forces along the axial direction are very different for different NA's. This is because the scattering force is only along the axial direction. The microspheres will be trapped at positions where the total optical force changes its sign. The scattering force affects the trapping position (Fig. 2.5a).

Figure 2.6 shows the calculated optical forces on microspheres in air exerted by a laser beam focused by an objective lens with NA = 0.95. The maximum value of the NA for an objective lens in air is 1.0, while it is 1.33 for an objective lens in water. Because of a larger relative refractive index in air than in water, the scattering force on a microsphere in air is much larger than that in water. This makes Fig. 2.6 appear very different from Fig. 2.5. The optical forces along the axial direction are asymmetric, because the scattering forces are in the forward direction.

Figure 2.7 shows more calculation results of optical forces on microspheres in air along the axial direction. For a $D = 3.0 \mu\text{m}$ microsphere (Fig. 2.7a), the minimum axial force (F_z^{\min}) is positive when NA = 0.85 or NA = 0.9, and is only slightly negative when NA = 0.95. Thus a laser beam focused by an objective lens with NA less than 0.95 can not trap a 3 μm silica microsphere. The situation becomes

better for smaller microspheres. The minimum axial force is negative for a $0.5\text{ }\mu\text{m}$ microsphere in a laser beam focused by objective lenses with all three different NA's (Fig. 2.7d).

The minimum axial forces on a microsphere in air as a function of the diameter of the microsphere are shown in Fig. 2.8. The minimum forces oscillate as the diameter of the microsphere changes. This is because of the interference between the scattered light and un-scattered light. The oscillation period is about half the wavelength of the laser inside of the microsphere, which is $\lambda/(2n_p) = 364\text{ nm}$. A microsphere can not be trapped if the minimum force is positive. For $\text{NA} = 0.85$, only microspheres with certain diameters can be trapped. This serves as a selection process and can be used for sorting microspheres. The size distribution of the trapped microspheres will be different from the size distribution of the microspheres before trapping. For example, if the microspheres prior trapping have a large diameter distribution ranging from 0.7 to $1.7\text{ }\mu\text{m}$, the diameter of microspheres trapped by a $\text{NA} = 0.85$ laser will always be about $1.4\text{ }\mu\text{m}$.

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