

Preface

Asymptotic geometric analysis is concerned with geometric and linear properties of finite-dimensional objects, normed spaces, and convex bodies, especially with asymptotics of their various quantitative parameters as the dimension tends to infinity. Deep geometric, probabilistic, and combinatorial methods developed here are used outside the field in many areas, related to the subject of the program.

The main tools of the theory belong to the concentration phenomenon and large deviation inequalities. Concentration of measure is, in fact, an isomorphic form of isoperimetric problems, which was first developed inside asymptotic geometric analysis and then became pertinent to many branches of mathematics as an efficient tool and useful concept. Some new techniques of the theory are connected with measure transportation methods and with related PDEs. The concentration phenomenon is well known to be closely linked with combinatorics (Ramsey theory), and such links have been recently better understood in the setting of infinite-dimensional transformation groups, by means of the so-called fixed point on compacta property: on the one hand, every classical Ramsey-type result is equivalent to the fixed point on compacta property of the group of automorphisms of a suitable structure and on the other hand, the fixed point on compacta property is often established by using concentration of measure in subgroups.

The last few years also witnessed the development of small ball probability estimates and their applications, especially for quantitative results on random matrices. A deep understanding of classical convexity and its analytic methods is necessary to advance new types of “isomorphic” results. It is now difficult to draw a borderline between asymptotic and classical convexity theories; and results of each are used in the other and also have numerous applications. Among the recent important ones are results of a geometric-probabilistic flavor on the volume distribution in convex bodies and central limit theorems for convex bodies and others, with close links with geometric inequalities and optimal transport.

More recently, similar results have been established for a larger category of log-concave distributions on Euclidean space, replacing uniform distributions on convex bodies. This is a remarkable extension of the whole theory which could be called “Geometrization of Probability” because it extends to the class of (log-concave)

probability measures many typical geometric notions and results. For example, the notion of polarity, the Blaschke–Santaló inequality and its inverse (by Bourgain–Milman), the Brunn–Minkowski inequality, the Urysohn inequality, and many others are formulated and proved now on this larger category.

The achievements of asymptotic geometric analysis demonstrate new and unexpected phenomena characteristic for high dimensions. These phenomena appear in a number of domains of mathematics and adjacent domains of science dealing with functions of a large number of variables. Besides the main subject of our program, asymptotic theory of normed spaces and convex bodies, it includes the branch of discrete mathematics known as asymptotic combinatorics, including problems of complexity and graph theory; a considerable part of probability dealing with large numbers of correlated random components, including large deviation and the theory of random matrices; and many others. The theory of computational complexity studies the inherent computational difficulty of various computational problems, mostly originated in combinatorial optimization. Complexity theory is, actually, a purely asymptotic field, as is the notion of complexity classes; the most basic notion here is formulated and perceived as an asymptotic notion, where the growing parameter is the size of the computational problem under investigation. The famous “P versus NP” problem asks in fact to compare two asymptotic complexity classes. In recent years several important breakthroughs in complexity theory were obtained by applying tools from asymptotic geometric analysis such as the concentration of measure phenomenon, spectral theory, and discrete harmonic analysis.

The thematic program on asymptotic geometric analysis took place at the Fields Institute for Research in Mathematical Sciences in July–December 2010. The main directions of research covered by the program included:

- Asymptotic theory of convexity and normed spaces
- Concentration of measure and isoperimetric inequalities with an optimal transportation approach
- Applications of the concept of concentration
- Connections with transformation groups and Ramsey theory
- Geometrization of probability
- Random matrices
- Connection with asymptotic combinatorics and complexity theory

Avi Wigderson (Institute for Advanced Study) delivered the Distinguished Lecture Series “Randomness, pseudorandomness and derandomization” and Shiri Artstein-Avidan (Tel-Aviv University) delivered the Coxeter Lecture Series “Abstract duality, the Legendre transform and a new duality transform,” “Order isomorphisms and the fundamental theorem of affine geometry,” and “Multiplicative transforms and characterization of the Fourier transform.”

Three workshops were held during the program:

1. Asymptotic Geometric Analysis and Convexity (September 13–17, 2010), organized by Monika Ludwig, Vitali Milman, and Nicole Tomczak-Jaegermann, preceded by a concentration period on convexity (September 8–10,

2010) and followed by a concentration period on asymptotic geometric analysis (September 20–22, 2010)

2. Concentration Phenomenon, Transformation Groups and Ramsey Theory (October 12–16, 2010), organized by Eli Glasner, Vladimir Pestov, and Stevo Todorćević
3. Geometric Probability and Optimal Transportation (November 1–5, 2010), organized by Bo'az Klartag and Robert McCann, preceded by a concentration period on partial differential equations and geometric analysis (October 25–29) and followed by a concentration period on nonlinear dynamics and applications (November 8–10)

To give an idea of program's scale, there were 426 participants, including 85 graduate students and 17 postdocs and 49 long-term participants (those who came for one month or more). The total number of participant days in the program was an impressive 5162.

The program organizers and editors of this volume are grateful to the Fields Institute for an excellent working environment and generous financial support. They also thank the US National Science Foundation for its support.

This volume contains a selection of papers by the participants of the thematic program, which reflects some of the main directions of the scientific activities.

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